Application of the FFT for computer aided determination of the frequency response of a discrete time system

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$$(u_k) \longrightarrow \boxed{H(z)} \longrightarrow (y_k)$$

DISCRETE TIME SYSTEM

INPUT SEQUENCE (u_k)

OUTPUT SEQUENCE (y_k)

$$u(z) = \mathcal{Z}\{(u_k)\} = \sum_{k=0}^{\infty} u_k z^{-k}$$

$$y(z) = \mathcal{Z}\{(y_k)\} = \sum_{k=0}^{\infty} y_k z^{-k}$$

$$H(z) = \frac{y(z)}{u(z)}$$

DISCRETE TIME TRANSFER FUNCTION

$$(\delta_k) = (1, 0, 0, \ldots) \quad \circ\!\!-\!\!\bullet \quad 1$$

$$(\sigma_k) = (1, 1, 1, \ldots) \quad \circ\!\!-\!\!\bullet \quad \frac{z}{z-1}$$

$$(u_k) = (\delta_k) \longrightarrow u(z) = 1$$

$$(y_k) = (h_k)$$

PULSE RESPONSE

$$(\delta_k) \longrightarrow \boxed{H(z)} \longrightarrow (h_k)$$

$$H(z) = \sum^{\infty} h_k z^{-k}$$

$k=0$

FREQUENCY RESPONSE

$z = e^{j\omega T}$ UNIT CIRCLE

$$H(z)\Big|_{z=e^{j\omega T}} = H\left(e^{j\omega T}\right) = \sum_{k=0}^{\infty} h_k \, e^{-j\omega T k}$$

- FREQUENCY DISCRETIZATION

$\omega \to n\omega_0 \qquad n = 0, 1, 2, \ldots$

- FINITE NUMBER OF ELEMENTS

$$\hat{H}_n = \sum_{k=0}^{N-1} h_k \, e^{-j\omega_0 T k n} \qquad \text{APPROXIMATION}$$

SET $\omega_0 = \dfrac{2\pi}{NT}$

$$\rightarrow \quad \hat{H}_n = \sum_{k=0}^{N-1} h_k \, e^{-j\frac{2\pi}{N} n k}$$

COMPARE TO DFT

$$X_n := \sum_{k=0}^{N-1} x_k \, \underbrace{e^{-j\frac{2\pi}{N} n k}}_{W}$$

$\Rightarrow$ PULSE RESPONSE $\rightarrow$ SEQUENCE WITH N-ELEMENS USE DFT

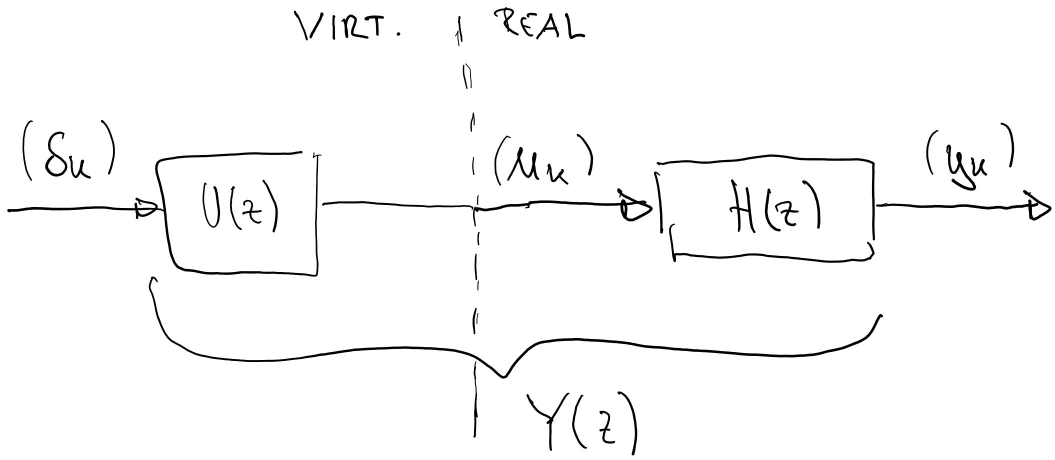
RESULT : APPROXIMATION OF FREQUENCY RESPONSE $\hat{H}_n = \hat{H}(n\omega_0)$ OF THE DISCRETE TIME SYSTEM FOR THE SAMPLING POINTS $n\omega_0$

$n = 0, 1, 2 \ldots \frac{N}{2}$, $\omega_0 = \frac{2\pi}{NT}$

PRACTICAL USEAGE

- DIRAC FUNCTION – TOO LESS ENERGY FOR REAL SYSTEMS

- EXTENSION OF THE PROBLEM



!! INPUT (δ_k) !!

$$Y(z) = \mathcal{Z}\{(y_k)\} = \sum_{k=0}^{\infty} y_k z^{-k}$$

$$U(z) = \mathcal{Z}\{(u_k)\} = \sum_{k=0}^{\infty} u_k z^{-k}$$

$$(y_k) \xrightarrow[\text{FFT}]{\text{N-POINT}} \hat{Y}_n$$

$$(\ldots) \quad \text{N-POINT} \quad \hat{\ }$$

$$(u_k) \xrightarrow[\text{FFT}]{\text{N-POINT}} \hat{U}_n$$

$$Y(z) = U(z)\, H(z) \rightarrow H(z) = \frac{Y(z)}{U(z)}$$

$$H\left(e^{j\omega T}\right) = \frac{Y\left(e^{j\omega T}\right)}{U\left(e^{j\omega T}\right)}$$

$$\boxed{\hat{H}_n = \frac{\hat{Y}_n}{\hat{U}_n}}$$

DIVISION OF COMPLEX NUMBERS

$\Rightarrow$ APPROXIMATION FOR $H\left(e^{j\omega T}\right)$

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