

Template for Term Papers, Bachelor's Theses, and Master's Theses

Term Paper

for the Seminar

“Theoretical and Empirical Microeconomics and Macroeconomics
with Implications for Social Policy All Around the World”

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1 Structure of the Thesis/Term Paper

1.1 Parts of the Manuscript

A thesis generally consists of the following parts:

- (1) a title page,
- (2) a table of contents,
- (3) optionally, a list of abbreviations, tables, and/or figures (these are generally unnecessary for theses and more relevant for books),
- (4) the main text (body),
- (5) a bibliography,
- (6) optionally, an appendix, and
- (7) a written declaration of authorship.

It may make sense to place the bibliography after potential appendices. Please ask your supervisor for their preference. The parts before the main text are called “front matter.”

1.2 Structure of the Main Text

For the main text of theses, two basic outlines are very common:

- Three-part structure: This structure divides an academic paper (including theses) into the three parts
 - (1) introduction,
 - (2) main text, and
 - (3) discussion/concluding remarks.
- Four-part structure/“IMRaD”¹: This structure divides an academic paper (including theses) into the four parts
 - (1) introduction,
 - (2) methods,
 - (3) results, and
 - (4) discussion (conclusion).

1. For detailed information, see <https://en.wikipedia.org/wiki/IMRAD> and <https://skillsearthsciences.sites.uu.nl/writing/structure/imrad/>.

The IMRaD structure is very common in the natural sciences and life sciences. It is also generally suited for manuscripts in economics. Economists, however, usually do not call the methods section “Methods” but give it a different name or even split it up into a sequence of sections, such as “Theoretical Model”/“Theoretical Predictions,” followed by “Empirical Strategy” and “Data”/“Dataset” (or the other way round) or by “Experimental Design” in the case that the paper features an experiment. Similarly, economists frequently split the results section into a sequence of sections, such as “Treatment Effect,” followed by “Analysis of Heterogeneity,” “Evidence on Mechanisms” and/or “Robustness Checks.” And some theoretical papers have the structure “Model,” “Results,” “Applications.” None, however, do away with the “Introduction” and “Discussion”/“Conclusion” section. It is also not uncommon—but by no means the standard—in economics to have a dedicated “Related Literature” section after the introduction.

Other structures are possible, of course. Please consult with your advisor to find a good structure for your manuscript.

1.3 Structuring Introduction and Discussion: Funnel and Reverse Funnel

A general structure that can be found in many research papers is the so-called X-shaped or “hourglass” structure.² This structure is brought about when the so-called “funnel” technique is used to write the introduction,³ and when a “reverse funnel” or “upside-down funnel” is used to write the discussion/concluding section:⁴

- (1) The paper starts with a *broad* perspective at the beginning of the introduction, which becomes *narrower* with every sentence, leading to the paper’s narrow research question.
- (2) The *narrowest* part is when the exact research question is posed, the research methodology is described in detail, and the results are derived/presented.
- (3) When the results are discussed and related to the literature in the concluding section, the perspective becomes *broad* again.

2. See https://skillsearthsciences.sites.uu.nl/writing/structure/#attachment_2387, Fig. 1, and <https://pmc.ncbi.nlm.nih.gov/articles/PMC5405644/pdf/JHRS-10-3.pdf>, Figure 1.

3. See <https://read.aupress.ca/read/read-think-write/section/9e945e9b-62cf-4dbb-b8cd-823549ede292#fig1401>, Figure 14.1, and <https://kib.ki.se/en/write-cite/academic-writing/structure-academic-texts>

4. See <https://read.aupress.ca/read/read-think-write/section/9e945e9b-62cf-4dbb-b8cd-823549ede292#fig1402>, Figure 14.2, and <https://kib.ki.se/en/write-cite/academic-writing/writing-conclusion>.

Let us take a look at the “Introduction” section from Sigfridsson and Ryde (1998) to see the funneling approach at work:⁵

Molecular simulation methods have become an important technique in many areas of chemistry through the recent advent of effective and wide-spread software for molecular mechanics, molecular dynamics, and Monte Carlo simulations, and procedures for the estimation of free energy differences. In all such methods, a proper description of the electrostatic interactions within the simulated system is of key importance, ... However, in most implementations, ..., the simplest possible solution is used, ... Thus, most classical simulation methods need a point-charge parameterisation of the molecules of interest.

Unfortunately, atomic charges are not observables, i.e. they cannot unambiguously be determined by experiments or quantum chemical calculations. Therefore, a large number of methods have been suggested for the estimation of point charges [1]. Several groups have tried to derive the charges directly from experimental quantities, ... Yet, relevant data is usually missing or too scarce to allow a determination of all charges in interesting molecules. Instead, most techniques derive atomic charges from quantum chemical calculations.

The simplest way to determine quantum chemical charges is the Mulliken population analysis. ... Although, Mulliken charges are known to strongly depend on the basis set [6] and to reproduce electrostatic moments poorly [1], they are still widely used due to their simplicity. Several other ... methods have been devised to cure the problems of the Mulliken charges [1], ...

...

In this paper, we make a critical analysis of the four most popular potential-based point-charge methods, ... It is shown that these methods may give widely different results and possible explanations for this are discussed. Two alternative methods for deriving atomic charges are suggested, which avoid the arbitrariness in the selection of potential points, and their performance is judged using ...

Paragraph 1

Broad introduction with general description of the research field (“many areas of chemistry”), research topic (“molecular simulation methods”), and recent development (“through the recent advent of ...”). Followed by a more detailed description of crucial aspects in this context (“In all such methods, a proper description ... is of key importance”) and of shortcomings of existing methods.

Paragraph 2

Narrowing down the topic: Description of the state of the art and of issues and limitations observed in the literature.

Paragraph 3

Further description of the state of the art: strengths and weaknesses of existing approaches.

Paragraphs 4–6

Further description of unresolved issues.

Paragraph 7

Narrow description of the specific contribution of this paper: Identification of undesirable arbitrary variation in simulation results of existing methods, which inspired development of an “alternative methods” that avoids this issue.

5. See https://lucris.lub.lu.se/ws/portalfiles/portal/135489685/25_charges.pdf.

Let us now inspect the “Concluding Remarks” subsection from Sigfridsson and Ryde (1998) to see the “reverse funneling” approach at work:

Potential-based charges strongly depend on the way the potential points are selected. We have presented and tested a new method that avoids such a dependence, CHELMO. It fits the charges directly to the electrostatic moments and it performs as well, or better, as the best electrostatic potential method judging from the calculated electrostatic potential and moments. ... The major disadvantage of the method is that it cannot be used for large molecules, since there is a limited number of moments. No more than 25 independent charges in a molecule can be determined if all moments up to hexadecapole moments are used. In practice it should not be used for molecules with more than about 20 atoms since otherwise the higher moments may be poorly reproduced ... This could be remedied if moments higher than hexadecapole moments are calculated but such moments are not included in the output of normal quantum chemical packages.

In order to get a method that can be used for larger system, we constructed the CHELP-BOW method, which estimates the charges from a Boltzmann-weighted fit to the electrostatic potential. It has the advantage over the other ... methods that ... The present implementation of the CHELP-BOW method is very simplified, but in a future publication we will refine and thoroughly test the method. However, the results are conclusive enough to show that the method ... has the desired behaviour. ... Clearly, this is the method we recommend for general use.

An important advantage of the methods developed in this paper is that they are general, i.e. they can be used with any quantum chemical program and they can use experimental data (...) as well. The only thing that has to be changed is the input section of the programs. Thus any quantum chemical basis sets can be used, and any level of theory that gives a wave function may be employed. Furthermore, the methods can easily be adapted to ...

Finally, a comment on the traditional electrostatic potential methods. ... Thus, we cannot see any justification to use CHELP charges except for backward comparisons.

Paragraph 1

Narrow recap of the first contribution of the paper: brief description of the issue and how it was solved by the new method. Followed by description of the limitations of the new method. Outlook how these limitations could be overcome.

Paragraph 2

Narrow recap of the central contribution of the paper and how it relates to the literature: how existing methods were further improved upon by the second method proposed in this paper. Followed by outlook on future advances and recommendation for researchers in the field.

Paragraph 3

Broader implications due to the generality of the new method.

Paragraph 4

Other broader implications.

1.4 Some Questions That May Help You Guide Writing Your Thesis

You may find the following list of questions helpful as a guide to making sure that your manuscript covers all important aspects. The questions are primarily intended for doctoral and advanced researchers, but they may also be helpful to students writing term papers and theses.

Your introduction might answer the following questions (not necessarily in this order, but this order will give rise to a “funnel” structure):

- (1) What do we already know?
- (2) What do we not know yet (the “knowledge gap”)?
- (3) Why is it interesting/relevant to close the gap?
- (4) *What is the research question? [R]*
- (5) Why is it interesting to answer the research question?
- (6) How do we contribute to closing the knowledge gap: How do we answer the research question? (Which method do we use?)
- (7) Why is the chosen method (more) suitable (than alternative methods) for answering the research question?

And your conclusion/discussion might answer the following questions (again, not necessarily in this order, but this order will give rise to a “reverse funnel” structure):

- (8) What was the knowledge gap before the current study? [Similar to the introduction.]
- (9) *How did we contribute to closing that gap, and what did we find? [A]* (Which method/dataset did we use, and what are the results?—Recap and take-home message.)
- (10) How do our results relate to (confirm/contradict/complement/extend) the results from previous and other contemporaneous studies?
- (11) What part of the gap is still open? (Phrased a bit more negatively: What are the limitations of our approach?)
- (12) Next steps/avenues for future research: How could we go about closing the remaining knowledge gap (by removing the limitations of our current approach)?⁶

In her book *The Little Book of Research Writing*,⁷ Varanya Chaubey proposes a simpler list of only three items—the “RAP method”:

6. John Cochrane might do away with question 12. See his “Writing Tips for Ph.D. Students,” https://www.fma.org/assets/docs/membercontent/writing_cochrane.pdf.

7. For a quick summary of the book, see https://mauve-porcupine-8992.squarespace.com/s/Chaubey_Research_Writing-bxddd.pdf. See also <https://www.econscribe.org/about> and <https://arxiv.org/pdf/2012.07787>.

- The “R” stands for “research question.”
- The “A” stands for “answer” (to the research question, of course).
- The “P” stands for “positioning statement.” (How does our manuscript relate to the literature: Does it corroborate or extend or contradict others’ findings?)

Relating the “RAP” approach to the the list of questions above, question 4 is the “R,” the answer to question 9 is the “A,” and the remaining questions spell out the “P.”

1.5 Structuring Paragraphs in Academic Manuscripts

Obviously, there is no single “right” way of constructing paragraphs. However, a few basic rules can help you write paragraphs that are *easy to comprehend* by your readers. A common approach is to compose sentences such that they reflect the following functional sequence:⁸

- (1) *topic sentence (TS)*—which, if necessary, connects to the previous paragraph;
- (2) *supporting sentences (SS)*—which elaborate on the topic sentence; and
- (3) *concluding sentence or connecting sentence (CS)*—which refers back to the topic sentence and/or provides a link to the subsequent paragraph.

There are various ways in which the supporting sentences can be filled with content. The University of Sheffield suggests⁹ that the supporting sentences could consist of

- (1) “explanation or definitions (optional),”
- (2) “evidence and examples,” and
- (3) “comment” (an interpretation and evaluation of the evidence by you).

The University of Hull and Newcastle University (UK) consider the mnemonic “PEEL” helpful, although they define its meaning in slightly different ways: “Point, Evidence, Explanation, Link”¹⁰ versus “Point, Evidence, Evaluation, Link”¹¹. One may view “Explanation”—“why the point is important and how it helps with your overall argument”—as narrower than “Evaluation”—which is a general critical reflection on the informativeness of the evidence and can include an explanation of its importance. “PEEL” relates to TS/SS/CS as follows:

8. See <https://libguides.newcastle.edu.au/writing-paragraphs/structure>, https://www.une.edu.au/library/students/academic-writing/write-paragraphs/paragraphs/Academic-paragraphs_v2.pdf, <https://learningessentials.auckland.ac.nz/writing-effectively/paragraph/>

9. See <https://sheffield.ac.uk/study-skills/writing/academic/paragraph>.

10. See <https://libguides.hull.ac.uk/writing/paras>.

11. See <https://www.ncl.ac.uk/academic-skills-kit/writing/academic-writing/paragraphing/>

- (1) “P,” the Point, corresponds to the topic sentence.
- (2) “E,” Evidence, is the first part of the supporting sentences.
- (3) “E,” Evaluation/Explanation, forms the second part of the supporting sentences.
- (4) “L,” Link, refers to the connecting sentence that links to the next paragraph.

The Wilfrid Laurier University (Canada) suggests a different mnemonic: “MEAL.”¹²

- (1) “M: Main Point Sentence” corresponds to the topic sentence.
- (2) “E: Evidence” is the first part of the supporting sentences.
- (3) “A: Analysis/Synthesis” forms the second part of the supporting sentences.
- (4) “L: Linking-Back Sentence” refers to the concluding sentence that summarizes the paragraph and links back to the topic sentence or even the topic of the entire paper.

The first three components are largely identical according to these approaches. A difference exists in the meaning of “L”: While “PEEL” stresses the link to the *next* step in the chain of thought, “MEAL” stresses underscoring the message of the *current* paragraph. That is, the latter advocates *closure*: summarizing the paragraph and linking *back* to the topic sentence.

Yet slightly different is the approach put forth by Academic Writing UK:¹³

- (1) Topic sentence—key topic in this paragraph.
- (2) Development—the main idea/topic discussed in more detail.
- (3) Example—support/evidence/data/statistics that show your development is valid/credible.
- (4) Summary—overall main point summarized/evaluated.

It is perfectly fine to mix the different approaches. For some paragraphs, a closer explanation of the topic sentence (“development”) may be necessary, while it is unnecessary for others. And in some paragraphs, you may want underscore the central message by reiterating it in the final sentence (concluding sentence), while for other paragraphs, you will use the final sentence to set the stage for the next step in your chain of reasoning (connecting sentence).

One thing that *all* these suggestions have in common is, however, that they start with a *topic sentence* and end with a *concluding/connecting sentence*. Hence, no matter how you fill the supporting sentences, it is probably best to start a paragraph with a topic sentence and end with a concluding/connecting sentence.

12. See <https://students.wlu.ca/academics/support-and-advising/student-success/assets/resources/writing/how-to-structure-an-academic-paragraph.html>.

13. See <https://academic-englishuk.com/paragraphing/>.

Let us illustrate the “MEAL”/“PEEL” structure, with an optional explanation/definition included, through some example sentences from Sigfridsson and Ryde (1998):

... Thus, most classical simulation methods need a point-charge parameterisation of the molecules of interest.	L: Concluding sentence
Unfortunately, atomic charges are not observables, i.e. they cannot unambiguously be determined by experiments or quantum chemical calculations. Therefore, a large number of methods have been suggested for the estimation of point charges [1]. Several groups have tried to derive the charges directly from experimental quantities, e.g. dipole moments, electrostatic potentials, or free energy differences [2–4]. Yet, relevant data is usually missing or too scarce to allow a determination of all charges in interesting molecules. Instead, most techniques derive atomic charges from quantum chemical calculations.	M/P: Topic sentence (reference to previous paragraph) Optional explanation/definition Supporting sentences: E: Evidence A/E: Analysis/Evaluation
The simplest way to determine quantum chemical charges is the Mulliken population analysis. ...	L: Connecting sentence (link to next paragraph) M/P: Topic sentence (reference to previous paragraph)

The rule to finish a paragraph with a concluding sentence entails, in particular, that a theoretical or empirical finding should never be the last thing that you mention in a paragraph. An “evidence” sentence should always be followed by an interpretation in which you tell the reader what we learn from that finding. Here is an example from Sigfridsson and Ryde (1998):

... With these radii, the charges almost coincide with those obtained by the CHELPG method (within 0.02 e). Thus, we can conclude that the main difference between the Merz–Kollman and the CHELPG method is the 1.4 times larger effective radii of the former method, whereas the sampling schemes are almost equivalent.	Evidence: Supporting sentence with empirical finding Concluding sentence with interpretation of that finding
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To reiterate, there is no single “right” way of constructing paragraphs. However, keeping the TS/SS/CS structure in mind as a guideline is bound to help you produce a text that enables readers to follow your chain of reasoning.

1.6 Additional Online Resources on Effective Writing in Economics

Additional useful recommendations and tips can be found in Plamen Nikolov's "Writing Tips for Crafting Effective Economics Research Papers – 2023-2024 Edition" (<https://docs.iza.org/dp16276.pdf>) and in John H. Cochrane's "Writing Tips for Ph.D. Students" (https://www.fma.org/assets/docs/membercontent/writing_cochrane.pdf).

2 Layout and Design of the Thesis

2.1 Introductory Remark

The following recommendations are based on the official guidelines¹⁴ of the examination office of the Department of Economics at the University of Bonn.

2.2 Typeface and Font Sizes

This template uses the Times typeface by default for the body text. Alternatively, you can choose a typeface of equal size. Equal size here means that the same amount of text (or less) fits on a page with the layout—the page size, margins, and line spacing—defined below.

It is probably a good choice to use a serif font. This is because papers and theses in economics often feature mathematical formulas and equations. With serif fonts, characters can usually be disambiguated relatively well, and often better than with sans-serif fonts (compare: AI/AI, Al/Al). Here are some examples of glyphs that are easy to confuse in particular fonts:

Digit one, lowercase l, uppercase I, vertical bar:

Times: 111| Charter: 111| Avant Garde: 111| Helvetica: 111| Fira Sans: 111|

Lowercase o, digit zero, uppercase O:

Times: o0O Cabin: oOO Alegria Sans: o o O Fira Mono: o00 Cascadia Code: o00

Italic Latin a, p, u, v, y, Y vs. italic Greek alpha, rho, upsilon, nu, gamma, Upsilon:

Times: *a α, p ρ, u υ, v ν, y γ, Y Υ* Fira Sans: *a α, p ρ, u υ, v ν, y γ, Y Υ*

14. See <https://www.econ.uni-bonn.de/examinations/de/informationen/bachelor/bachelorarbeit/dokumente/ba-merkblatt-2016-05-23.pdf> and <https://www.econ.uni-bonn.de/examinations/en/information/master-economics/master-thesis/documents/ma-master-thesis-style-guide-2014-06-10.pdf>.

When using Times or Times New Roman as the typeface, the following font sizes apply:

- The font size is 11 pt for body text.
- The font sizes, font weights, and font shapes for headings are as follows:
 - Level-1 headings (\section): 14 pt, boldface.
 - Level-2 headings (\subsection): 12 pt, boldface.
 - Level-3 headings (\subsubsection): 11 pt (like body text), italic.
 - Level-4 headings (\paragraph): 11 pt, upright (like body text). (Level-4 headings are usually unnecessary in documents that have the scope of theses.)
- The font size is 10 pt for tables (not including the table of contents), table titles, figure captions, list of references, and appendix.
- The font size is 9 pt for footnotes, table notes, and figure notes.

If you use a different typeface, adjust the font sizes such that the same amount of text fits on a page as if you had used Times New Roman with the font sizes mentioned above.

2.3 Page Size and Page Margins

The examination office's guidelines require that bachelor's and master's theses be printed on A4 paper (29.7 cm width, 21.0 cm height).

The following margins are required when using a font size of 11 pt to give about the same amount of text per page as it is produced when following the guidelines to the letter:

- The sum of the left and right margin has to equal 6.5 cm (default: 2.5 cm left margin, 4.0 cm right margin; the latter is relatively wide in order to provide space for annotations by reviewers). Consequently, the text width amounts to 14.5 cm.
- The sum of the top and bottom margin has to equal 5.6 cm (default: 2.8 cm top margin, 2.8 cm bottom margin). Consequently, the text height amounts to 24.1 cm.

2.4 Line Spacing

- Line spacing is 19.5 pt for the body text. The reason for the generous—but not particularly aesthetic—line spacing is that many reviewers like to place annotations in-between the lines of text. Apart from that, manuscripts in economics often include mathematical expressions with fractions, subscripts, superscripts, sums, integrals, etc. which all require generous line spacing.

- This results in 35 lines of text per page ($35 \times 19.5 \text{ pt} = 682.5 \text{ pt} = 24.08 \text{ cm}$), as “one-and-a-half line spacing” would produce in Microsoft Word.
- There is no additional vertical space between paragraphs. Paragraphs that do not follow a heading should have a first-line indent (that corresponds to the line height of 19.5 pt).
- The line spacing for headings is also 19.5 pt, in combination with vertical space above and below as follows:
 - Level-1 headings (\section) are spaced 39 pt above and 19.5 pt below.
 - Level-2 headings (\subsection) are spaced 19.5 pt above and 19.5 pt below.
 - Level-3 headings (\subsubsection) are spaced 9.75 pt above and 9.75 pt below.
 - Level-4 headings (\paragraph) are spaced 9.75 pt above and 0 pt below. (Level-4 headings are usually unnecessary in documents that have the scope of theses.)
- The line spacing is 15 pt for tables (including table of contents), table titles, figure captions, list of references, and appendix.
- The line spacing is 13.5 pt for footnotes, table notes, and figure notes.

2.5 Page Numbering

Page numbering starts with the first page of the main text. The main text is numbered using Arabic numerals. (After the main text, page numbering can be continued with Arabic numerals. For appendices, a prefix like “A” can be added to the page numbers.)

For the front matter, page numbers can be omitted, or Roman numerals can be used. In this case, the (Roman) numbering starts with the cover page. (However, no page number is printed on the cover page itself, as it is evident that it is the first page.)

3 Citing Other Authors and Sources

3.1 In-Text Citations

3.1.1 General Rules

Both literal citations from other texts as well as the paraphrasing of other authors’ ideas must be identified as such. The cited author(s) is (are) indicated right before or after the citation, including the year of the manuscript that you are citing/paraphrasing.

3.1.2 Paraphrasing Other Authors' Thoughts

Here is an example of a paraphrasing in-text citation:

Sarfraz and Razzak (2002) suggest an algorithm to automatically capture the outlines of fonts.

The author's name/authors' names can be embedded in the sentence, as in the example above, or surrounded by parentheses:

Many researchers in computer science have worked on the problem of converting the outlines of objects to mathematically describable curves, such as Bézier curves (see, e.g., Sarfraz and Razzak 2002).

Paraphrasing citations of this type usually start with “see.”

3.1.3 Literal Citations

Literal citations must be indicated via quotation marks. Use double quotation marks for this purpose. For quotes within direct quotes, use single quotation marks. For literal citations, the page number must also be provided, if possible. Here is an example of a literal citation:

Sarfraz and Razzak (2002, 795) state that “another major difference lies in the curve model for the description of the design curve. The outline capturing technique, instead of traditional Bézier cubics, is based upon a generalized Hermite cubic.” They conclude (p. 796): “Accordingly, the design curve will be GC^1 continuous.”

Omissions must be indicated by ellipses (“...”), as in the following example:

Sarfraz and Razzak (2002, 795) state: “Another major difference lies in the curve model for the description of the design curve. The outline capturing technique, ..., is based upon a generalized Hermite cubic.”

Changes must be placed in square brackets, as in the following example:

Sigfridsson and Ryde (1998, 377) find that the “CHELMO [Charges from Electrostatic Moments] method gives the best multipole moments for small and medium-sized polar systems.”

Longer quotations—as a rule of thumb, anything that spans more than two lines of text—should be formatted as block quotes:

Sigfridsson and Ryde (1998, 377) summarize their findings as follows:

The CHELMO method gives the best multipole moments for small and medium-sized polar systems, whereas the CHELP-BOW charges reproduce best the total interaction energy in actual simulations. Among the standard methods, the Merz–Kollman charges give the best moments and potentials, but they show an appreciable dependence on the orientation of the molecule.

Even longer quotations—which span more than one paragraph—are typeset as follows:

Sarfraz and Razzak (2002, 801) describe the results of their approach as follows:

The scheme and the algorithm has been implemented and tested for various shapes. Visually quite elegant results have been observed.

The algorithm has been tested for four fonts in Figs. 4(a), 5(a), 6(a) and (7a). The fonts in Figs. 4(a) and 5(a) are Arabic, whereas the images in Figs. 6(a) and 7(a) are Greek and Kanji characters respectively.

Fig. 4(b) is the outline of Fig. 4(a) using modified Avrahami and Pratt algorithm [24]. Fig. 4(c) displays the initial characteristic points in Fig. 4(b) using the method of Davis [23]. Figs. 4(d) and (e) demonstrate the initial characteristic points as well as the intermediate characteristic points which have been obtained after minimizing the errors between the original outline and the computed outline.

3.2 List of References

All works referenced in the manuscript—and only those—must be included in the list of references. It is either placed right after the main text, before the appendix, or at the very end, following all appendices. The publications should be arranged by author (or editor) in alphabetical order. Some examples of entries in the reference list can be found at the end of this guide. If you refer to Internet sources, the complete web address and the date on which you accessed it should be mentioned in the reference list.

I recommend generating citations following the guidelines of the *Chicago Manual of Style*. The reasons for this recommendation are as follows:

- The citation styles of several economics journals (such as the *American Economic Review* and the *Journal of Political Economy*) are based on the *CMOS* citation style.
- The *CMOS* citation style is very well documented online.¹⁵
- The *CMOS* citation style is implemented in both Zotero and BibLaTeX.¹⁶

The *CMOS* style posits that titles of books and journals be italicized, while titles of journal articles and newspaper articles be placed in double quotation marks.

3.3 Additional Recommendations Regarding Citations and Formatting

3.3.1 Citations Embedded in a Sentence

The settings in this document output citations as they would be generated by the popular `natbib` package. That is, `\cite`, `\citet`, and `\textcite` yield the same result:

Sarfraz and Razzak (2002), Sarfraz and Razzak (2002), Sarfraz and Razzak (2002)

Citing the same author multiple times:

Aristotle (1907, 1929, 1968) ...

15. https://www.chicagomanualofstyle.org/tools_citationguide/citation-guide-2.html.

16. See <https://www.zotero.org/styles?q=chicago> and <https://ctan.org/pkg/biblatex-chicago>, respectively.

If a citation includes a “von” part and occurs at the beginning of a sentence, it has to be capitalized, which is done via the `\Citet` command:

Van Gennep (1909) ..., van Gennep (1909) ...

Citing a work with more than three authors via `\cite`, `\citep`, `\citet`, etc. abbreviates the author list to first author plus “et al.” by default:

The book *Federal Appointments Process* (Gerhardt 2000) has a single author.
 The report “A Hybrid Hierarchical Model of a Multiple Virtual Storage (MVS) Operating System” (Chiu and Chow 1978) has two authors.
 The *LaTeX Companion* (Goossens, Mittelbach, and Samarin 1994) has three authors.
 The article “Alkanethiolate gold cluster molecules” (Hostetler et al. 1998) has many authors and was published in *Langmuir*.

The full list of author names is printed when using the starred commands `\citep*`, `\citet*`, etc:

The article “Alkanethiolate gold cluster molecules” (Hostetler, Wingate, Zhong, Harris, Vachet, Clark, Londono, Green, Stokes, Wignall, Glish, Porter, Evans, and Murray 1998) has many authors and was published in *Langmuir*.

3.3.2 Citations Included in Parentheses

A citation in parentheses:

This process has been described in the literature in detail (see Gerhardt 2000, and the references therein).

An alternative way to include additional text in the parentheses is the following:

Its properties are covered in publications and reports from various disciplines (for instance, Chiu and Chow 1978; Padhye, Firoiu, and Towsley 1999; Sarfraz and Razzak 2002; Markey 2005 are of relevance).

3.3.3 Citing Only Parts of Works

To cite particular pieces of information of a work, several commands are available, such as `\citeauthor`, `\citeyear`, `\citetitle`, etc.:

The book *TeXbook*, written by Donald E. Knuth, was published in 1984. It is available online via <https://ctan.org/pkg/texbook>.

4 Recommendations for Typesetting Mathematical Expressions

4.1 Equations and Equation Numbering

Short equations can be typeset in-line:

Let $\pi(x) = p(x)x - c(x)$ denote the profit of a monopolist, where $c(x)$ is the cost of producing quantity x .

Equations that are taller than a normal line of text and important equations should be typeset as display equations:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k} \quad (1)$$

All display equations should be numbered because this makes it easier to refer to them:

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots \quad (2)$$

Equation (2) posits that ...

Some authors prefer to number only those equations that they actually reference in the body text. This practice is debatable, however, since the very purpose of the numbering is to make an equation easy to find. When encountering an unnumbered equation, one does not know

whether the numbered equation that one is looking for can be found above or below the un-numbered equation. Hence, please number *all* equations. (In the same vein, we also numbers *all* pages, because selectively numbering only particular pages would make little sense.)

4.2 Italic Shape for Variables

All variables should be consistently italicized. This makes it easier to identify them and to comprehend the text. In LaTeX, this is achieved by using math mode (e.g., $\$x\$$ or $\backslash(x\backslash)$ to produce x):

Let p denote the price, and c marginal cost.

4.3 Upright Shape for Mathematical Constants and Functions with Established Meaning

In line with ISO norm 80000-2:2019(E), functions, numerical constants, and operators with an established, “well-defined” meaning should *not* be italicized but set upright to disambiguate them from variables. “Well-defined” here refers to the meaning being the same across contexts. This applies to, for instance, the exponential function (exp) and the logarithmic function (log, ln), to Euler’s constant (e) and the circular constant (π), as well as to the symbols for the differential (d) and the expected value (E). The ISO 80000-2:2019(E) standard^{17,18} states:

4 Variables, functions and operators

It is customary to use different sorts of letters for different sorts of entities, e.g. x , y , ... for numbers or elements of some given set, f , g for functions, etc. This makes formulas more readable and helps in setting up an appropriate context.

Variables such as x , y , etc., and running numbers, such as i in $\sum_i x_i$ are printed in italic type. Parameters, such as a , b , etc., which may be considered as constant in a particular context, are printed in italic type. The same applies to functions in general, e.g. f , g .

An explicitly defined function not depending on the context is, however, printed in upright type, e.g. sin, exp, ln, Γ . Mathematical constants, the values of which never

17. See <https://cdn.standards.iteh.ai/samples/64973/329519100abd447ea0d49747258d1094/ISO-80000-2-2019.pdf>, p. 1. See also the descriptions and discussions in <https://tug.org/tugboat/tb18-1/tb54becc.pdf> and in <https://nhigham.com/2016/01/28/typesetting-mathematics-according-to-the-iso-standard/>.

18. Note that the decimal separator used by the ISO is a comma (as it is standard, e.g., in German texts) rather than a period (as it is standard in, e.g., English texts). The ISO 80000-1 standard permits both.

change, are printed in upright type, e.g. $e = 2,718\,281\,828\dots$; $\pi = 3,141\,592\dots$; $i^2 = -1$. Well-defined operators are also printed in upright type, e.g. **div**, δ in δx and each d in df/dx . Some transforms use special capital letters (...).

Numbers expressed in the form of digits are always printed in upright type, e.g. 351 204; 1,32; $7/8$.

Binary operators, for example $+$, $-$, $/$, shall be preceded and followed by thin spaces. This rule does not apply in case of unary operators, as in $-17,3$.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\omega t + \phi)$. If the symbol for the function consists of two or more letters and the argument contains no operation symbol, such as $+$, $-$, \times , or $/$, the parentheses around the argument may be omitted. In these cases, there shall be a thin space between the symbol for the function and the argument, e.g. $\text{int } 2,4$; $\sin n\pi$; $\text{arcosh } 2A$; $\text{Ei } x$.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$; do not write $\cos x + y$, which could be mistaken for $\cos(x + y)$.

A comma, semicolon or other appropriate symbol can be used as a separator between numbers or expressions. The comma is generally preferred, except when numbers with a decimal comma are used.

If an expression or equation must be split into two or more lines, the following method shall be used:

— Place the line breaks immediately before one of the symbols $=$, $+$, $-$, \pm , or \mp , or, if necessary, immediately before one of the symbols \times , $:$, or $/$.

The symbol shall not be given twice around the line break; two minus signs could for example give rise to sign errors. If possible, the line break should not be inside of an expression in parentheses.

Here are some examples:

Profit π (variable) vs. radial constant π (numerical constant).

Effort e (variable) vs. Euler's number e (numerical constant).

Expected value $E[X]$, variance $\text{Var}[X]$, covariance $\text{Cov}[X, Y]$, probability $\text{Pr}[X \leq x]$.

Exponential function and logarithm: $\exp x$, $\log y$, $\lg y$, $\ln y$.

Sine and cosine function: $\sin \theta$, $\cos \theta$.

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right). \quad (3)$$

Variable d vs. differential d , difference operator Δ , limit \lim .

$$E[X] := \int_{-\infty}^{\infty} x f(x) dx; \quad f'(x) := \frac{df(x)}{dx} := \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}. \quad (4)$$

Following the ISO recommendations, vectors and matrices should be typeset using bold-face. Also for vectors and matrices, italics should be used when they denote variables, and upright font should be used for vector-/matrix-valued operators and functions with a “well-defined” meaning:

A multivariate random variable X has the expected value $\mathbf{E}[X]$ (a vector of scalar expected values). The variance–covariance matrix Σ is a symmetric $(n \times n)$ matrix with the following entries:

$$\Sigma = \mathbf{Cov}[X] = \begin{bmatrix} \text{Var}[X_1] & \text{Cov}[X_1, X_2] & \cdots & \text{Cov}[X_1, X_n] \\ \text{Cov}[X_2, X_1] & \text{Var}[X_2] & \cdots & \text{Cov}[X_2, X_n] \\ \vdots & & \ddots & \vdots \\ \text{Cov}[X_n, X_1] & \text{Cov}[X_n, X_2] & \cdots & \text{Var}[X_n] \end{bmatrix}. \quad (5)$$

Descriptive subscripts and superscripts should be printed upright so that they can be disambiguated from exponential operations etc.:

In the equation $\hat{\beta} = (X^T X)^{-1} X^T y$, the (upright) uppercase letter T denotes the matrix transpose.

By contrast, in the expression $X = (x_t)_{t=1}^T$, the (italic) uppercase letter T denotes the total number of observations, with individual observations indexed by lowercase t .

5 Portability

5.1 Compatibility with Other Typesetting Software

The settings in this template regarding the font family (by default, Times), font sizes, and line spacing were chosen such that a virtually identical layout can be achieved with other typesetting software such as Microsoft Word and Apple Pages and the open-source alternatives LibreOffice Writer (<https://www.libreoffice.org>) and Typst (<https://typst.app>). This means taking into account, for instance, that Word restricts font sizes to multiples of 0.5 pt.

5.2 Clean Code/Semantic Coding: Separating Content and Formatting

All written texts are structured into words and sentences. If they are long enough, texts are also structured into paragraphs. Academic texts, in addition, feature sections, subsections, and often subsubsections, as well as figures, tables, lists of references, and potentially appendices. These structural elements are associated with headings/titles and captions.

The structure of a document is conveyed to the reader through *formatting*. Effective formatting of academic documents, thus, requires that all elements of a particular type—say, section headings—be identifiable as such. Identifiability is achieved through styling that is *consistent within type* and *distinct across types*. For example, all section headings must be formatted identically, and at the same time, they must look different from subsection headings.

As a consequence, one should refrain from using manual, discretionary formatting whenever possible. In line with this principle, the source code of this template refers to the document's structure as much as possible. This is also called “semantic coding”: using code that describes the *meaning* of the content and not *how* those elements should look.

- Examples for *semantic* commands are `\author`, `\title`, `\section`, `\emph`, `\url`, `\cite`, `\begin{figure} ... \end{figure}`, `\caption`, `\begin{quote} ... \end{quote}`.
- Examples for LaTeX commands that are *not semantic*—and should be avoided—are `\textbf`, `\textit`, `\newpage`, `\pagebreak`, `\linebreak`, `\`, `\noindent`, `\smallskip`, `\centering`, `\noindent`, `\hspace`, `\vspace`, `\cellcolor`, `\multirow`.

The reason why one should stick to using semantic commands is that they keep the code *portable*: Semantic code can be *reused* across document without any changes (or at least without major adjustments). This is not then case with nonsemantic code, that is, code that includes manual formatting instructions. Instructions like `\bigskip`, `\hspace`, `\` (even worse, `\` `\`), etc. may result in decent formatting in a particular document but are bound to lead to undesirable formatting in another document. Moreover, manual formatting is error-prone, as it easily leads to inconsistent styling.

This entails two things: First, the formatting should be determined, as much as possible, in the *preamble* of the LaTeX document. This comes at the cost, of course, of a rather long preamble. Second, instead of defining custom commands, one should *redefine* standard LaTeX commands or use (widely adopted) packages that are available on CTAN (<https://ctan.org>).

This template follows the above principles, resulting in clean, portable code: The source code of the body of this template (apart from the boxes with examples) consists almost completely of semantic, basic LaTeX, with a few commands from popular packages on top. And

while the preamble of this document is rather long—to achieve the formatting described in [subsection 5.1](#)—the packages that are absolutely necessary for successful compilation are few:

- `amsmath`,
- `amsthm`,
- `babel` with options `USenglish` and `ngerman`,
- `biblatex-chicago` with options `authordate`, `backend = biber`, and `natbib`;
- `bm`,
- `enumitem`,
- `fontenc` with options `LGR` and `T1`,
- `hyperref`,
- `isodate`,
- `listings`,
- `mathtools`,
- `microtype`,
- `newtxmath` and `newtxtext`,
- `printlen`,
- `ragged2e`,
- `relsize`,
- `soul`,
- `tabularray` with `\UseTblrLibrary{booktabs, siunitx}`,
- `tcolorbox` with option `most`,
- `titlecaps`,
- `xcolor` with options `svgnames` and `x11names`.

5.3 Using BibLaTeX (Rather Than BibTeX)

This template uses the modern BibLaTeX framework (the *biblatex-chicago* package, <https://ctan.org/pkg/biblatex-chicago>, to be precise) instead of the vintage BibTeX framework to manage the references included in the document. The reason for this choice is that BibLaTeX is much more flexible than BibTeX and also handles multi-language references much better.

Moreover, in line with the principle describe above, BibLaTeX permits much better separation of a document's content (in this case, the *.bib* file) and its formatting (of the citations and the bibliography) than BibTeX. In particular, suppressing output of information that is present in a *.bib* file is cumbersome with BibTeX: It amounts to editing the “bibliography style” *.bst*

file, and *.bst* files have a syntax that is completely different from LaTeX and difficult to learn. It is usually easier to just remove the information from the *.bib* file. This, however, impacts the portability of the *.bib* file: In other circumstances one might want that very information to be present. BibLaTeX simplifies skipping information that is present in the *.bib* file.

An example: Researchers frequently present their research in the form of posters. Space on posters is usually quite limited. Hence, one might not want authors' first names to be abbreviated on a poster. In the associated paper, however, the names should be printed in full to minimize ambiguity. With BibTeX, this requires time-consuming adjustments of the *.bst* file. With BibLaTeX, it is much easier: In the *.tex* file for the poster, one can include the command

```
\ExecuteBibliographyOptions{giveninits = true}
```

Similarly, in a paper, the list of references should include URLs or DOIs (digital object identifiers) whenever they exist, to make it easy for readers to locate a cited work. On a poster, by contrast, one might want to suppress DOIs and URLs. With BibTeX, this would require time-consuming adjustments of the *.bst* file. With BibLaTeX, one can simply do, for instance,

```
\AtEveryBibitem{%
  \ifentrytype{article}{\clearfield{doi}\clearfield{url}}{}%
}
```

Using BibLaTeX requires using the program *biber* instead of *bibtex* for compiling the bibliography. When using the Overleaf online service, using *biber* instead of *bibtex* is as simple as it gets: Overleaf automatically chooses *biber* to compile the bibliography when it encounters BibLaTeX's `backend = biber` option.¹⁹ And when using the TeXstudio editor (<https://texstudio.org>), an easy way to make this happen is to include the “magic comment”

```
% !BIB program = biber
```

at the beginning of the *.tex* file.

19. This applies to the commercial version (<https://www.overleaf.com>) as well as to free versions <https://overleaf-students.astro.uni-bonn.de/> and https://uni-bonn.sciebo.de/apps/overleaf_nextcloud/launcher/launch.

5.4 Using the *tabularray* Package (Rather Than *tabularx/tabulary/tabu*, *booktabs*, *longtable*, *xltabular*, *multirow*, *makecell*, *colortbl*, *threeparttable*, ...)

The principle of separating the content from its formatting as much as possible can also be applied to tables. The *tabularray* package (<https://ctan.org/pkg/tabularray>) allows you to do just that. *tabularray* has been around since May 2021, so it is relatively new—but it has already matured into a package that is extremely useful. *tabularray* replaces (or loads) all the other table-related LaTeX packages that you may have used so far—like *tabularx/tabulary/tabu*, *longtable*, *xltabular*, *multirow*, *makecell*, *booktabs*, *colortbl*, *threeparttable*, ...

Separating the contents of a table from its formatting means that no formatting instructions whatsoever need to be placed in the table cells—neither for drawing rules between table rows nor for highlighting entries nor for increasing spacing nor for changing alignment, and so on. Instead, the placement of rules at the top and bottom of a table or between particular rows, the specification of column widths, the specification of font sizes and cell alignment, desired highlighting via boldface or a background color, etc., can all be coded as arguments to the table environment. This is beneficial in at least three aspects:

- The source code of the table's contents is very clean. This particularly applies to animated Beamer presentations: no need to put `\pause\only\cellcolor/...` in table cells.
- It makes table contents *portable*: A table's contents can be reused easily across multiple documents that are formatted in different ways.
- It also drastically simplifies the inclusion of table contents that are produced by an external program such as Python, R, or Stata.

Further additional helpful features of *tabularray* are:

- *tabularray* makes it easy to bottom-align or top-align particular table rows or individual cells entries, while keeping a different alignment for the other rows/cells.
- Table entries that consist of multiple lines of text become easy to produce: Just enclose them in curly braces, like in this example: `... & {Line 1 \\ Line 2} & ... \\`. No need to use `\multirow` or `\makecell` any more.
- You can set document-wide formatting defaults to achieve consistent formatting of all tables. For instance, you can set defaults such that all tables feature the same top and bottom rules. Or, you could make all column heads be typeset in boldface document-wide.

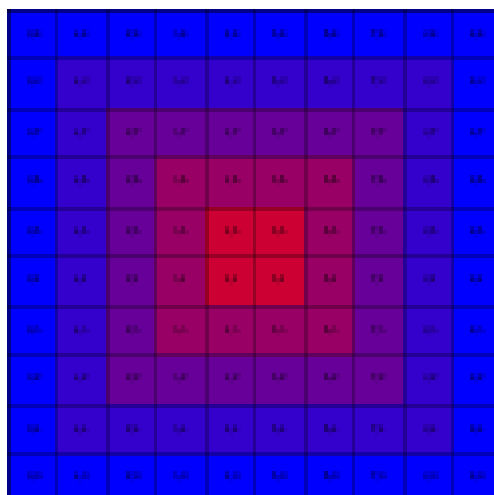


Figure 1. An example PNG image. Figure captions go *below* the figures and close with a period.

Note: Figure captions should be short; by default they should not span more than a single line. Any additional information that is necessary for understanding a figure should go in a figure note, as illustrated here.

When using `\UseTblrLibrary{siunitx}`, `tabularray` loads the `siunitx` package. This is another useful package which permits separating content from formatting. In particular, the `S` column type is able to round numbers to a specified precision. This means that one can include table content produced by an external program with as many decimal digits as that program produces and have LaTeX take care of the rounding.

6 Tests, Including Example Figures

6.1 Tests of List Formatting

If the order of the list entries does not matter—that is, the list is a collection of items without a hierarchy—use an unordered list, that is, a list with bullet points:

- Nested bulleted list (please avoid more than two list levels):
 - A journey into the unknown.
 - The art of tea brewing.
 - Exploring the depths of the ocean.
 - The mysteries of ancient civilizations.
 - Innovations in renewable energy.
 - The beauty of a starry night.

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- The art of tea brewing.
- Exploring the depths of the ocean.
- The mysteries of ancient civilizations.
- Innovations in renewable energy.
- The beauty of a starry night.

If the order of the list entries matters—such as when describing a sequence of events, a hierarchy, or items sorted by priority—use an ordered list, that is, a numbered list:

(1) Nested numbered list (please avoid more than two list levels!):

- a. We first provide an overview of the foundational concepts in LaTeX, including its purpose as a typesetting system that is particularly well-suited for producing complex documents such as academic papers and theses. LaTeX allows for precise control over document structure and formatting, making it a preferred choice for many professionals in academia and research.
- b. We delve into²⁰ the various environments available in LaTeX, such as the `itemize` and `enumerate` environments. Each environment serves a different purpose, with `itemize` creating bullet points and `enumerate` generating numbered lists.
- c. We discuss the importance of packages in LaTeX, which enhance its functionality. Packages like `amsmath` for advanced mathematical typesetting and `graphicx` for including images are essential for expanding the capabilities of basic LaTeX. Users should familiarize themselves with how to include and utilize these packages to maximize their LaTeX experience.
- d. Lastly, we explore the common pitfalls and troubleshooting tips for LaTeX users. Issues such as compilation errors, misformatted text, and missing packages can often arise. Knowing how to read error messages and where to find help, such as online forums and documentation, can significantly improve the user experience and help resolve issues efficiently.

(2) We first provide an overview of the foundational concepts in LaTeX, including its purpose as a typesetting system that is particularly well-suited for producing complex documents such as academic papers and theses. LaTeX allows for precise control over document structure and formatting, making it a preferred choice for many professionals in academia and research.

20. This text was AI-generated ...

- (3) We delve into²¹ the various environments available in LaTeX, such as the `itemize` and `enumerate` environments. Each environment serves a different purpose, with `itemize` creating bullet points and `enumerate` generating numbered lists.
- (4) We discuss the importance of packages in LaTeX, which enhance its functionality. Packages like `amsmath` for advanced mathematical typesetting and `graphicx` for including images are essential for expanding the capabilities of basic LaTeX. Users should familiarize themselves with how to include and utilize these packages to maximize their LaTeX experience.
- (5) Lastly, we explore the common pitfalls and troubleshooting tips for LaTeX users. Issues such as compilation errors, misformatted text, and missing packages can often arise. Knowing how to read error messages and where to find help, such as online forums and documentation, can significantly improve the user experience and help resolve issues efficiently.

Apple. A sweet and crunchy fruit that comes in various colors such as red, green, and yellow. Apples are rich in dietary fiber and vitamin C, making them a healthy snack. They are also used in a variety of recipes, including pies, salads, and juices.²²

Banana. A long, yellow fruit that is soft and sweet when ripe. Bananas are known for being an excellent source of potassium, which helps maintain proper muscle and nerve function. They are often eaten as a quick snack or used in smoothies, baked goods, and desserts.²³

Cherry. A small, round fruit that is typically red or dark purple in color and contains a hard pit in the center. Cherries are known for their sweet-tart flavor and are commonly consumed fresh, or used in jams, pies, and juices. They are also rich in antioxidants and anti-inflammatory compounds.²⁴

21. This text was AI-generated ...

22. This description was AI-generated.

23. This description was AI-generated, too.

24. This description was AI-generated, too.

Line 01—This is the first line on the page.

Line 02. Checking the alignment of body text with the baseline grid.

Line 03

Line 04

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6.2 Test of Theorems, Lemmas, Hypotheses, Results, Proofs

Conjecture 1 (Poincaré conjecture). *Every three-dimensional topological manifold which is closed, connected, and has trivial fundamental group is homeomorphic to the three-dimensional sphere.*

Corollary 2. *Let the performance measurement system \mathbf{P} be minimal and balanced. If not all measures of \mathbf{P} are verifiable, the first-best effort cannot be induced by an explicit contract.*

Lemma 3. *To implement a given action a at minimal cost, higher-powered incentives are necessary under a more risky performance measure.*

Proof. Obvious from (IC'). ■

Lemma 4. *The optimal capacity choice of firm i is given by*

$$k^* = \begin{cases} 1 & \text{for } \underline{c} \leq c_i < \hat{c}, \\ 0 & \text{for } \hat{c} \leq c_i \leq \bar{c}, \end{cases} \quad \text{where } \hat{c} = H^{-1}(1 / \theta).$$

Proof. See [Appendix A](#). ■

Proposition 5. *Let \mathbf{P} be balanced with respect to the principal's objective V . Then the following statements hold:*

- (i) *The first-best solution can be achieved by a linear contract based on \mathbf{P} .*
- (ii) *If \mathbf{P} is also minimal, all measures have nonzero value in the optimal contract.*

Proposition 6. *If the principal strictly prefers the implemented action in a model with a finite action space, there is a positive risk–incentive relationship.*

Proposition 7. *If the agent is risk neutral, there is a positive risk–incentive relationship as long as the probability of a high payment exceeds $1/2$.*

Theorem 8 (Lindeberg–Lévy Central Limit Theorem). Let X_1, X_2, \dots, X_n be i.i.d. random variables with expected value $E[X_i] = \mu < \infty$ and variance $0 < \text{Var}[X_i] = \sigma^2 < \infty$. Then, the random variable

$$Z_n = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{X_1 + X_2 + \dots + X_n - n\mu}{\sqrt{n}\sigma}$$

converges in distribution to the standard normal random variable as n goes to infinity, that is

$$\lim_{n \rightarrow \infty} \Pr[Z_n \leq x] = \Phi(x), \quad \text{for all } x \in \mathbb{R},$$

where $\Phi(x)$ is the standard normal CDF.

Assumption 1 (MLRP). The signals y_i fulfill the monotone likelihood ratio property: $p_i(y_i | a_i) / p_i(y_i | a_i)$ is increasing in y_i .

Assumption 2 (CDFC). The signals y_i fulfill the convexity of the distribution function condition: $\partial^2 F_i(y_i | a_i) / \partial a_i^2 \geq 0$.

Definition 1. We define a performance signal \hat{s} to be *more risky* (less informative with respect to the agent's action) than a signal s , if \hat{a} is a garbling of s , i.e., if there exists a number $b \in (0, 1/2]$ such that

$$\hat{p}(a) = (1 - b) p(a) + b (1 - p(a)) = b + (1 - 2b) p(a).$$

Example 1. Let the agent's action space be $\{a_1, a_2\}$ with $v(a_1) = 0$ and $v(a_2) = 4$, the agent's utility be given by $U(w, a) = \sqrt{w} - c(a)$ with $c(a_1) = 0$ and $c(a_2) = 1$, and success probabilities be $p(a_1) = 0.25$ and $p(a_2) = 0.75$. Then, high effort a_2 is implemented by wages $w_L = 0$ and $w_H = 4$. The expected compensation cost is $E[w] = 0.25 \times 0 + 0.75 \times 4 = 3$, and the principal's net profit is $4 - 3 = 1$. The principal strictly prefers a_2 to a_1 , since implementing a_1 yields a net profit of 0.

Now consider a garbling of the form proposed in our definition of riskiness. Success probabilities become $\hat{p}(a_1) = 0.25 + 0.5b$ and $\hat{p}(a_2) = 0.75 - 0.5b$. High effort is implemented by wages $w_L = 0$ and $w_H = 1 / (0.5 - b)^2$. As $1 / (0.5 - b)^2 > 4$, $\forall b \in (0, 1/2)$, incentives become higher powered. The principal's net profit under a_2 is $4 - (0.75 - 0.5b) / (0.5 - b)^2$, which is larger than his zero profit under a_1 for $b < (7 - \sqrt{33}) / 16$ or $b > (7 + \sqrt{33}) / 16$. For these levels of b , the principal still implements a_H after the garbling, and there is a positive risk–incentive relationship.

Hypothesis 1. Subjects allocate more money to payoffs that are concentrated on a single date than to equal-sized payoffs that are dispersed over multiple earlier dates, $d^{\text{CB}} > 0$ (in contrast to standard discounting).

Hypothesis 2. The effect described in [Hypothesis 1](#) is the more pronounced, the more dispersed a payoff is, that is, $d_8^{\text{CB}} > d_4^{\text{CB}} > d_2^{\text{CB}} > 0$.

Result 1 (Test of [Hypothesis 1](#)). On average, subjects allocate more money to payoffs that are concentrated rather than dispersed, that is, our measure of concentration bias, \hat{d} , is significantly larger than zero.

Result 2 (Test of [Hypothesis 2](#)). Our measure of concentration bias is the greater, the more dispersed payments in the CONC-DISP and DISP-CONC condition are, that is, $\hat{d}_8 > \hat{d}_4 > \hat{d}_2 > 0$.

Remark 1 (Formatting of theorem-like segments). The formatting of theorems and theorem-like segments of academic papers varies widely between journals and publishers. There is no agreed-upon standard. Some journals indent these segments, others do not; some print lemmas, propositions, and theorems in italics, others use upright letters. Therefore, the categorization and formatting of the different theorem-like environments in this template follows the formatting suggested by the American Mathematical Society in <https://mirrors.ctan.org/macros/latex/required/amscls/doc/amsthdoc.pdf>:

These default settings are provided:

- plain: italic text, extra space above and below;
- definition: upright text, extra space above and below;
- remark: upright text, no extra space above or below.

...

The following list summarizes the types of structures which are normally associated with each theorem style.

plain	Theorem, Lemma, Corollary, Proposition, Conjecture, Criterion, Assertion
definition	Definition, Condition, Problem, Example, Exercise, Algorithm, Question, Axiom, Property, Assumption, Hypothesis
remark	Remark, Note, Notation, Claim, Summary, Acknowledgment, Case, Conclusion

0,4	1,4	2,4	3,4	4,4
0,3	1,3	2,3	3,3	4,3
0,2	1,2	2,2	3,2	4,2
0,1	1,1	2,1	3,1	4,1
0,0	1,0	2,0	3,0	4,0

Figure 2. An example cropped PDF image.

Note: Figures as well as tables should be placed at the top of a page. If the top is already occupied, they can also be placed at the bottom of a page. This corresponds to LaTeX's default placement order. That is, figures and tables should *not* be placed in the midst of body text.

6.3 Tests of Cross-Referencing

Section 4, section 6, subsection 6.1, subsubsection 7.5.1, paragraph 7.5.2.1, Appendix A, ??.

Figure 1, Figure 2, Table 1, Table 2.

Conjecture 1, Corollary 2, Proposition 5, Lemma 4, Lemma 3, Proposition 6, Proposition 7, Theorem 8.

Hypothesis 1, Hypothesis 2, Result 1 Result 2.

Assumption 1, Assumption 2, Definition 1, Example 1, Remark 1.

7 Text Samples, Including Example Tables

7.1 Overview

The whole Latin alphabet in a short sentence: Amazingly few discotheques provide jukeboxes. All letters between other letters and some punctuation: Incredibly, he makes a major life-change! For example: "I'll require that the system have two sizes." All digits and f-ligatures:

Table 1. My first table (table titles go *above* the tables and do *not* have a closing period)

m	$\Re\{\mathfrak{X}(m)\}$	$-\Im\{\mathfrak{X}(m)\}$	$\mathfrak{X}(m)$	$\frac{\mathfrak{X}(m)}{23}$	A_m	$\varphi(m) / ^\circ$	$\varphi_m / ^\circ$
1	16.128	+8.872	16.128	1.402	1.373	−146.6	−137.6
2	3.442	−2.509	3.442	0.299	0.343	133.2	152.4
3	1.826	−0.363	1.826	0.159	0.119	168.5	−161.1
4	0.993	−0.429	0.993	0.086	0.08	25.6	90
5	1.29	+0.099	1.29	0.112	0.097	−175.6	−114.7
6	0.483	−0.183	0.483	0.042	0.063	22.3	122.5
7	0.766	−0.475	0.766	0.067	0.039	141.6	−122
8	0.624	+0.365	0.624	0.054	0.04	−35.7	90
9	0.641	−0.466	0.641	0.056	0.045	133.3	−106.3
10	0.45	+0.421	0.45	0.039	0.034	−69.4	110.9
11	0.598	−0.597	0.598	0.052	0.025	92.3	−109.3

Source: Add the source of your data if you are using data that someone else collected. This table was adapted from <https://tex.stackexchange.com/a/112382/156280>.

Notes: Table titles should be short; by default they should not span more than a single line. Any additional information that is necessary for understanding a table should go in a table note, as illustrated here. A useful tool for generating LaTeX tables is <https://www.tablesgenerator.com>. The tables in this template use the environments provided by the *tabularray* package (<https://ctan.org/pkg/tabularray>).

Fifty-five fjord truffles offer sufficient flavor, although $537 + 489 = 1026$. All French and German special characters: Le cœur déçu mais l'âme plutôt naïve, Louÿs rêva de crapaüter en canoë au delà des îles, près du mälström où brûlent les novæ. Die Faltung einer Gauß-Kurve mit einer Lorentz-Kurve:

$$\int_{-\infty}^{\infty} \frac{y e^{-t^2}}{(x-t)^2 + y^2} dt = \pi \Re[w(x + i y)], \quad w(z) = e^{-z^2} \operatorname{erfc}(-i z), \quad x \in \mathbb{R}, y > 0. \quad (6)$$

The cross product $\alpha \times \beta$ of two vectors α and β is defined by

$$\alpha \times \beta := |\alpha| |\beta| \sin(\theta) \hat{n}. \quad (7)$$

The remaining text samples in this section were generated with the help of <https://www.blindtextgenerator.de>.

7.2 Kafka in \normalsize

Jemand musste Josef K. verleumdet haben, denn ohne dass er etwas Böses getan hätte, wurde er eines Morgens verhaftet. „Wie ein Hund!“ sagte er, es war, als sollte die Scham ihn über-

leben. Als Gregor Samsa eines Morgens aus unruhigen Träumen erwachte, fand er sich in seinem Bett zu einem ungeheueren Ungeziefer verwandelt. Und es war ihnen wie eine Bestätigung ihrer neuen Träume und guten Absichten, als am Ziele ihrer Fahrt die Tochter als erste sich erhob und ihren jungen Körper dehnte. „Es ist ein eigentümlicher Apparat“, sagte der Offizier zu dem Forschungsreisenden und überblickte mit einem gewissermaßen bewundernden Blick den ihm doch wohlbekannten Apparat. Sie hätten noch ins Boot springen können, aber der Reisende hob ein schweres, geknotetes Tau vom Boden, drohte ihnen damit und hielt sie dadurch von dem Sprunge ab. In den letzten Jahrzehnten ist das Interesse an Hungerkünstlern sehr zurückgegangen. Aber sie überwandten sich, umdrängten den Käfig und ... 0123456789

7.3 Kafka in \small

Jemand musste Josef K. verleumdet haben, denn ohne dass er etwas Böses getan hätte, wurde er eines Morgens verhaftet. „Wie ein Hund!“ sagte er, es war, als sollte die Scham ihn überleben. Als Gregor Samsa eines Morgens aus unruhigen Träumen erwachte, fand er sich in seinem Bett zu einem ungeheueren Ungeziefer verwandelt. Und es war ihnen wie eine Bestätigung ihrer neuen Träume und guten Absichten, als am Ziele ihrer Fahrt die Tochter als erste sich erhob und ihren jungen Körper dehnte. „Es ist ein eigentümlicher Apparat“, sagte der Offizier zu dem Forschungsreisenden und überblickte mit einem gewissermaßen bewundernden Blick den ihm doch wohlbekannten Apparat. Sie hätten noch ins Boot springen können, aber der Reisende hob ein schweres, geknotetes Tau vom Boden, drohte ihnen damit und hielt sie dadurch von dem Sprunge ab. In den letzten Jahrzehnten ist das Interesse an Hungerkünstlern sehr zurückgegangen. Aber sie überwandten sich, umdrängten den Käfig und ... 0123456789

7.4 Kafka in \footnotesize

Jemand musste Josef K. verleumdet haben, denn ohne dass er etwas Böses getan hätte, wurde er eines Morgens verhaftet. „Wie ein Hund!“ sagte er, es war, als sollte die Scham ihn überleben. Als Gregor Samsa eines Morgens aus unruhigen Träumen erwachte, fand er sich in seinem Bett zu einem ungeheueren Ungeziefer verwandelt. Und es war ihnen wie eine Bestätigung ihrer neuen Träume und guten Absichten, als am Ziele ihrer Fahrt die Tochter als erste sich erhob und ihren jungen Körper dehnte. „Es ist ein eigentümlicher Apparat“, sagte der Offizier zu dem Forschungsreisenden und überblickte mit einem gewissermaßen bewundernden Blick den ihm doch wohlbekannten Apparat. Sie hätten noch ins Boot springen können, aber der Reisende hob ein schweres, geknotetes Tau vom Boden, drohte ihnen damit und hielt sie dadurch von dem Sprunge ab. In den letzten Jahrzehnten ist das Interesse an Hungerkünstlern sehr zurückgegangen. Aber sie überwandten sich, umdrängten den Käfig und ... 0123456789

Table 2. Example of a regression table—alignment at the decimal point via the *siunitx* package

	Choice List				Combined
	A	B	C	D	
	(1)	(2)	(3)	(4)	(5)
Treatment	−0.390 (+0.352)	−0.228 (−0.205)	−0.729* [+0.377]	−0.449* [−0.245]	−0.453** {+0.204}
Female	0.948*** (0.354)	0.061 (0.233)	0.188 (0.372)	0.305 (0.226)	0.385* (0.222)
Female × Treatment	0.169 (0.514)	0.251 (0.325)	0.892* (0.533)	0.454 (0.341)	0.439 (0.307)
Final high school grade	−0.101 (0.198)	0.013 (0.144)	0.076 (0.224)	0.117 (0.146)	0.039 (0.133)
Trait self-control	−0.016 (0.016)	0.002 (0.010)	−0.016 (0.015)	0.000 (0.010)	−0.007 (0.009)
Constant	2.357*** (0.239)	1.512*** (0.144)	−0.322 (0.265)	2.158*** (0.161)	1.437*** (0.152)
Observations	303	289	295	304	1191
R^2	0.057	0.008	0.039	0.043	0.024
Treatment × (1 + Female)	−0.221	0.023	0.163	0.004	−0.014
$p_F[\text{Treatment} \times (1 + \text{Female}) = 0]$	0.327	0.008	0.192	0.000	0.003

Notes: Dependent variable: m_{\sim} . Robust standard errors (cluster-corrected for column 5) in parentheses. Missing observations ($N < 308$) due to exclusion of trials in which subjects behaved irrationally (i.e., chose a dominated option). The regressors Final high school grade and Trait self-control are mean-centered. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

7.5 The Story of *Lorem Ipsum*

7.5.1 The Beginning

Far far away, behind the word mountains, far from the countries Vokalia and Consonantia, there live the blind texts. Separated they live in Bookmarksgrove right at the coast of the Semantics, a large language ocean. A small river named Duden flows by their place and supplies it with the necessary regelialia. It is a paradisematic country, in which roasted parts of sentences fly into your mouth. Even the all-powerful Pointing has no control about the blind texts it is an almost unorthographic life One day however a small line of blind text by the name of Lorem Ipsum decided to leave for the far World of Grammar.

Table 3. Initial capital structures of large projects

Debt-to-Assets Ratio	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	Total
< 50%	–%	10%	5%	7%	11%	2%	–%	–%	–%	–%	3%	2%
50%–59.9%	20%	5%	16%	7%	11%	8%	13%	11%	9%	15%	8%	11%
60%–69.9%	–%	10%	5%	13%	11%	10%	16%	16%	17%	11%	15%	13%
70%–79.9%	–%	10%	37%	13%	15%	8%	24%	29%	16%	14%	35%	22%
80%–89.9%	40%	10%	5%	17%	7%	24%	18%	18%	21%	6%	8%	15%
> 90%	40%	55%	32%	43%	46%	49%	29%	26%	38%	24%	35%	37%
Mean	85%	80%	77%	80%	78%	85%	79%	79%	82%	77%	80%	80%
Median	85%	94%	76%	81%	80%	87%	79%	77%	82%	75%	72%	80%
No. of Projects	5	20	19	30	46	51	68	38	58	54	26	415

Notes: Adapted from <https://tex.stackexchange.com/a/373932/156280>. Large projects are defined as having a capitalization of at least \$1bn.

The Big Oxmox (Wilde 1899) advised her not to do so, because there were thousands of bad Commas, wild Question Marks and devious Semikoli, but the Little Blind Text didn't listen. She packed her seven versalia, put her initial into the belt and made herself on the way. When she reached the first hills of the Italic Mountains, she had a last view back on the skyline of her hometown Bookmarksgrove, the headline of Alphabet Village and the subline of her own road, the Line Lane. Pityful a rethoric question ran over her cheek, then she continued her way. On her way she met a copy. The copy warned the Little Blind Text, that where it came from it would have been rewritten a thousand times and everything that was left from its origin would be the word “and” and the Little Blind Text should turn around and return to its own, safe country.

7.5.2 The Main Text

7.5.2.1 The Duden River

But nothing the copy said could convince her and so it didn't take long until a few insidious Copy Writers ambushed her, made her drunk with Longe and Parole and dragged her into their agency, where they abused her for their projects again and again. And if she hasn't been rewritten, then they are still using her. Far far away, behind the word mountains, far from the countries Vokalia and Consonantia, there live the blind texts.²⁵

25. The quick, brown fox jumps over a lazy dog (Knuth 1986a, 1986b). DJs flock by when MTV ax quiz prog. Junk MTV quiz graced by fox whelps. Bawds jog, flick quartz, vex nymphs. Waltz, bad nymph, for quick jigs vex! Fox nymphs grab quick-jived waltz. Brick quiz whangs jumpy veldt fox. Bright vixens jump; dozy fowl quack.

Separated they live in Bookmarksgrove right at the coast of the Semantics, a large language ocean. A small river named Duden flows by their place and supplies it with the necessary regelialia. It is a paradisematic country, in which roasted parts of sentences fly into your mouth. Even the all-powerful Pointing has no control about the blind texts it is an almost unorthographic life One day however a small line of blind text by the name of Lorem Ipsum decided to leave for the far World of Grammar. The Big Oxmox advised her not to do so, because there were thousands of bad Commas, wild Question Marks and devious Semikoli, but the Little Blind Text didn't listen. She packed her seven versalia, put her initial into the belt and made herself on the way. When she reached the first hills of the Italic Mountains, she had a last view back on the skyline of her hometown Bookmarksgrove, the headline of Alphabet Village and the subline of her own road, the Line Lane. Pityful a rethoric question ran over her cheek, then she continued her way.

On her way she met a copy. The copy warned the Little Blind Text, that where it came from it would have been rewritten a thousand times and everything that was left from its origin would be the word "and" and the Little Blind Text should turn around and return to its own, safe country. But nothing the copy said could convince her and so it didn't take long until a few insidious Copy Writers ambushed her, made her drunk with Longe and Parole and dragged her into their agency, where they abused her for their projects again and again. And if she hasn't been rewritten, then they are still using her.

Far far away, behind the word mountains, far from the countries Vokalia and Consonantia, there live the blind texts. Separated they live in Bookmarksgrove right at the coast of the Semantics, a large language ocean. A small river named Duden flows by their place and supplies it with the necessary regelialia. It is a paradisematic country, in which roasted parts of sentences fly into your mouth. Even the all-powerful Pointing has no control about the blind texts it is an almost unorthographic life One day however a small line of blind text by the name of Lorem Ipsum decided to leave for the far World of Grammar. The Big Oxmox advised her

Quick wafting zephyrs vex bold Jim. Quick zephyrs blow, vexing daft Jim. Sex-charged fop blew my junk TV quiz. How quickly daft jumping zebras vex. Two driven jocks help fax my big quiz. Quick, Baz, get my woven flax jodhpurs! "Now fax quiz Jack!" my brave ghost pled. Five quacking zephyrs jolt my wax bed. "But I must explain to you how all this mistaken idea of denouncing pleasure and praising pain was born and I will give you a complete account of the system, and expound the actual teachings of the great explorer of the truth, the master-builder of human happiness. No one rejects, dislikes, or avoids pleasure itself, because it is pleasure, but because those who do not know how to pursue pleasure rationally encounter consequences that are extremely painful. Nor again is there anyone who loves or pursues or desires to obtain pain of itself, because it is pain, but because occasionally circumstances occur in which toil and pain can procure him some great pleasure. To take a trivial example, which of us ever undertakes laborious physical exercise, except to obtain some advantage from it? (Cicero 1995, 210.)

Table 4. Project funding by source

	1995	1996	1997	1998	1999	2000	2001	2002	Total
Bank Loans	23.33	42.83	67.43	56.65	72.39	110.89	108.48	62.20	557.88
Bonds	3.79	4.79	7.70	9.79	19.79	20.81	25.00	13.80	109.26
Development Agencies	17.59	18.96	22.05	20.97	16.62	17.69	18.75	18.75	162.63
Total Debt	44.71	66.58	96.98	87.41	108.80	149.39	152.23	94.75	829.77
Equity	19.16	28.54	41.56	37.46	46.70	64.02	65.24	40.61	355.68
Total	63.88	95.12	138.54	124.87	155.68	213.40	217.47	135.36	1185.63
Bank Loans	37%	45%	49%	45%	46%	52%	50%	46%	47%
Bonds	6%	5%	5%	8%	13%	10%	11%	10%	9%
Development Agencies	28%	20%	16%	17%	11%	8%	9%	14%	14%
Total Debt	70%	70%	70%	70%	70%	70%	70%	70%	70%
Equity	30%	30%	30%	30%	30%	30%	30%	30%	30%

Notes: Absolute amounts in US\$ billions. Adapted from <https://tex.stackexchange.com/a/373932/156280>.

not to do so, because there were thousands of bad Commas, wild Question Marks and devious Semikoli, but the Little Blind Text didn't listen.

7.5.2.2 The Seven Versalia

She packed her seven versalia, put her initial into the belt and made herself on the way. When she reached the first hills of the Italic Mountains, she had a last view back on the skyline of her hometown Bookmarksgrove, the headline of Alphabet Village and the subline of her own road, the Line Lane. Pityful a rethoric question ran over her cheek, then she continued her way. On her way she met a copy (Shore 1991).

The copy warned the Little Blind Text, that where it came from it would have been rewritten a thousand times and everything that was left from its origin would be the word “and” and the Little Blind Text should turn around and return to its own, safe country. But nothing the copy said could convince her and so it didn't take long until a few insidious Copy Writers ambushed her, made her drunk with Longe and Parole and dragged her into their agency, where they abused her for their projects again and again. And if she hasn't been rewritten, then they are still using her.

Far far away, behind the word mountains, far from the countries Vokalia and Consonantia, there live the blind texts. Separated they live in Bookmarksgrove right at the coast of the

Table 5. Global project bank facility lead arrangers

Rank	Lead Arranger	Number of Deals	Amount	Market Share	Equator Principles Adoption
1	State Bank of India	52	\$21,631.6	10.1%	N/A
2	Mitsubishi UFJ Financial	88	\$9,486.1	4.4%	Dec. 2005
3	Sumitomo Mitsui	71	\$8,188.1	3.8%	Jan. 2006
4	Credit Agricole	60	\$6,506.4	3.1%	Jun. 2005
5	Mizuho Financial	55	\$5,797.5	2.7%	Oct. 2003
6	Société Generale	55	\$5,760.5	2.7%	Sep. 2007
7	BNP Paribas	55	\$5,390.8	2.5%	Oct. 2008
8	Axis Bank	18	\$5,216.9	2.4%	N/A
9	IDBI Bank	10	\$5,162.3	2.4%	N/A
10	ING	49	\$4,916.1	2.3%	Jun. 2003
	Others	102	\$135,430.4	63.6%	
	Total Market	615	\$213,486.7	100.0%	

Note: Adapted from <https://tex.stackexchange.com/a/373932/156280>.

Semantics, a large language ocean. A small river named Duden flows by their place and supplies it with the necessary regelialia. It is a paradisematic country, in which roasted parts of sentences fly into your mouth.

Even the all-powerful Pointing has no control about the blind texts it is an almost unorthographic life One day however a small line of blind text by the name of Lorem Ipsum decided to leave for the far World of Grammar. The Big Oxmox advised her not to do so, because there were thousands of bad Commas, wild Question Marks and devious Semikoli, but the Little Blind Text didn't listen. She packed her seven versalia, put her initial into the belt and made herself on the way.

7.5.3 The End

When she reached the first hills of the Italic Mountains, she had a last view back on the skyline of her hometown Bookmarksgrove, the headline of Alphabet Village and the subline of her own road, the Line Lane. Pityful a rethoric question ran over her cheek, then she continued her way. On her way she met a copy. The copy warned the Little Blind Text, that where it came from it would have been rewritten a thousand times and everything that was left from its origin would be the word “and” and the Little Blind Text should turn around and return to its own,

Table 6. An additional table to check the horizontal spacing of columns

Test	Test 1	Test 12	Test 123	Test 1234	Test 12345	Test 123456	Test 1234567
------	--------	---------	----------	-----------	------------	-------------	--------------

Note: One can nest traditional environments (e.g., `tabular`, `tabular*`) in `tabularray` environments. This way, the outer formatting can be kept consistent (e.g., automatically adding `\toprule` and `\bottomrule`), while at the same time, formatting via the traditional environments can be used (e.g., `@{\extracolsep{\fill}}`).

safe country. But nothing the copy said could convince her and so it didn't take long until a few insidious Copy Writers ambushed her, made her drunk with Longe and Parole and dragged her into their agency, where they abused her for their projects again and again. And if she hasn't been rewritten, then they are still using her. Far far away, behind the word mountains, far from the countries Vokalia and Consonantia, there live the blind texts. Separated they live in Bookmarksgrove right at the coast of the Semantics, a large language ocean.

A small river named Duden flows by their place and supplies it with the necessary regelialia. It is a paradisematic country, in which roasted parts of sentences fly into your mouth. Even the all-powerful Pointing has no control about the blind texts it is an almost unorthographic life One day however a small line of blind text by the name of Lorem Ipsum decided to leave for the far World of Grammar. The Big Oxmox advised her not to do so (see, e.g., Baez and Lauda 2004), because there were thousands of bad Commas, wild Question Marks and devious Semikoli, but the Little Blind Text didn't listen. She packed her seven versalia, put her initial into the belt and made herself on the way.

When she reached the first hills of the Italic Mountains, she had a last view back on the skyline of her hometown Bookmarksgrove, the headline of Alphabet Village and the subline of her own road, the Line Lane. Pityful a rethoric question ran over her cheek, then she continued her way. On her way she met a copy. The copy warned the Little Blind Text, that where it came

Table 7. A table to illustrate rounding with the help of the *siunitx* package

Expandable column (X)	right-aligned	center-aligned	left-aligned	S (from <i>siunitx</i>)
Some text	1.23	4.56	7.89	9 012 345 678.9012
Some text	1.23	4.56	7.89	9 012 345 678.9012
Sum	2.46	9.12	15.78	18 024 691 357.8025

Notes: We place this table at the bottom of the page because the top is already occupied by another table.

Check the source code of this table to see that the S column type provided by the *siunitx* package takes care of the rounding in the right-most column.

Table 8. A *tabularray*-based table with natural column widths and a total width of less than `\textwidth`

Test	Test 1	Test 12	Test 123	Test 1234	Test 12345	Test 123456
$f(x)$	$\frac{\sqrt{3}}{n}$	Test	Content spanning multiple cells			Row 1
						Row 2

Notes: As this example shows, *tabularray* can produce table titles and notes whose width is identical to that of the table's body, just as the *threeparttable* package would produce them. So, no *threeparttable* needed, *tabularray* does it all.

You might agree, however, that tables which have the same width as the body text look better in most cases.

Source: You can combine several table notes, for instance, to indicate the source(s) of the data that you are presenting or analyzing.

from it would have been rewritten a thousand times and everything that was left from its origin would be the word “and” and the Little Blind Text should turn around and return to its own, safe country.

References

- Aristotle. 1907. *De Anima*. Edited by Robert Drew Hicks. Cambridge: Cambridge University Press. [14]
- Aristotle. 1929. *Physics*. Translated by P. H. Wicksteed and F. M. Cornford. New York: G. P. Putnam. [14]
- Aristotle. 1968. *Poetics*. Edited by D. W. Lucas. Clarendon Aristotle. Oxford: Clarendon Press. [14]
- Baez, John C., and Aaron D. Lauda. 2004. “Higher-Dimensional Algebra V: 2-Groups.” *Theory and Applications of Categories* 12:423–491. arXiv: [math/0307200v3](https://arxiv.org/abs/math/0307200v3). [43]
- Chiu, Willy W., and We Min Chow. 1978. “A Hybrid Hierarchical Model of a Multiple Virtual Storage (MVS) Operating System.” Research report RC-6947. IBM. [15]
- Cicero, Marcus Tullius. 1995. *De natura deorum. Über das Wesen der Götter* [in Latin and German]. Edited and translated by Ursula Blank-Sangmeister. With an afterword by Klaus Thraede. Stuttgart: Reclam. [40]
- Gerhardt, Michael J. 2000. *The Federal Appointments Process: A Constitutional and Historical Analysis*. Durham and London: Duke University Press. [15]
- Goossens, Michel, Frank Mittelbach, and Alexander Samarin. 1994. *The LaTeX Companion*. 1st ed. Reading, Mass.: Addison-Wesley. [15]
- Hostetler, Michael J., Julia E. Wingate, Chuan-Jian Zhong, Jay E. Harris, Richard W. Vachet, Michael R. Clark, J. David Londono, et al. 1998. “Alkanethiolate gold cluster molecules with core diameters from 1.5 to 5.2 nm: Core and monolayer properties as a function of core size.” *Langmuir* 14 (1): 17–30. [15]

- Knuth, Donald E. 1984. *The T_EXbook*. Vol. A of *Computers & Typesetting*. Reading, Mass.: Addison-Wesley. [16]
- Knuth, Donald E. 1986a. *T_EX: The Program*. Vol. B of *Computers & Typesetting*. Reading, Mass.: Addison-Wesley. [39]
- Knuth, Donald E. 1986b. *The METAFONTbook*. Vol. C of *Computers & Typesetting*. Reading, Mass.: Addison-Wesley. [39]
- Markey, Nicolas. 2005. “Tame the BeaST: The B to X of BibTeX,” October 16, 2005. Accessed October 1, 2006. http://mirror.ctan.org/info/bibtex/tamethebeast/ttb_en.pdf. [15]
- Padhye, Jitendra, Victor Firoiu, and Don Towsley. 1999. “A Stochastic Model of TCP Reno Congestion Avoidance and Control.” Technical report 99-02. Amherst, Mass.: University of Massachusetts. [15]
- Sarfraz, M., and M. F. A. Razzak. 2002. “Technical section: An algorithm for automatic capturing of the font outlines.” *Computers and Graphics* 26 (5): 795–804. [12–15]
- Shore, Bradd. 1991. “Twice-Born, Once Conceived: Meaning Construction and Cultural Cognition.” *American Anthropologist*, n.s., 93, no. 1 (March): 9–27. [41]
- Sigfridsson, Emma, and Ulf Ryde. 1998. “Comparison of methods for deriving atomic charges from the electrostatic potential and moments.” *Journal of Computational Chemistry* 19 (4): 377–395. [https://doi.org/10.1002/\(SICI\)1096-987X\(199803\)19:4<377::AID-JCC1>3.0.CO;2-P](https://doi.org/10.1002/(SICI)1096-987X(199803)19:4<377::AID-JCC1>3.0.CO;2-P). [3, 4, 8, 13]
- Van Gennep, Arnold. 1909. *Les rites de passage*. Paris: Nourry. [15]
- Wilde, Oscar. 1899. *The Importance of Being Earnest: A Trivial Comedy for Serious People*. English and American drama of the Nineteenth Century. Leonard Smithers and Company. Google Books: [4HIWAAAAYAAJ](#). [39]

Appendix A Additional Proofs

Proof of Lemma 4. Since expected profit is linear in k_i , the firm chooses the maximum capacity level $k = 1$ when (3) is positive and the minimum capacity $k_i = 0$ when (3) is negative. Taking into account that in equilibrium the condition $B^{-1}(b_i^*) = c_i$ must hold for all i , the critical value \hat{c} for the indifferent bidder is obtained by solving (3) for c_i . ■

Appendix B Example of a Long Table That Spans More Than One Page

Table B.1. Parameter values used in the time experiment

Decision situation #	Base payment at earlier date b_t (€)	Base payment at later date b_{t+k} (€)	Curvature z	Front-end delay t (weeks)	Delay k (weeks)	Budget m (€)	Price ratio $p_{t+k} / p_t = 1 / R$
1	1.50	1.50	0.0	1	5	20.00	1.42857
2	1.50	1.50	0.0	1	5	17.50	1.25000
3	1.50	1.50	0.0	1	5	15.56	1.11111
4	1.50	1.50	0.0	1	5	14.70	1.05000
5	1.50	1.50	0.0	1	5	14.00	1.00000
6	1.50	1.50	0.0	1	5	14.00	0.95238
7	1.50	1.50	0.0	1	5	14.00	0.90000
8	1.50	1.50	0.0	1	5	14.00	0.80000
9	1.50	1.50	0.0	1	5	14.00	0.70000
10	1.50	1.50	0.0	1	5	25.00	1.25000
11	1.50	1.50	0.0	1	5	21.00	1.05000
12	1.50	1.50	0.0	1	5	20.00	1.00000
13	1.50	1.50	0.0	1	5	20.00	0.95238
14	1.50	1.50	0.0	1	5	20.00	0.80000
15	1.50	1.50	0.0	1	10	20.00	1.42857
16	1.50	1.50	0.0	1	10	17.50	1.25000
17	1.50	1.50	0.0	1	10	15.56	1.11111
18	1.50	1.50	0.0	1	10	14.70	1.05000
19	1.50	1.50	0.0	1	10	14.00	1.00000
20	1.50	1.50	0.0	1	10	14.00	0.95238
21	1.50	1.50	0.0	1	10	14.00	0.90000
22	1.50	1.50	0.0	1	10	14.00	0.80000
23	1.50	1.50	0.0	1	10	14.00	0.70000
24	1.50	1.50	0.0	1	10	25.00	1.25000
25	1.50	1.50	0.0	1	10	21.00	1.05000
26	1.50	1.50	0.0	1	10	20.00	1.00000
27	1.50	1.50	0.0	1	10	20.00	0.95238
28	1.50	1.50	0.0	1	10	20.00	0.80000
29	1.50	1.50	0.0	0	5	14.70	1.05000
30	1.50	1.50	0.0	0	5	14.00	0.95238
31	1.50	1.50	0.0	0	5	21.00	1.05000
32	1.50	1.50	0.0	0	5	20.00	0.95238
33	1.50	1.50	0.0	0	10	14.70	1.05000
34	1.50	1.50	0.0	0	10	14.00	0.95238

Continued on next page.

Table B.1 (continued)

Decision situation #	Base payment at earlier date b_t (€)	Base payment at later date b_{t+k} (€)	Curvature z	Front-end delay t (weeks)	Delay k (weeks)	Budget m (€)	Price ratio $p_{t+k} / p_t = 1 / R$
35	1.50	1.50	0.0	0	10	21.00	1.05000
36	1.50	1.50	0.0	0	10	20.00	0.95238
37	1.50	1.50	0.4	1	5	20.00	1.42857
38	1.50	1.50	0.4	1	5	17.50	1.25000
39	1.50	1.50	0.4	1	5	15.56	1.11111
40	1.50	1.50	0.4	1	5	14.70	1.05000
41	1.50	1.50	0.4	1	5	14.00	1.00000
42	1.50	1.50	0.4	1	5	14.00	0.95238
43	1.50	1.50	0.4	1	5	14.00	0.90000
44	1.50	1.50	0.4	1	5	14.00	0.80000
45	1.50	1.50	0.4	1	5	14.00	0.70000
46	1.50	1.50	0.4	1	5	25.00	1.25000
47	1.50	1.50	0.4	1	5	21.00	1.05000
48	1.50	1.50	0.4	1	5	20.00	1.00000
49	1.50	1.50	0.4	1	5	20.00	0.95238
50	1.50	1.50	0.4	1	5	20.00	0.80000
51	1.50	1.50	0.4	1	10	20.00	1.42857
52	1.50	1.50	0.4	1	10	17.50	1.25000
53	1.50	1.50	0.4	1	10	15.56	1.11111
54	1.50	1.50	0.4	1	10	14.70	1.05000
55	1.50	1.50	0.4	1	10	14.00	1.00000
56	1.50	1.50	0.4	1	10	14.00	0.95238
57	1.50	1.50	0.4	1	10	14.00	0.90000
58	1.50	1.50	0.4	1	10	14.00	0.80000
59	1.50	1.50	0.4	1	10	14.00	0.70000
60	1.50	1.50	0.4	1	10	25.00	1.25000
61	1.50	1.50	0.4	1	10	21.00	1.05000
62	1.50	1.50	0.4	1	10	20.00	1.00000
63	1.50	1.50	0.4	1	10	20.00	0.95238
64	1.50	1.50	0.4	1	10	20.00	0.80000
65	1.50	1.50	0.4	0	5	14.70	1.05000
66	1.50	1.50	0.4	0	5	14.00	0.95238
67	1.50	1.50	0.4	0	5	21.00	1.05000
68	1.50	1.50	0.4	0	5	20.00	0.95238
69	1.50	1.50	0.4	0	10	14.70	1.05000
70	1.50	1.50	0.4	0	10	14.00	0.95238
71	1.50	1.50	0.4	0	10	21.00	1.05000
72	1.50	1.50	0.4	0	10	20.00	0.95238

Continued on next page.

Table B.1 (continued)

Decision situation #	Base payment at earlier date b_t (€)	Base payment at later date b_{t+k} (€)	Curvature z	Front-end delay t (weeks)	Delay k (weeks)	Budget m (€)	Price ratio $p_{t+k} / p_t = 1 / R$
73	1.50	1.50	0.0	1	10	150.00	1.25000
74	1.50	1.50	0.0	1	10	126.00	1.05000
75	1.50	1.50	0.0	1	10	120.00	1.00000
76	1.50	1.50	0.0	1	10	120.00	0.95238
77	1.50	1.50	0.0	1	10	120.00	0.80000
78	1.50	1.50	0.4	1	10	150.00	1.25000
79	1.50	1.50	0.4	1	10	126.00	1.05000
80	1.50	1.50	0.4	1	10	120.00	1.00000
81	1.50	1.50	0.4	1	10	120.00	0.95238
82	1.50	1.50	0.4	1	10	120.00	0.80000

Note: Taken from https://www.econtribute.de/RePEc/ajk/ajkdps/ECONtribute_336_2024.pdf.

Table B.2. An additional table to check the table counter increment and the vertical spacing

Test	Test 1	Test 12	Test 123	Test 1234	Test 12345	Test 123456	Test 1234567
------	--------	---------	----------	-----------	------------	-------------	--------------

Appendix C An Example Code Listing

Taken from https://holgergerhardt.github.io/scbr/2024-10-13_estimation-illustration.R:

```
1 # This file illustrates the most important parts of the estimation of
2 # preference parameters performed by Holger Gerhardt & Rafael Suchy (2024),
3 # "Estimating Preference Parameters from Strictly Concave Budget Restrictions,"
4 # https://www.econtribute.de/RePEc/ajk/ajkdps/ECONtribute_336_2024.pdf.
5
6 # Version: 2024-10-13
7 # The results reported in this file were produced using R version 4.4.1
8 # (https://cloud.r-project.org/bin/macosx/big-sur-arm64/base/R-4.4.1-arm64.pkg)
9 # and the up-to-date versions of the packages mentioned below.
10
11
12
13
14 # PREAMBLE -----
15
16
17 # Clean up environment
18 rm(list = ls())
19
20 options(max.print = 9999) # Increase limit for omitting entries in output
21 options(scipen = 999) # Increase limit of using scientific notation
22 round_prec <- 4 # Number of decimal places for reporting the parameter estimates
23
24 # Required packages
25 packages_required <- c(
26   "AER", # For "tobit()" function
27   "cli", # For colored error and warning messages
28   "msm", # For "deltamethod()" function
29   "maxLik" # For maximum likelihood estimation
30 )
31 # Install packages that are not installed yet
32 install.packages(setdiff(packages_required, rownames(installed.packages())))
33 # For updating already installed packages, use the following
```

```

34 # install.packages(packages_required)
35 # Load the required packages
36 for (name in packages_required) {
37   library(name, character.only = TRUE)
38 }
39
40 # Alternatively, use groundhog:
41 # install.packages("groundhog")
42 # library("groundhog")
43 # groundhog.library(packages_required, "2024-10-10")
44
45
46
47
48 # BUDGET RESTRICTIONS -----
49
50
51 # General functional form, based on Equation (9) from
52 # https://www.econtribute.de/RePEc/ajk/ajkdps/ECONtribute_336_2024.pdf:
53 #  $c_t^{1+z} + (1/R)^{1+z} c_{t+1}^{1+z} = m^{1+z}$ .
54 # Thus, linear budget restrictions (LBRs) for  $z = 0$ , and
55 # strictly concave budget restrictions (SCBRs) for  $z > 0$ .
56 c_2 <- function(c_1, m, R, z, b_1, b_2) {
57   R * (m^(1+z) - c_1^(1+z))^(1/(1+z))
58 }
59
60 # Experimental parameters
61 # (see Table E.1 in https://www.econtribute.de/RePEc/ajk/ajkdps/ECONtribute_336_2024.pdf)
62 # The presentation of the different budget restrictions in the experiment by Gerhardt & Suchy
63 # was randomized as described in the manuscript. We abstract from this randomization here.
64 budget_restrictions <- matrix(c(
65   c(01, 1.50, 1.50, 0.0, 1, 05, 020.00, 0.70000, 1.42857, 1, 1, 0.01, 02.5, 025),
66   c(02, 1.50, 1.50, 0.0, 1, 05, 017.50, 0.80000, 1.25000, 1, 1, 0.01, 02.5, 025),
67   c(03, 1.50, 1.50, 0.0, 1, 05, 015.56, 0.90000, 1.11111, 1, 1, 0.01, 02.5, 025),
68   c(04, 1.50, 1.50, 0.0, 1, 05, 014.70, 0.95238, 1.05000, 1, 1, 0.01, 02.5, 025),
69   c(05, 1.50, 1.50, 0.0, 1, 05, 014.00, 1.00000, 1.00000, 1, 1, 0.01, 02.5, 025),
70   c(06, 1.50, 1.50, 0.0, 1, 05, 014.00, 1.05000, 0.95238, 1, 1, 0.01, 02.5, 025),
71   c(07, 1.50, 1.50, 0.0, 1, 05, 014.00, 1.11111, 0.90000, 1, 1, 0.01, 02.5, 025),
72   c(08, 1.50, 1.50, 0.0, 1, 05, 014.00, 1.25000, 0.80000, 1, 1, 0.01, 02.5, 025),
73   c(09, 1.50, 1.50, 0.0, 1, 05, 014.00, 1.42857, 0.70000, 1, 1, 0.01, 02.5, 025),
74   c(10, 1.50, 1.50, 0.0, 1, 05, 025.00, 0.80000, 1.25000, 1, 1, 0.01, 02.5, 025),
75   c(11, 1.50, 1.50, 0.0, 1, 05, 021.00, 0.95238, 1.05000, 1, 1, 0.01, 02.5, 025),
76   c(12, 1.50, 1.50, 0.0, 1, 05, 020.00, 1.00000, 1.00000, 1, 1, 0.01, 02.5, 025),
77   c(13, 1.50, 1.50, 0.0, 1, 05, 020.00, 1.05000, 0.95238, 1, 1, 0.01, 02.5, 025),
78   c(14, 1.50, 1.50, 0.0, 1, 05, 020.00, 1.25000, 0.80000, 1, 1, 0.01, 02.5, 025),
79   c(15, 1.50, 1.50, 0.0, 1, 10, 020.00, 0.70000, 1.42857, 1, 1, 0.01, 02.5, 025),
80   c(16, 1.50, 1.50, 0.0, 1, 10, 017.50, 0.80000, 1.25000, 1, 1, 0.01, 02.5, 025),
81   c(17, 1.50, 1.50, 0.0, 1, 10, 015.56, 0.90000, 1.11111, 1, 1, 0.01, 02.5, 025),
82   c(18, 1.50, 1.50, 0.0, 1, 10, 014.70, 0.95238, 1.05000, 1, 1, 0.01, 02.5, 025),
83   c(19, 1.50, 1.50, 0.0, 1, 10, 014.00, 1.00000, 1.00000, 1, 1, 0.01, 02.5, 025),
84   c(20, 1.50, 1.50, 0.0, 1, 10, 014.00, 1.05000, 0.95238, 1, 1, 0.01, 02.5, 025),
85   c(21, 1.50, 1.50, 0.0, 1, 10, 014.00, 1.11111, 0.90000, 1, 1, 0.01, 02.5, 025),
86   c(22, 1.50, 1.50, 0.0, 1, 10, 014.00, 1.25000, 0.80000, 1, 1, 0.01, 02.5, 025),
87   c(23, 1.50, 1.50, 0.0, 1, 10, 014.00, 1.42857, 0.70000, 1, 1, 0.01, 02.5, 025),
88   c(24, 1.50, 1.50, 0.0, 1, 10, 025.00, 0.80000, 1.25000, 1, 1, 0.01, 02.5, 025),
89   c(25, 1.50, 1.50, 0.0, 1, 10, 021.00, 0.95238, 1.05000, 1, 1, 0.01, 02.5, 025),
90   c(26, 1.50, 1.50, 0.0, 1, 10, 020.00, 1.00000, 1.00000, 1, 1, 0.01, 02.5, 025),
91   c(27, 1.50, 1.50, 0.0, 1, 10, 020.00, 1.05000, 0.95238, 1, 1, 0.01, 02.5, 025),
92   c(28, 1.50, 1.50, 0.0, 1, 10, 020.00, 1.25000, 0.80000, 1, 1, 0.01, 02.5, 025),
93   c(29, 1.50, 1.50, 0.0, 0, 05, 014.70, 0.95238, 1.05000, 1, 1, 0.01, 02.5, 025),
94   c(30, 1.50, 1.50, 0.0, 0, 05, 014.00, 1.05000, 0.95238, 1, 1, 0.01, 02.5, 025),
95   c(31, 1.50, 1.50, 0.0, 0, 05, 021.00, 0.95238, 1.05000, 1, 1, 0.01, 02.5, 025),
96   c(32, 1.50, 1.50, 0.0, 0, 05, 020.00, 1.05000, 0.95238, 1, 1, 0.01, 02.5, 025),
97   c(33, 1.50, 1.50, 0.0, 0, 10, 014.70, 0.95238, 1.05000, 1, 1, 0.01, 02.5, 025),
98   c(34, 1.50, 1.50, 0.0, 0, 10, 014.00, 1.05000, 0.95238, 1, 1, 0.01, 02.5, 025),
99   c(35, 1.50, 1.50, 0.0, 0, 10, 021.00, 0.95238, 1.05000, 1, 1, 0.01, 02.5, 025),
100  c(36, 1.50, 1.50, 0.0, 0, 10, 020.00, 1.05000, 0.95238, 1, 1, 0.01, 02.5, 025),
101  c(37, 1.50, 1.50, 0.4, 1, 05, 020.00, 0.70000, 1.42857, 1, 2, 0.01, 02.5, 025),
102  c(38, 1.50, 1.50, 0.4, 1, 05, 017.50, 0.80000, 1.25000, 1, 2, 0.01, 02.5, 025),
103  c(39, 1.50, 1.50, 0.4, 1, 05, 015.56, 0.90000, 1.11111, 1, 2, 0.01, 02.5, 025),
104  c(40, 1.50, 1.50, 0.4, 1, 05, 014.70, 0.95238, 1.05000, 1, 2, 0.01, 02.5, 025),
105  c(41, 1.50, 1.50, 0.4, 1, 05, 014.00, 1.00000, 1.00000, 1, 2, 0.01, 02.5, 025),
106  c(42, 1.50, 1.50, 0.4, 1, 05, 014.00, 1.05000, 0.95238, 1, 2, 0.01, 02.5, 025),
107  c(43, 1.50, 1.50, 0.4, 1, 05, 014.00, 1.11111, 0.90000, 1, 2, 0.01, 02.5, 025),
108  c(44, 1.50, 1.50, 0.4, 1, 05, 014.00, 1.25000, 0.80000, 1, 2, 0.01, 02.5, 025),
109  c(45, 1.50, 1.50, 0.4, 1, 05, 014.00, 1.42857, 0.70000, 1, 2, 0.01, 02.5, 025),
110  c(46, 1.50, 1.50, 0.4, 1, 05, 025.00, 0.80000, 1.25000, 1, 2, 0.01, 02.5, 025),
111  c(47, 1.50, 1.50, 0.4, 1, 05, 021.00, 0.95238, 1.05000, 1, 2, 0.01, 02.5, 025),
112  c(48, 1.50, 1.50, 0.4, 1, 05, 020.00, 1.00000, 1.00000, 1, 2, 0.01, 02.5, 025),
113  c(49, 1.50, 1.50, 0.4, 1, 05, 020.00, 1.05000, 0.95238, 1, 2, 0.01, 02.5, 025),
114  c(50, 1.50, 1.50, 0.4, 1, 05, 020.00, 1.25000, 0.80000, 1, 2, 0.01, 02.5, 025),
115  c(51, 1.50, 1.50, 0.4, 1, 10, 020.00, 0.70000, 1.42857, 1, 2, 0.01, 02.5, 025),
116  c(52, 1.50, 1.50, 0.4, 1, 10, 017.50, 0.80000, 1.25000, 1, 2, 0.01, 02.5, 025),
117  c(53, 1.50, 1.50, 0.4, 1, 10, 015.56, 0.90000, 1.11111, 1, 2, 0.01, 02.5, 025),
118  c(54, 1.50, 1.50, 0.4, 1, 10, 014.70, 0.95238, 1.05000, 1, 2, 0.01, 02.5, 025),
119  c(55, 1.50, 1.50, 0.4, 1, 10, 014.00, 1.00000, 1.00000, 1, 2, 0.01, 02.5, 025),
120  c(56, 1.50, 1.50, 0.4, 1, 10, 014.00, 1.05000, 0.95238, 1, 2, 0.01, 02.5, 025),
121  c(57, 1.50, 1.50, 0.4, 1, 10, 014.00, 1.11111, 0.90000, 1, 2, 0.01, 02.5, 025),
122  c(58, 1.50, 1.50, 0.4, 1, 10, 014.00, 1.25000, 0.80000, 1, 2, 0.01, 02.5, 025),
123  c(59, 1.50, 1.50, 0.4, 1, 10, 014.00, 1.42857, 0.70000, 1, 2, 0.01, 02.5, 025),
124  c(60, 1.50, 1.50, 0.4, 1, 10, 025.00, 0.80000, 1.25000, 1, 2, 0.01, 02.5, 025),
125  c(61, 1.50, 1.50, 0.4, 1, 10, 021.00, 0.95238, 1.05000, 1, 2, 0.01, 02.5, 025),
126  c(62, 1.50, 1.50, 0.4, 1, 10, 020.00, 1.00000, 1.00000, 1, 2, 0.01, 02.5, 025),
127  c(63, 1.50, 1.50, 0.4, 1, 10, 020.00, 1.05000, 0.95238, 1, 2, 0.01, 02.5, 025),
128  c(64, 1.50, 1.50, 0.4, 1, 10, 020.00, 1.25000, 0.80000, 1, 2, 0.01, 02.5, 025),

```

```

129 c(65, 1.50, 1.50, 0.4, 0, 05, 014.70, 0.95238, 1.05000, 1, 2, 0.01, 02.5, 025),
130 c(66, 1.50, 1.50, 0.4, 0, 05, 014.00, 1.05000, 0.95238, 1, 2, 0.01, 02.5, 025),
131 c(67, 1.50, 1.50, 0.4, 0, 05, 021.00, 0.95238, 1.05000, 1, 2, 0.01, 02.5, 025),
132 c(68, 1.50, 1.50, 0.4, 0, 05, 020.00, 1.05000, 0.95238, 1, 2, 0.01, 02.5, 025),
133 c(69, 1.50, 1.50, 0.4, 0, 10, 014.70, 0.95238, 1.05000, 1, 2, 0.01, 02.5, 025),
134 c(70, 1.50, 1.50, 0.4, 0, 10, 014.00, 1.05000, 0.95238, 1, 2, 0.01, 02.5, 025),
135 c(71, 1.50, 1.50, 0.4, 0, 10, 021.00, 0.95238, 1.05000, 1, 2, 0.01, 02.5, 025),
136 c(72, 1.50, 1.50, 0.4, 0, 10, 020.00, 1.05000, 0.95238, 1, 2, 0.01, 02.5, 025),
137 c(73, 1.50, 1.50, 0.0, 1, 10, 150.00, 0.80000, 1.25000, 1, 3, 0.10, 10.0, 160),
138 c(74, 1.50, 1.50, 0.0, 1, 10, 126.00, 0.95238, 1.05000, 1, 3, 0.10, 10.0, 160),
139 c(75, 1.50, 1.50, 0.0, 1, 10, 120.00, 1.00000, 1.00000, 1, 3, 0.10, 10.0, 160),
140 c(76, 1.50, 1.50, 0.0, 1, 10, 120.00, 1.05000, 0.95238, 1, 3, 0.10, 10.0, 160),
141 c(77, 1.50, 1.50, 0.0, 1, 10, 120.00, 1.25000, 0.80000, 1, 3, 0.10, 10.0, 160),
142 c(78, 1.50, 1.50, 0.4, 1, 10, 150.00, 0.80000, 1.25000, 1, 4, 0.10, 10.0, 160),
143 c(79, 1.50, 1.50, 0.4, 1, 10, 126.00, 0.95238, 1.05000, 1, 4, 0.10, 10.0, 160),
144 c(80, 1.50, 1.50, 0.4, 1, 10, 120.00, 1.00000, 1.00000, 1, 4, 0.10, 10.0, 160),
145 c(81, 1.50, 1.50, 0.4, 1, 10, 120.00, 1.05000, 0.95238, 1, 4, 0.10, 10.0, 160),
146 c(82, 1.50, 1.50, 0.4, 1, 10, 120.00, 1.25000, 0.80000, 1, 4, 0.10, 10.0, 160)
147 ), nrow = 82, byrow = TRUE)
148 colnames(budget_restrictions) <- c(
149   "DecNum", "BasePayOne", "BasePayTwo", "Curvature", "FED", "Delay", "Budget",
150   "PriceOneDivPriceTwo", "PriceTwoDivPriceOne", "ProbOne", "Block", "StepSize",
151   "TickDist", "PlotMax"
152 )
153
154 # Generate data frame so that we can include all possible allocations that could be selected
155 # from the different budget restrictions
156 budget_restrictions_df <- as.data.frame(budget_restrictions)
157 # Add necessary columns, initialized with NA
158 budget_restrictions_df$c_1_series <- list(NA)
159 budget_restrictions_df$c_2_series <- list(NA)
160 budget_restrictions_df$c_series <- list(NA)
161 budget_restrictions_df$lb <- NA
162 budget_restrictions_df$ub <- NA
163
164 # In the actual experiment, the sooner payment was displayed on the vertical axis, and the later
165 # payment was displayed on the horizontal axis. For convenience, we display c_1 on the horizontal
166 # and c_2 on the vertical axis here.
167
168 # Populate data frame
169 for (i in 1:dim(budget_restrictions_df)[1]) {
170   c_1_series <- seq(
171     0, budget_restrictions_df$Budget[i], budget_restrictions_df$StepSize[i]
172   )
173   c_2_series <- round(c_2(
174     c_1_series,
175     budget_restrictions_df$Budget[i],
176     budget_restrictions_df$PriceOneDivPriceTwo[i],
177     budget_restrictions_df$Curvature[i],
178     budget_restrictions_df$BasePayOne[i],
179     budget_restrictions_df$BasePayTwo[i]
180   ), 2) # Rounding to 2 decimal places because we displayed amounts with a precision of €0.01
181   # Keep only c_1-c_2 pairs for c_2 >= baseline payment
182   c_1_series <- c_1_series[c_2_series >= budget_restrictions_df$BasePayTwo[i]]
183   c_2_series <- c_2_series[1:length(c_1_series)]
184   # Keep only c_1-c_2 pairs for c_1 >= baseline payment
185   c_2_series <- c_2_series[c_1_series >= budget_restrictions_df$BasePayOne[i]]
186   c_1_series <- c_1_series[c_1_series >= budget_restrictions_df$BasePayOne[i]]
187   # Add horizontal/vertical segment at the position of the baseline payments
188   c_1_series <- c(0, c_1_series, max(c_1_series))
189   c_2_series <- c(max(c_2_series), c_2_series, 0)
190   budget_restrictions_df[[i, "c_1_series"]] <- as.list(c_1_series)
191   budget_restrictions_df[[i, "c_2_series"]] <- as.list(c_2_series)
192   budget_restrictions_df[[i, "C_series"]] <- as.list(c_1_series / c_2_series)
193   budget_restrictions_df[[i, "lb"]] <- budget_restrictions_df[[i, "BasePayTwo"]] / max(c_2_series)
194   budget_restrictions_df[[i, "ub"]] <- max(c_1_series) / budget_restrictions_df[[i, "BasePayTwo"]]
195 }
196 rm(c_1_series, c_2_series, i)
197
198
199
200
201 # PLOT BUDGET RESTRICTIONS -----
202
203
204 plotBRs <- function(df, curv) {
205   df_aux <- df[df$Curvature == curv, ]
206   plot_max <- max(df_aux[, "PlotMax"])
207   plot(
208     NaN, NaN,
209     xlim = c(0, plot_max),
210     ylim = c(0, plot_max),
211     axes = FALSE,
212     xlab = "", ylab = "",
213     asp = 1,
214     main = bquote("Budget restrictions with curvature" ~ italic(z) ~ "=" ~ .(curv))
215   )
216   axis(1, seq(0, plot_max, 20), col = "gray25", col.axis = "gray25", pos = 0)
217   text(x = plot_max / 2, y = -22.5, bquote(italic(c)[italic(t)]), xpd = TRUE)
218   axis(2, seq(0, plot_max, 20), col = "gray25", col.axis = "gray25", pos = 0)
219   text(x = -22.5, y = plot_max / 2, bquote(italic(c)[italic(t) + italic(k)]), xpd = TRUE, srt = 90)
220   for (i in 1:dim(df_aux)[1]) {
221     lines(unlist(df_aux[[i, "c_1_series"]]), unlist(df_aux[[i, "c_2_series"]]), col = "navy")
222   }
223 }

```

```

224
225 for (curv in unique(budget_restritions_df$Curvature)) {
226   plotBRs(budget_restritions_df, curv)
227 }
228 rm(curv)
229
230
231
232
233 # SIMULATE CHOICES -----
234
235
236 # For replicability of the results, use particular seed for the pseudorandom draws
237 set.seed(42)
238 # For new draws, remove the fixed seed via the following line
239 # set.seed(NULL)
240
241 # Optimal payment ratio:
242 # Equation (25) in https://www.econtribute.de/RePEc/ajk/ajkdps/ECONtribute_336_2024.pdf
243 C_star <- function(t, k, R, z, beta, delta, rho) {
244   (1 / (beta*(t == 0) * delta*k * R*(1 + z)))^(1 / (rho + z))
245 }
246
247 # Add noise with Gaussian distribution:
248 # additive normally distributed noise on log(C_star) => multiplicative log-normal noise on C_star
249 C_star_noisy <- function(t, k, R, z, beta, delta, rho, sigma) {
250   C_star(t, k, R, z, beta, delta, rho) * exp(rnorm(length(t), mean = 0, sd = sigma))
251 }
252
253 # Determine optimal (noisy) points on all BRs
254 opt_noisy_points_on_BR <- function(df_ind, beta, delta, rho, sigma) {
255   ones <- rep(1, dim(df_ind)[1])
256   beta_vec <- beta * ones
257   delta_vec <- delta * ones
258   rho_vec <- rho * ones
259   sigma_vec <- sigma * ones
260   # If rho is so small (negative) that the curvature of the utility function exceeds the curvature
261   # of the budget restriction, replace rho by a value that is ever so slightly larger than
262   # the curvature of the budget restriction. This way, the condition for an interior allocation
263   # can still be applied, ideally leading to finite values (instead of NaNs), which can then be
264   # converted to corner allocations.
265   rho_vec[rho_vec <= -df_ind$Curvature] <-
266     -df_ind$Curvature[rho_vec <= -df_ind$Curvature] + 0.000001
267   C_star_noisy_list <- C_star_noisy(
268     df_ind$FED,
269     df_ind$Delay,
270     df_ind$PriceOneDivPriceTwo,
271     df_ind$Curvature,
272     beta_vec, delta_vec, rho_vec, sigma_vec
273 )
274 # Initialize vectors with NA
275 opt_noisy_C <- opt_noisy_C_idx <- opt_noisy_c_1 <- opt_noisy_c_2 <-
276   rep(NA, length(C_star_noisy_list))
277 # Populate vectors by iterating of the budget restrictions
278 for (i in 1:length(C_star_noisy_list)) {
279   # Find the point among the discrete points on the current budget restriction that is
280   # closest to the (continuous) theoretical prediction
281   dist <- abs(C_star_noisy_list[i] - unlist(df_ind[i, "C_series"]))
282   # Use this allocation as the simulated choice
283   opt_noisy_C_idx[i] <- which.min(dist)
284   opt_noisy_C[i] <- unlist(df_ind[i, "C_series"])[opt_noisy_C_idx[i]]
285   opt_noisy_c_1[i] <- unlist(df_ind[i, "c_1_series"])[opt_noisy_C_idx[i]]
286   opt_noisy_c_2[i] <- unlist(df_ind[i, "c_2_series"])[opt_noisy_C_idx[i]]
287   # Convert predicted allocation beyond the baseline payments to corner allocation
288   max_C <- unlist(df_ind[i, "C_series"])[length(unlist(df_ind[i, "C_series"])) - 1]
289   min_C <- unlist(df_ind[i, "C_series"])[2]
290   if (opt_noisy_C[i] > max_C || is.nan(min(dist))) {
291     opt_noisy_C[i] <- df_ind[[i, "ub"]]
292     opt_noisy_c_1[i] <- max(unlist(df_ind[[i, "c_1_series"]]))
293     opt_noisy_c_2[i] <- df_ind[[i, "BasePayTwo"]]
294   }
295   if (opt_noisy_C[i] < min_C) {
296     opt_noisy_C[i] <- df_ind[[i, "lb"]]
297     opt_noisy_c_1[i] <- df_ind[[i, "BasePayOne"]]
298     opt_noisy_c_2[i] <- max(unlist(df_ind[[i, "c_2_series"]]))
299   }
300 }
301 rm(dist)
302 return(c(
303   "C" = list(opt_noisy_C),
304   "c_1" = list(opt_noisy_c_1),
305   "c_2" = list(opt_noisy_c_2)
306 ))
307 }
308
309 # Simulate participants with the following preference and noise parameters
310 # id, beta, delta, rho, sigma
311 params_sim <- matrix(c(
312   1, 1.00, 1.00, 0.000, 0.00, # Will be hard to estimate with z = 0 but easy with z = 0.4
313   2, 1.00, 0.99, 0.000, 0.25, # Will be hard to estimate with z = 0 but easy with z = 0.4
314   3, 0.80, 0.99, 0.000, 0.25, # Will be hard to estimate with z = 0 but easy with z = 0.4
315   4, 0.80, 0.99, 0.100, 0.50, # Should be estimable with both z = 0 and z = 0.4
316   5, 0.85, 0.95, 0.150, 0.50, # Should be estimable with both z = 0 and z = 0.4
317   6, 1.00, 1.00, 1.000, 0.00, # Should be estimable with both z = 0 and z = 0.4
318   7, 1.00, 0.99, 100.0, 0.25, # Should be estimable with both z = 0 and z = 0.4

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319 8, 1.00, 1.00, 200.0, 0.05, # Should be estimable with both z = 0 and z = 0.4
320 9, 1.00, 1.00, 200.0, 0.00 # May fail to converge (rho -> Inf), since c_1 = c_2 for all choices
321 ), ncol = 5, byrow = TRUE)
322 colnames(params_sim) <- c("id", "beta", "delta", "rho", "sigma")
323
324 ids <- unique(params_sim[, "id"])
325
326 # Create new empty data frame
327 df <- budget_restrictions_df[FALSE, ]
328
329 # Populate the data frame with the (noisy) choices of the simulated participants and
330 # plot the simulated choices
331 for (id in ids) {
332   df_ind <- budget_restrictions_df
333   # Add column with IDs
334   df_ind$id <- id
335   # Store simulated choices
336   points_sim <- opt_noisy_points_on_BR(
337     df_ind,
338     params_sim[params_sim[, "id"] == id, "beta"],
339     params_sim[params_sim[, "id"] == id, "delta"],
340     params_sim[params_sim[, "id"] == id, "rho"],
341     params_sim[params_sim[, "id"] == id, "sigma"]
342   )
343   df_ind$payment_1 <- unlist(points_sim["c_1"])
344   df_ind$payment_2 <- unlist(points_sim["c_2"])
345   df_ind$payment_ratio <- unlist(points_sim["C"])
346   # Append to the existing data frame
347   df <- rbind(df, df_ind)
348   # Generate a plot per level of curvature of the budget restrictions
349   for (curv in unique(df_ind$Curvature)) {
350     plotBRs(df_ind, curv)
351     points(
352       x = unlist(points_sim["c_1"])[df_ind$Curvature == curv],
353       y = unlist(points_sim["c_2"])[df_ind$Curvature == curv],
354       col = "navy",
355       pch = 20,
356     )
357     mtext(bquote(
358       "Simulated choices for ID" ~ .(id) * ":" ~
359       italic(β) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "beta"]) * ", " ~
360       italic(δ) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "delta"]) * ", " ~
361       italic(ρ) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "rho"]) * ", " ~
362       italic(σ) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "sigma"])
363     ), side = 3, col = "navy", line = 0.25)
364     Sys.sleep(0.5)
365   }
366 }
367 rm(curv, df_ind, id)
368 rm(points_sim)
369
370
371
372
373 # TOBIT ESTIMATION -----
374
375
376 # Generating the explanatory variables, see
377 # https://www.econtribute.de/RePEc/ajk/ajkdps/ECONtribute_336_2024.pdf, eq. (26) on p. 15:
378 df$cov_1 <-
379   -as.integer(df$FED == 0) # -I[t = 0], coefficient: gamma_beta
380 df$cov_2 <-
381   -df$Delay # -k, coefficient: gamma_delta
382 df$cov_3 <-
383   -(1 + df$Curvature) * log(df$PriceOneDivPriceTwo) # -(1 + z) ln(R), coefficient: gamma_rho
384
385 ids <- unique(df$id)
386 tobit_estimates_report <- list()
387
388 # Estimate separately for each curvature level of the budget restrictions
389 for (curv in unique(df$Curvature)) {
390   # Initialize collection of estimates with NA
391   tobit_estimates <- matrix(rep(NA, length(ids) * 5), ncol = 5)
392   tobit_estimates <- cbind(ids, tobit_estimates)
393   colnames(tobit_estimates) <- c("id", "beta", "delta", "rho", "sigma", "logL")
394   # Estimate each individual separately
395   for (id in ids) {
396     print(paste0("Tobit estimation, Subject ID: ", id, "; BR curvature: ", curv))
397     df_ind <- df[df$id == id & df$Curvature == curv, ]
398     # Plot simulated choices
399     plotBRs(df_ind, curv)
400     points(
401       df_ind$payment_1,
402       df_ind$payment_2,
403       col = "navy",
404       pch = 20
405     )
406     mtext(bquote(
407       "Simulated choices for ID" ~ .(id) * ":" ~
408       italic(β) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "beta"]) * ", " ~
409       italic(δ) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "delta"]) * ", " ~
410       italic(ρ) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "rho"]) * ", " ~
411       italic(σ) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "sigma"])
412     ), side = 3, col = "navy", line = 0.25)
413     tryCatch(

```

```

414 {
415   withCallingHandlers(
416     # Attempt Tobit regression
417     {
418       model <- tobit(
419         log(payment_ratio) ~ cov_1 + cov_2 + cov_3 - 1,
420         # "+" here because we included the minus sign above when creating the regressors
421         left = log(df_ind$lb),
422         right = log(df_ind$sub),
423         data = df_ind
424       )
425     },
426     # Show potential warning messages of the Tobit regression
427     warning = function(w) {
428       message(style_bold(col_blue("WARNING: ", conditionMessage(w))))
429       invokeRestart("muffleWarning")
430     }
431   )
432   vcov_matrix <- vcov(model)
433   coeffs <- model$coefficients
434   est_beta <-
435     round(exp(as.numeric(coeffs[["cov_1"]]) / as.numeric(coeffs[["cov_3"]])), round_prec)
436   # beta = exp(gamma_beta / gamma_rho), eq. (27) in Gerhardt & Suchy (2024)
437   est_delta <-
438     round(exp(as.numeric(coeffs[["cov_2"]]) / as.numeric(coeffs[["cov_3"]])), round_prec)
439   # delta = exp(gamma_delta / gamma_rho), eq. (28)
440   est_rho <-
441     round(1 / as.numeric(coeffs[["cov_3"]]) - curv, round_prec)
442   # rho = (1 / gamma_rho) - z, eq. (29)
443   est_sigma <- round(model$scale, round_prec)
444   tobit_estimates[tobit_estimates[, "id"] == id, "beta"] <- est_beta
445   tobit_estimates[tobit_estimates[, "id"] == id, "delta"] <- est_delta
446   tobit_estimates[tobit_estimates[, "id"] == id, "rho"] <- est_rho
447   tobit_estimates[tobit_estimates[, "id"] == id, "sigma"] <- est_sigma
448   tobit_estimates[tobit_estimates[, "id"] == id, "logL"] <- round(logLik(model), round_prec)
449   # Use delta method to calculate standard errors of structural parameters
450   est_beta_se <- deltamethod(list(~ exp(x1 / x3)), coeffs, vcov(model)[1:3, 1:3])
451   est_delta_se <- deltamethod(list(~ exp(x2 / x3)), coeffs, vcov(model)[1:3, 1:3])
452   est_rho_se <- deltamethod(list(~ 1 / x3 - curv), coeffs, vcov(model)[1:3, 1:3])
453   # Add best-fitting allocations to plot (remove random component, i.e., sigma = 0)
454   points(
455     unlist(opt_noisy_points_on_BR(df_ind, est_beta, est_delta, est_rho, sigma = 0)[["c_1"]]),
456     unlist(opt_noisy_points_on_BR(df_ind, est_beta, est_delta, est_rho, sigma = 0)[["c_2"]]),
457     col = "#FFA500",
458     pch = 20
459   )
460   mtext(bquote(
461     "Parameter estimates:" ~
462     hat(italic(beta)) ~ "=" ~ .(est_beta) * ", " ~
463     hat(italic(delta)) ~ "=" ~ .(est_delta) * ", " ~
464     hat(italic(rho)) ~ "=" ~ .(est_rho) * ", " ~
465     hat(italic(sigma)) ~ "=" ~ .(est_sigma)
466   ), side = 3, col = "#FFA500", line = -1)
467 },
468 # If Tobit regression fails, issue error message
469 error = function(e) {
470   message(style_bold(bg_red(col_br_white("ERROR: ", conditionMessage(e))))
471 }
472 )
473 Sys.sleep(0.25) # Short break to update plot
474 }
475 # Collect and display Tobit estimates
476 tobit_estimates_report[[toString(curv)]] <- tobit_estimates
477 print(cbind(params_sim, tobit_estimates_report[[toString(curv)]][, 2:6]))
478 }
479 # Collect estimates
480 all_estimates_report <- tobit_estimates_report
481 rm(curv, id, df_ind, tobit_estimates)
482 rm(coeffs, vcov_matrix)
483 rm(est_beta, est_beta_se, est_delta, est_delta_se, est_rho, est_rho_se, est_sigma)
484
485
486
487
488 # NONLINEAR MAXIMUM LIKELIHOOD ESTIMATION -----
489
490
491 # Log-likelihood contribution of a single observation according to eq. (30) in
492 # https://www.econtribute.de/RePEc/ajk/ajkdps/ECONtribute_336_2024.pdf
493 LL_contrib <- function(C_star_obs, t, k, lb, ub, R, z, beta, delta, rho, sigma) {
494   C_star_pred <- C_star(t, k, R, z, beta, delta, rho)
495   if (
496     beta < 0 || delta < 0 || sigma < 0 || rho < -z
497     # Parameters must not become negative, and
498     # curvature of utility beyond curvature of BR cannot be identified
499   ) {
500     lnf <- -999999
501   } else {
502     # Interior solution
503     lnf <- log(dnorm((log(C_star_obs) - log(C_star_pred)) / sigma) / sigma)
504     # If observation is lower bound
505     if (C_star_obs < lb + 0.00001) {
506       lnf <- log(pnorm((log(lb) - log(C_star_pred)) / sigma))
507     }
508     # If observation is upper bound

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509   if (C_star_obs > ub - 0.00001) {
510     lnf <- log(pnorm((log(C_star_pred) - log(ub)) / sigma))
511   }
512 }
513 # Rule out NaNs and infinite values
514 if (is.na(lnf) || is.nan(lnf) || lnf == -Inf) {
515   lnf <- -999999
516 }
517 return(lnf)
518 }
519
520 # Log-likelihood contributions of all observations collected in vector
521 # (required by some optimization methods)
522 LL_contrib_vec <- function(C_star_obs, t, k, lb, ub, R, z, beta, delta, rho, sigma) {
523   ln <- unlist(sapply(
524     1:length(C_star_obs),
525     function(i) {
526       LL_contrib(
527         C_star_obs[i],
528         t[i], k[i], lb[i], ub[i], R[i], z[i],
529         beta, delta, rho, sigma
530       )
531     }
532   ))
533   return(as.vector(ln))
534 }
535
536 ids <- unique(df$id)
537 mle_estimates_report <- list()
538
539 # Estimate separately for each curvature level of the budget restrictions
540 for (curv in unique(df$Curvature)) {
541   # Initialize collection of estimates with NA
542   mle_estimates <- matrix(rep(NA, length(ids) * 5), ncol = 5)
543   mle_estimates <- cbind(ids, mle_estimates)
544   colnames(mle_estimates) <- c("id", "beta", "delta", "rho", "sigma", "logL")
545   # Estimate each individual separately
546   for (id in ids) {
547     print(paste0("NL-ML estimation, Subject ID: ", id, "; BR curvature: ", curv))
548     df_ind <- df[df$id == id & df$Curvature == curv, ]
549     # Objective function for maxLik must not contain any arguments except params
550     LL_contrib_vec_filled <- function(params) {
551       LL_contrib_vec(
552         df_ind$payment_ratio,
553         df_ind$FED, df_ind$Delay, df_ind$lb, df_ind$ub,
554         df_ind$PriceOneDivPriceTwo, df_ind$Curvature,
555         params[1], params[2], params[3], params[4]
556       )
557     }
558     # Set the initial values for the numerical nonlinear estimation procedure
559     # By default, take the outcome of the Tobit regression
560     tobit_for_init <- tobit_estimates_report[[toString(curv)]]
561     init_vals <- round(c(
562       tobit_for_init[tobit_for_init[, "id"] == id, "beta"],
563       tobit_for_init[tobit_for_init[, "id"] == id, "delta"],
564       tobit_for_init[tobit_for_init[, "id"] == id, "rho"],
565       tobit_for_init[tobit_for_init[, "id"] == id, "sigma"]
566     ), 2)
567     # If the Tobit regression yields nonsensical results, set different initial value
568     if (init_vals["beta"] <= 0 | is.na(init_vals["beta"])) {
569       init_vals["beta"] <- 0.95
570     }
571     if (init_vals["delta"] <= 0 | is.na(init_vals["delta"])) {
572       init_vals["delta"] <- 0.95
573     }
574     if (is.na(init_vals["rho"])) {
575       init_vals["rho"] <- 0.05
576     } else if (init_vals["rho"] <= -curv) {
577       init_vals["rho"] <- 1
578     }
579     if (init_vals["sigma"] <= 0 | is.na(init_vals["sigma"])) {
580       init_vals["sigma"] <- 0.1
581     }
582     # As is frequently the case with numerical optimization procedures, individual-specific
583     # initial values may be required, e.g., if Tobit did not converge. This is particularly likely
584     # with linear budget restrictions and less so with strictly concave budget restrictions.
585     if (id == 1 && curv == 0) {
586       init_vals[c("beta", "delta", "rho", "sigma")] = c(1, 1, 0.01, 0.005)
587     }
588     if (id %in% c(2, 3) && curv == 0) {
589       init_vals[c("beta", "delta", "rho", "sigma")] = c(0.995, 0.995, 0.01, 0.2)
590     }
591     # sum(LL_contrib_vec_filled(init_vals)) # Helpful for finding initial values
592     # Plot simulated choices
593     plotBRs(df_ind, curv)
594     points(
595       df_ind$payment_1,
596       df_ind$payment_2,
597       col = "navy",
598       pch = 20
599     )
600     mtext(bquote(
601       "Simulated choices for ID" ~ .(id) * ":" ~
602       italic(β) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "beta"]) * ", " ~
603       italic(δ) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "delta"]) * ", " ~

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604     italic(p) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "rho"]) * ", " ~
605     italic(o) ~ "=" ~ .(params_sim[params_sim[, "id"] == id, "sigma"])
606   ), side = 3, col = "navy", line = 0.25)
607   tryCatch(
608     {
609       withCallingHandlers(
610         # Attempt NL-MLE
611         {
612           mle_estim <- maxLik(
613             logLik = LL_contrib_vec_filled,
614             start = init_vals,
615             # method = "BFGS", # Broyden/Fletcher/Goldfarb/Shanno
616             # method = "BFGSR", # Broyden/Fletcher/Goldfarb/Shanno
617             # method = "BHHH", # Berndt/Hall/Hall/Hausman
618             method = "NR", # Newton/Raphson
619             # method = "SANN", # Simulated Annealing
620             control = list(
621               gradtol = 10^-8, # Return code 1 (normal convergence)
622               tol = 10^-8, # Return code 2 (normal convergence)
623               steptol = -1, # Return code 3
624               iterlim = 1000, # 10^6, # Return code 4
625               reltol = 10^-8 # Return code 8 (normal convergence)
626             )
627           )
628           est_se <- stdEr(mle_estim, eigentol = 10^(-15))
629         },
630         # Show potential warning messages of the NL-MLE
631         warning = function(w) {
632           message(style_bold(col_blue("WARNING: ", conditionMessage(w))))
633           invokeRestart("muffleWarning")
634         }
635       )
636       # Add best-fitting allocations to plot (remove random component, i.e., sigma = 0)
637       points(
638         unlist(opt_noisy_points_on_BR(
639           df_ind,
640           mle_estim$estimate["beta"], mle_estim$estimate["delta"], mle_estim$estimate["rho"],
641           sigma = 0
642         )["c_1"]),
643         unlist(opt_noisy_points_on_BR(
644           df_ind,
645           mle_estim$estimate["beta"], mle_estim$estimate["delta"], mle_estim$estimate["rho"],
646           sigma = 0
647         )["c_2"]),
648         col = "#FF4500",
649         pch = 20
650       )
651       mle_estim$estimate <- round(mle_estim$estimate, round_prec)
652       mtext(bquote(
653         "Parameter estimates:" ~
654         hat(italic(beta)) ~ "=" ~ .(mle_estim$estimate["beta"]) * ", " ~
655         hat(italic(delta)) ~ "=" ~ .(mle_estim$estimate["delta"]) * ", " ~
656         hat(italic(rho)) ~ "=" ~ .(mle_estim$estimate["rho"]) * ", " ~
657         hat(italic(sigma)) ~ "=" ~ .(mle_estim$estimate["sigma"])
658       ), side = 3, col = "#FF4500", line = -1)
659       mle_estimates[mle_estimates[, "id"] == id, "beta"] <- mle_estim$estimate["beta"]
660       mle_estimates[mle_estimates[, "id"] == id, "delta"] <- mle_estim$estimate["delta"]
661       mle_estimates[mle_estimates[, "id"] == id, "rho"] <- mle_estim$estimate["rho"]
662       mle_estimates[mle_estimates[, "id"] == id, "sigma"] <- mle_estim$estimate["sigma"]
663       mle_estimates[mle_estimates[, "id"] == id, "logL"] <- round(mle_estim$maximum, round_prec)
664     },
665     # If NL-MLE fails, issue error message
666     error = function(e) {
667       message(style_bold(bg_red(col_br_white("ERROR: ", conditionMessage(e))))
668     )
669   )
670   Sys.sleep(0.25) # Short break to update plot
671 }
672 # Collect NL-MLE estimates
673 mle_estimates_report[[toString(curv)]] <- mle_estimates
674 cols = c("beta", "delta", "rho", "sigma", "logL")
675 # Collect and display all estimates
676 all_estimates_report[[toString(curv)]] <- cbind(
677   params_sim,
678   tobit_estimates_report[[toString(curv)]][, cols],
679   mle_estimates_report[[toString(curv)]][, cols]
680 )
681 colnames(all_estimates_report[[toString(curv)]] <-
682   c(
683     "id",
684     "beta_sim", "delta_sim", "rho_sim", "sigma_sim",
685     "beta_tobit", "delta_tobit", "rho_tobit", "sigma_tobit", "logL_tobit",
686     "beta_mle", "delta_mle", "rho_mle", "sigma_mle", "logL_mle"
687   )
688   print(all_estimates_report[[toString(curv)]]
689 )
690 rm(curv, id, df_ind, mle_estimates, est_se, tobit_for_init)
691 all_estimates_report[["0"]][, 1:10]
692 # This should yield
693 #
694 #   id beta_sim delta_sim rho_sim sigma_sim beta_tobit delta_tobit rho_tobit sigma_tobit logL_tobit
695 # [1,] 1      1.00      1.00      0.00      0.00      NA           NA           NA           0.0006      32.1004
696 # [2,] 2      1.00      0.99      0.00      0.25      0.9557      0.9910      0.0000      155.4723     -9471.4181
697 # [3,] 3      0.80      0.99      0.00      0.25      0.9557      0.9910      0.0000      155.4723     -9471.4181
698 # [4,] 4      0.80      0.99      0.10      0.50      0.7998      0.9907      0.0961      0.3942      -15.9651

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699 # [5,] 5      0.85      0.95      0.15      0.50      0.8554      0.9514      0.1556      0.4335      -14.5728
700 # [6,] 6      1.00      1.00      1.00      0.00      1.0000      1.0000      0.9999      0.0001      324.9009
701 # [7,] 7      1.00      0.99      100.00      0.25      0.6308      0.9941      -3.9738      0.1959      8.6606
702 # [8,] 8      1.00      1.00      200.00      0.05      0.1828      1.0957      57.7050      0.0496      64.9979
703 # [9,] 9      1.00      1.00      200.00      0.00      0.9814      1.0025      196.4053      0.0007      240.3492
704 # With linear budget restrictions:
705 # IDs 1, 2, and 3: Tobit does not converge for linear utility.
706 # Tobit converges to strongly convex utility instead of strongly concave utility for ID 7.
707 # ID 8: Strongly concave utility is hard to estimate in the presence of noise.
708
709 all_estimates_report[["0"]][, c(1:5, 11:15)]
710 # This should yield
711 #      id beta_sim delta_sim rho_sim sigma_sim beta_mle delta_mle rho_mle sigma_mle logL_mle
712 # [1,] 1      1.00      1.00      0.00      0.00      1.0000      1.0000      0.0110      0.0003      34.8556
713 # [2,] 2      1.00      0.99      0.00      0.25      0.9905      0.9900      0.0001      0.1853      0.0000
714 # [3,] 3      0.80      0.99      0.00      0.25      0.9905      0.9900      0.0001      0.1853      0.0000
715 # [4,] 4      0.80      0.99      0.10      0.50      0.7998      0.9907      0.0961      0.3942     -15.9651
716 # [5,] 5      0.85      0.95      0.15      0.50      0.8554      0.9514      0.1556      0.4335     -14.5728
717 # [6,] 6      1.00      1.00      1.00      0.00      1.0000      1.0000      0.9999      0.0001      324.9009
718 # [7,] 7      1.00      0.99      100.00      0.25      555.2478      1.0991      57.9643      0.2008       7.6510
719 # [8,] 8      1.00      1.00      200.00      0.05      0.1745      1.0984      59.2789      0.0496      64.9978
720 # [9,] 9      1.00      1.00      200.00      0.00      0.9800      1.0000      196.4100      0.0049      179.5666
721 # With linear budget restrictions (LBRs):
722 # IDs 1, 2, and 3: Estimation becomes possible with NL-MLE by searching for suitable initial values,
723 # which, however, is cumbersome and difficult to automate.
724 # ID 7: Our NL-MLE routine converges to strongly concave utility, thereby fixing the weak point
725 # of Tobit. At the same time, with strongly concave utility all allocations are characterized by
726 # c_1 = c_2, with beta and delta hardly having any influence. Hence, identification of beta and
727 # delta becomes very hard.
728
729 all_estimates_report[["0.4"]][, 1:10]
730 # This should yield
731 #      id beta_sim delta_sim rho_sim sigma_sim beta_tobit delta_tobit rho_tobit sigma_tobit logL_tobit
732 # [1,] 1      1.00      1.00      0.00      0.00      1.0000      1.0000     -0.0002      0.0004      260.7333
733 # [2,] 2      1.00      0.99      0.00      0.25      0.9697      0.9903     -0.0093      0.2442     -0.3814
734 # [3,] 3      0.80      0.99      0.00      0.25      0.8085      0.9913      0.0063      0.2156       4.7404
735 # [4,] 4      0.80      0.99      0.10      0.50      0.8394      0.9889      0.0885      0.5054     -30.1979
736 # [5,] 5      0.85      0.95      0.15      0.50      0.9753      0.9509      0.0958      0.4442     -24.9066
737 # [6,] 6      1.00      1.00      1.00      0.00      0.9997      1.0000      0.9981      0.0004      258.3382
738 # [7,] 7      1.00      0.99      100.00      0.25      0.8767      0.9071      12.5038      0.2460     -0.6843
739 # [8,] 8      1.00      1.00      200.00      0.05      1.2020      1.0142      18.5775      0.0513      63.6310
740 # [9,] 9      1.00      1.00      200.00      0.00      0.9419      1.0080      196.3481      0.0006      248.9800
741 # With strictly concave budget restrictions (SCBRs):
742 # IDs 1, 2, and 3: Tobit converges for linear utility. Thus, SCBRs fix the weak point of LBRs.
743
744 all_estimates_report[["0.4"]][, c(1:5, 11:15)]
745 # This should yield
746 #      id beta_sim delta_sim rho_sim sigma_sim beta_mle delta_mle rho_mle sigma_mle logL_mle
747 # [1,] 1      1.00      1.00      0.00      0.00      1.0000      1.0000     -0.0002      0.0004      260.7333
748 # [2,] 2      1.00      0.99      0.00      0.25      0.9697      0.9903     -0.0093      0.2442     -0.3814
749 # [3,] 3      0.80      0.99      0.00      0.25      0.8085      0.9913      0.0063      0.2156       4.7404
750 # [4,] 4      0.80      0.99      0.10      0.50      0.8394      0.9889      0.0885      0.5054     -30.1979
751 # [5,] 5      0.85      0.95      0.15      0.50      0.9753      0.9509      0.0958      0.4442     -24.9066
752 # [6,] 6      1.00      1.00      1.00      0.00      0.9997      1.0000      0.9981      0.0004      258.3382
753 # [7,] 7      1.00      0.99      100.00      0.25      0.8767      0.9071      12.5038      0.2460     -0.6843
754 # [8,] 8      1.00      1.00      200.00      0.05      1.2019      1.0142      18.5773      0.0513      63.6310
755 # [9,] 9      1.00      1.00      200.00      0.00      0.9400      1.0100      196.3500      0.0047      181.4055
756 # This illustrates that with the chosen optimization method ("NR"), our NL-MLE routine delivers
757 # (virtually) the same results as Tobit.

```