4.2-1 解：$A_{11}=1$. $A_{12}=3$. $A_{21}=7$. $A_{22}=5$.

$B_{11}=6$. $B_{12}=8$ $B_{21}=4$ $B_{22}=2$.

$S_1 = B_{12}-B_{22} = 8-2 = 6$.

$S_2 = A_{11}+A_{12} = 1+3 = 4$.

$S_3 = A_{21}+A_{22} = 7+5 = 12$.

$S_4 = B_{21}-B_{11} = 4-6 = -2$.

$S_5 = A_{11}+A_{22} = 1+5 = 6$

$S_6 = B_{11}+B_{22} = 6+2 = 8$.

$S_7 = A_{12}-A_{22} = 3-5 = -2$.

$S_8 = B_{21}+B_{22} = 4+2 = 6$.

$S_9 = A_{11}-A_{21} = 1-7 = -6$

$S_{10} = B_{11}+B_{12} = 6+8 = 14$.

$P_1 = A_{11}S_1 = 1\times 6 = 6$.

$P_2 = S_2 B_{22} = 4\times 2 = 8$

$P_3 = S_3 B_{11} = 12\times 6 = 72$.

$P_4 = A_{22}S_4 = 5\times(-2) = -10$.

$P_5 = S_5 S_6 = 6\times 8 = 48$

$P_6 = S_7 S_8 = -2\times 6 = -12$

$P_7 = S_9 S_{10} = -6\times 14 = -84$.

$C_{11} = P_5+P_4-P_2+P_6 = 48+(-10)-8+(-12) = 18$

$C_{12} = P_1+P_2 = 6+8 = 14$

$C_{21} = P_3+P_4 = 72-10 = 62$.

$C_{22} = P_5+P_1-P_3-P_7 = 48+6-72-(-84) = 66$

故 $C = \begin{pmatrix} 18 & 14 \\ 62 & 66 \end{pmatrix}$.

4.2-2.

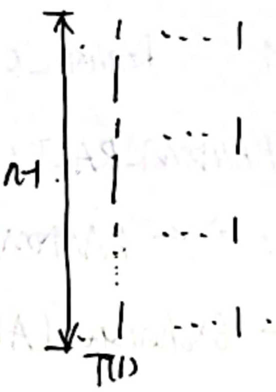
```
Strassen(A, B):
C ← n×n的矩阵.
if. A.row = 1:
    C = A × B.
else partition(A, B, C).
    S1 = B12 - B22
    S2 = A11 + A12.
    S3 = A21 + A22.
    S4 = B21 - B11
    S5 = A11 + A22.
    S6 = B11 + B22
    S7 = A12 - A22.
    S8 = B21 + B22.
    S9 = A11 - A21
    S10 = B11 + B12
    P1 = Strassen(A11, S1).
    P2 = Strassen(S2, B22).
    P3 = Strassen(S3, B11).
    P4 = Strassen(A22, S4).
    P5 = Strassen(S5, S6).
    P6 = Strassen(S7, S8).
    P7 = Strassen(S9, S10).
    C11 = P5 + P4 - P2 + P6.
    C12 = P1 + P2.
    C21 = P3 + P4
    C22 = P5 + P1 - P3 - P7.
return C.
```

4.3-6. 证明: $T(n) \le c(n-a)\lg(n-a)$.

$$T(n) = 2T(\tfrac{n}{2}+17)+n.$$
$$\le 2c(\tfrac{n}{2}+17-a)\lg(\tfrac{n}{2}+17-a)+n.$$
$$= c(n+34-2a)\lg\frac{n+34-2a}{2}+n.$$
$$= c(n+34-2a)\lg(n+34-2a)-c(n+34-2a)+n.$$
$$\le c(n+34-2a)\lg(n+34-2a)$$

只要 $2a-34 \ge a$ 即 $a \ge 34$ 即可.

4.4-4. 递归树如下: 树高 $n-1$ 层. 每层代价为 1. 整树代价为 $1+1+\cdots+\theta(1)=O(n)$



猜测 $T(n)=O(n)$, 即 $T(n) \le cn$

代入法求证: $T(n) \le cn$.

$$T(n) = T(n-1)+1$$
$$\le c(n-1)+1$$
$$= cn-c+1$$
$$\le cn.$$

只要 $c \ge 1$ 即可. 可令 $c=1$.

4.5-3. 证明: $T(n)=T(\frac{n}{2})+\theta(1)$.

$a=1$, $b=2$. 可得: $n^{\log_b a}=n^{\log_2 1}=n^0=1$.

$f(n)=\theta(1)=O(1)=O(n^{\log_b a})=O(1)$.

则 $T(n)=\theta(n^{\log_b a}\lg n)=\theta(\lg n)$ 证毕.

快排：

```
QUICK_SORT(A, p, r):
if p < r:
    q ← PARTITION(A, p, r)
    QUICK_SORT(A, p, q-1)
    QUICK_SORT(A, q+1, r)

PARTITION(A, p, r):
x ← A[r]  //主元
i ← p-1
for j ← p to r-1
    if A[j] ≤ x:
        i ← i+1
        exchange(A[i], A[j]).
exchange(A[i+1], A[r])
return i+1.
```

最坏情况 $O(n^2)$：输入有序.

平均情况 $O(n\lg n)$：平衡划分.

随机化快排：

```
RANDOM_QUICK_SORT(A, p, r)
if p<r:
    q ← RANDOM_PART(A, p, r).
    RANDOM_QUICK_SORT(A, p, q-1)
    RANDOM_QUICKSORT(A, q+1, r)

RANDOM_PART(A, p, r):
i ← RANDOM(p, r)
exchange(A[r], A[i])
return PARTITION(A, p, r)
```

期望时间复杂度 $O(n\lg n)$.