Exercises I.

1. Find a basis of the algebra

$$\langle 1, a, x, y \mid ax - xa = x,\ ay - ya = -y,$$
$$x^2 = y^2 = 0,\ xy + yx = 1 \rangle$$

2. Let V be a vector space over a field F. A mapping $N: V \to F$ is called a <u>quadratic form</u> if

1) $N(\alpha v) = \alpha^2 N(v)$ for all $\alpha \in F, v \in V$;

2) $N(v, w) = N(v+w) - N(v) - N(w)$ is a bilinear form.

The algebra $Cl(V, N) = \langle 1, V \mid v^2 = N(v) \cdot 1,\ \cancel{vw + wv = N(v,w)}\ v \in V \rangle$ is called the Clifford algebra of the form N. Find a basis of $Cl(V, N)$.

3. Find a normal form in the semigroup

$$\langle x, y \mid yx = 1 \rangle.$$

4. Prove that every associative algebra A has a presentation $A = \langle X \mid R = 0 \rangle$, such that the set R is closed with respect to compositions.

5. Let L be a Lie algebra. Let $U(L)$ be the universal enveloping algebra of L. Prove that if $a, b \in U(L)$ are nonzero elements then $ab \neq 0$.