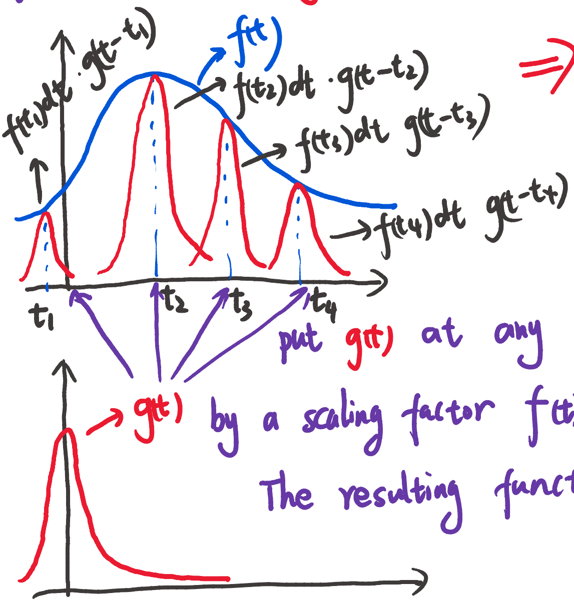
What is convolution? $F(t) = f(t) * g(t)$

Intuitively, convolution can be thought of as a "blending" or "mixing" operation between two functions, where one function is sliding over the other with scaling.



The idea is same as the convolution neural networks (CNNs).

put $g(t)$ at any position of $f(t)$ multiplied by a scaling factor $f(t)dt$, then add up them. The resulting function is $F(t) = f(t) * g(t)$

How to compute $F(t)$?

For any t_0, $F(t_0)$ is a value of adding up the contributions to t_0 from scaled $g(t)$ at all points of $f(t)$

any t

So, we have

$$F(t) = f(t) * g(t) = \int_{-\infty}^{+\infty} \underbrace{f(\tau)\, g(t-\tau)}_{} \, d\tau$$

This is the contribution (value) from the scaled $g(t)$ at the position τ of $f(t)$ to point t of $F(t)$.

Fortunately, we can easily prove $f(t) * g(t) = g(t) * f(t)$.

$$\int_{-\infty}^{+\infty} f(\tau) g(t-\tau) d\tau = \int_{+\infty}^{-\infty} f(t-\theta) g\theta \cdot d(-\theta) = \int_{-\infty}^{+\infty} f(t-\theta) g(\theta) d\theta$$

let $\theta = t-\tau$

$$= \int_{-\infty}^{+\infty} f(t-\tau) g(\tau) d\tau$$

For impulse function $\delta(t)$, we have

$$f(t) * \delta(t-t_0) = f(t-t_0)$$

性质

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各种卷积算子都满足下列性质：

令 $x(t) * h(t) = y(t)$，a 为任意常数或复常数，则卷积有如下性质：

性质名称	函数的卷积积分	序列的卷积和
交换律	$x(t) * h(t) = h(t) * x(t)$	$x[k] * h[k] = h[k] * x[k]$
分配律	$x(t) * [g(t) + h(t)] = x(t) * g(t) + x(t) * h(t)$	$x[k] * (g[k] + h[k]) = x[k] * g[k] + x[k] * h[k]$
结合律	$[x(t) * g(t)] * h(t) = x(t) * [g(t) * h(t)]$	$x[k] * g[k] * h[k] = x[k] * g[k] * h[k]$
数乘结合律	$a[x(t) * h(t)] = [ax(t)] * h(t) = x(t) * ah(t)$	$a \cdot \{x[k] * h[k]\} = \{a \cdot x[k]\} * h[k] = x[k] * \{a \cdot h[k]\}$
平移特性	$x(t-t_1) * h(t-t_2) = y(t-t_1-t_2)$	x[k-n]*h[k-l] = y[k-(n+l)]
微分特性 (差分特性)	$y'(t) = x'(t) * h(t) = x(t) * h'(t)$	$\nabla x[k] * h[k] = x[k] * \nabla h[k] = \nabla y[k]$ $\Delta x(k) * h(k) = x[k] * \Delta h[k] = \Delta y[k]$
积分特性 (求和特性)	$y^{(-1)}(t) = x^{(-1)}(t) * h(t) = x(t) * h^{(-1)}(t)$	$x[k] * \sum_{n=-\infty}^{k} h[n] = \left(\sum_{n=-\infty}^{k} x[n]\right) * h[k] = \sum_{n=-\infty}^{k} y[n]$
等效特性	$y(t) = x^{(-1)}(t) * h'(t) = x'(t) * h^{(-1)}(t)$	$\nabla x[k] * \sum_{n=-\infty}^{k} h[n] = \left(\sum_{n=-\infty}^{k} x[n]\right) * \nabla h[k] = y[k]$

其中，$x^{(-1)}(t) = \int_{-\infty}^{t} x(\tau) d\tau$，$\nabla x[k] = x[k] - x[k-1]$，$\Delta x[k] = x[k+1] - x[k]$

另外，对于单位阶跃函数 $\varepsilon(t) = \left\{\begin{smallmatrix} 1 & (t>0) \\ 0 & (t<0) \end{smallmatrix}\right\}$ 与单位冲激函数 $\delta(t) = \left\{\begin{smallmatrix} \infty & (t=0) \\ 0 & (t\neq0) \end{smallmatrix}\right\} \left(\int_{-\infty}^{+\infty} \delta(t) dt = 1\right)$、

单位阶跃序列 $\varepsilon[k] = \left\{\begin{smallmatrix} 1 & (k\geq0) \\ 0 & (k\leq-1) \end{smallmatrix}\right\}$ 与单位冲激序列 $\delta[k] = \left\{\begin{smallmatrix} 1 & (k=0) \\ 0 & (k\neq0) \end{smallmatrix}\right\}$ 而言，卷积还具有下列性质：

性质名称	函数的卷积积分	序列的卷积和
	$x(t) * \delta(t) = x(t)$	$x[k] * \delta[k] = x[k]$
延时特性	$x(t) * \delta(t-t_0) = x(t-t_0)$	$x[k] * \delta[k-n] = x[k-n]$
微分特性 (差分特性)	$x(t) * \delta'(t) = x'(t)$	$x[k] * \nabla\delta[k] = \nabla x[k]$ $x[k] * \Delta\delta[k] = \Delta x[k]$
积分特性 (求和特性)	$x(t) * \varepsilon(t) = \int_{-\infty}^{t} x(\tau) d\tau = x^{(-1)}(t)$	$x[k] * \varepsilon[k] = \sum_{n=-\infty}^{k} x[n]$ [3]

More details refer to BaiduBaike or Wikipedia.