CS 61B
Spring 2021

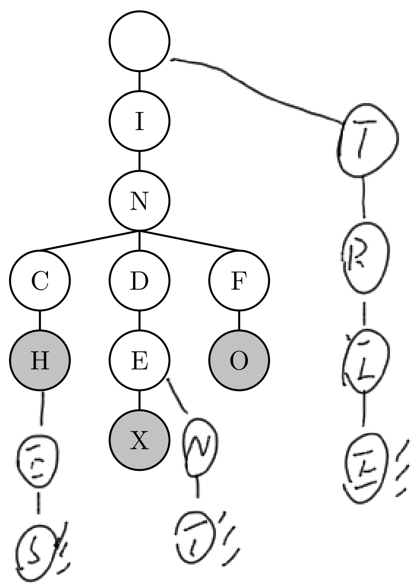
Tries and More Graphs

Discussion 11: April 5th, 2021

1 Trie Your Best

(a) What strings are stored in the trie below? Now insert the strings *indent, inches, and trie* into the trie.

Inch
Index
Info



(b) What is the runtime to find out if a given string is in the tree? What is the runtime to add a string to the tree? Describe your answers in terms of N, the number of words in the trie. You may assume the max length of any word in the trie is a constant.

$\Theta(1)$

(c) *Extra:* How could you modify a trie so that you can efficiently determine the number of words with a specific prefix in the trie? Describe your runtime of your solution.

每个结点额外储存个数信息。插入单词后，将访问过的节点上的对应值+1，查询时返回prefix的最末一个字母对应的结点的个数信息。

2 A Tree Takes on Graphs

Your friend at Stanford has made some statements about graphs, but you believe they are all false. Provide counterexamples to each of the statements below:

(a) "Every graph has one unique MST."

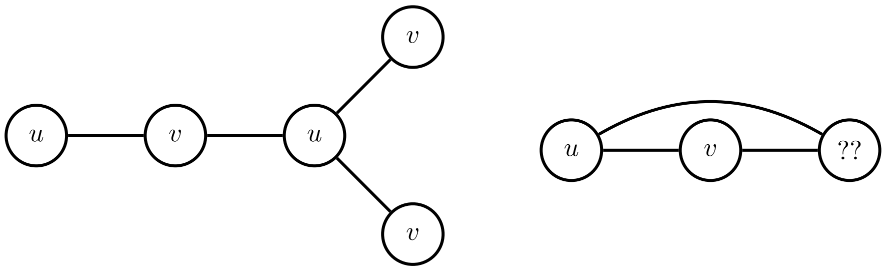
(b) "No matter what heuristic you use, A* search will always find the correct shortest path."

要保证总是找到 admissible 和 consistent.

(c) "If you add a constant factor to each edge in a graph, Dijkstra's algorithm will return the same shortest paths tree."

3 Graph Algorithm Design

(a) An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets U and V such that every edge connects an item in U to an item in V. For example below, the graph on the left is bipartite, whereas on the graph on the right is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?
Hint: Can you modify an algorithm we already know?



将起始点标为u
它邻居标为v
而后它的邻居再标为u
类推.
若要标为u的点
标上v. 或反之, 则
不是二分图
无论BFS还是DFS
O(E+V)

(b) Consider the following implementation of DFS, which contains a crucial error:

```
create the fringe, which is an empty Stack
push the start vertex onto the fringe and mark it
while the fringe is not empty:
    pop a vertex off the fringe and visit it
    for each neighbor of the vertex:
        if neighbor not marked:
            push neighbor onto the fringe
            mark neighbor
```

First, identify the bug in this implementation. Then, give an example of a graph where this algorithm may not traverse in DFS order.
Hint: When should we be marking vertices?

在访问节点时标记. 而不是在入栈时.

对每个顶点s,
用BFS找s到其他点
的最短路径.

(c) *Extra:* Provide an algorithm that finds the shortest cycle (in terms of the number of edges used) in a directed graph in $O(EV)$ time and $O(E)$ space, assuming $E > V$.

检查其他点有没有
到s的边. 有的话
就找到了一个循环. 长度为dist[v]+1
对每个顶点重复找出最小长度. V·O(E+V) = O(EV+V²) = O(EV)