CS 61B Spring 2021

MSTs, Dijkstra's, and Tries

Exam Prep Discussion 11: April 5, 2021

1 Oracle Dijkstra's

In some graph G, we are given a sorted list of nodes, sorted by their distances from some start vertex A. Design an *efficient* algorithm to find the shortest paths tree starting from A.

Hint: Your algorithm should be more efficient than Dijkstra's.

略去优先队列的使用.按序运行Dijkstra即可.

2 Multiple MSTs

Recall a graph can have multiple MSTs if there are multiple spanning trees of minimum weight.

(a) For each subpart below, select the correct option and justify your answer. If you select "never" or "always," provide a short explanation. If you select "sometimes", provide two graphs that fulfill the given properties — one with multiple MSTs and one without. Assume G is an undirected, connected graph.

1. If **none** the edge weights are **identical**, there will

   - ◯ never be multiple MSTs in G. ✓
   - ◯ sometimes be multiple MSTs in G.
   - ◯ always be multiple MSTs in G.

   Justification:

2. If **some** of the edge weights are **identical**, there will

   - ◯ never be multiple MSTs in G.
   - ◯ sometimes be multiple MSTs in G. ✓
   - ◯ always be multiple MSTs in G.

   Justification:

3. If **all** of the edge weights are **identical**, there will

   - ◯ never be multiple MSTs in G.
   - ◯ sometimes be multiple MSTs in G. ✓ 有时一棵树本身就是MST
   - ◯ always be multiple MSTs in G.

   Justification:

(b) Suppose we have a connected, undirected graph G with N vertices and N edges, where all the **edge weights are identical**. Find the maximum and minimum number of MSTs in G and explain your reasoning.

Minimum: __3______
Maximum: __N____

Justification:

所有边相等. MST = ST
创建一个有N-1条边的连通图 即树
向树中添加一条边 产生一个环. 环最短3. 最大N
移除环中任一条边可创建一个ST. 最少3个. 最大N个

(c) It is possible that Prim's and Kruskal's find **different** MSTs on the same graph `G` (as an added exercise, construct a graph where this is the case!). Given any graph `G` with integer edge weights, modify G to **ensure** that Prim's and Kruskal's will always find the same MST. You may not modify Prim's or Kruskal's.

Hint: Look at subpart 1 of part a.

如何修改图使所有边权相异.
假定所有边权为整数 那么就可以给每个边权添加一个
0~1的小数, 不会改变MST.
只需要让偏移量唯一.
一种可能是 偏移量为 $0, \frac{1}{E}, \frac{2}{E}, \cdots, \frac{E-1}{E}$
E为边数

3 Graph Algorithm Design

For each of the following scenarios, write a brief description for an algorithm for finding the MST in an undirected, connected graph G.

(a) If all edges have edge weight 1. Hint: Runtime is O(V+E)

DFS产生的树即可. 或BFS.

(b) If all edges have edge weight 1 or 2. Hint: Use your algorithm from part (a)

先移除边权为2，运行(a)，获得一些连通分量

连通分量作为新图的节点. 查看权为2的边

若两端在新图中不连通，加上一条边.

最后在新图上运行(a)

4 A Wordsearch

Given an N by N wordsearch and N words, devise an algorithm to solve the wordsearch in O(N^3). For simplicity, assume no word is contained within another, i.e. if the word "bear" is given, "be" wouldn't also be given.

If you are unfamiliar with wordsearches or want to gain some wordsearch solving intuition, see below for an example wordsearch. Note that the below wordsearch doesn't follow the precise specification of an N by N wordsearch with N words, but your algorithm should work on this wordsearch regardless.

Example Wordsearch:

C	M	U	H	O	S	A	E	D
T	R	A	T	H	A	N	K	A
O	C	Y	E	S	R	T	U	T
N	I	R	S	A	I	O	L	S
Y	R	R	M	T	N	N	H	R
Y	E	A	E	V	A	R	U	E
A	A	A	I	M	E	L	C	R
N	H	D	J	Y	U	A	C	I
T	Y	S	A	A	R	S	U	C
A	R	S	I	G	Y	E	S	A

ajay	anton
crystal	eric
grace	isha
luke	naama
rica	sarina
sherry	shreyas
sohum	sumer
tony	vidya

Hint: Add the words to a Trie, and you may find the `longestPrefixOf` operation helpful. Recall that `longestPrefixOf` accepts a `String key` and returns the longest prefix of `key` that exists in the `Trie`, or **`null`** if no prefix exists.