

第四次作业

[返回](#)

姓名: Hollow Man 班级: 班级6

Hollow Man

一.简答题 (共7题,100.0分)

1 7-1.pdf

我的答案:

$$\begin{aligned}
 (1) E(X) &= -1 \times \frac{1}{3} + 0 \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} + 1 \times \frac{1}{12} + 2 \times \frac{1}{4} = \frac{1}{3} \\
 (2) E(-X+1) &= -E(X) + 1 = -\frac{1}{3} + 1 = \frac{2}{3} \\
 (3) E(X^2) &= (-1)^2 \times \frac{1}{3} + 0^2 \times \frac{1}{6} + \left(\frac{1}{2}\right)^2 \times \frac{1}{6} + 1^2 \times \frac{1}{12} + 2^2 \times \frac{1}{4} = \frac{35}{24} \\
 (4) D(X) &= E(X^2) - [E(X)]^2 = \frac{35}{24} - \left(\frac{1}{3}\right)^2 = \frac{97}{72}
 \end{aligned}$$

2 7-2.pdf

我的答案:

2. 设商品量的最大值点为 x , 收益为随机变量 Y , $Y=g(x)$.

$$f(x) = \begin{cases} \frac{1}{2000}, & 2000 < x < 4000 \\ 0, & \text{其他} \end{cases}$$

$$Y = g(x) = \begin{cases} 3x_0, & x > x_0 \\ 3x - (x_0 - x), & x \leq x_0 \end{cases}$$

则 $E(Y) = \int_{2000}^{x_0} (4x - x_0) \frac{1}{2000} dx + \int_{x_0}^{4000} 3x_0 \frac{1}{2000} dx$

$$= \frac{1}{1000} (-x_0^2 + 7000x_0 - 4 \times 10^6)$$

$$\therefore [E(Y)]'_{x_0} = (-2x_0 + 7000) \times \frac{1}{1000} = 0$$

$$\therefore x_0 = 3500$$

又 $\therefore [E(Y)]''_{x_0} = -\frac{2}{1000} \therefore x_0 = 3500$ 为最大值点.

\therefore 组织货源 3500t 时收益最大.

3 7-3.pdf

我的答案:

$$\begin{aligned}
 3. (1) E(x) &= \int_0^{+\infty} x e^{-x} dx \\
 &= -x e^{-x} \Big|_0^{+\infty} + \int_0^{+\infty} e^{-x} dx \\
 &= -e^{-x} \Big|_0^{+\infty} = 1 \\
 (2) E(2x) &= 2E(x) = 2 \\
 (3) E(x + e^{-2x}) &= E(x) + E(e^{-2x}) = 1 + \int_0^{+\infty} e^{-2x} e^{-x} dx \\
 &= 1 + \int_0^{+\infty} e^{-3x} dx = 1 + \frac{1}{3} = \frac{4}{3} \\
 (4) D(x) &= E(x^2) - [E(x)]^2 \\
 &= \int_0^{+\infty} x^2 e^{-x} dx - 1^2 = -x^2 e^{-x} \Big|_0^{+\infty} + 2 \int_0^{+\infty} x e^{-x} dx - 1^2 \\
 &= -2x e^{-x} \Big|_0^{+\infty} + 2 \int_0^{+\infty} e^{-x} dx - 1^2 = 2 - 1 = 1
 \end{aligned}$$

4 7-4.pdf

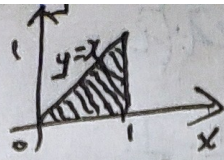
我的答案:

$$\begin{aligned}
 4. \text{设死亡人数为 } X, X \sim B(3000, 0.01) \\
 \text{当 } 2000X > 10 \times 3000 \text{ 时} \\
 \text{即 } X > 15 \text{ 时, 保险公司亏本} \\
 \therefore P(X > 15) &= P\left(\frac{X - np}{\sqrt{np(1-p)}} > \frac{15 - np}{\sqrt{np(1-p)}}\right) = P\left(\frac{X - 3}{\sqrt{3 \times 0.999}} > \frac{15 - 3}{\sqrt{3 \times 0.999}}\right) \\
 &= P\left(\frac{X - 3}{\sqrt{3 \times 0.999}} > 6.9367\right) \\
 &= 1 - \Phi(6.9367) = 0
 \end{aligned}$$

5 7-5.pdf

我的答案:

5. $0 \leq y \leq x \leq 1$ 区域如图所示:



$$\therefore f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_0^x 12y^2 dx = 4x^3, & 0 \leq x \leq 1 \\ \text{其他} \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_y^1 12y^2 dx = 12y^2(1-y), & 0 \leq y \leq 1 \\ \text{其他} \end{cases}$$

$$(1) E(X) = \int_0^1 x \cdot 4x^3 dx = \frac{4}{5}$$

$$(2) E(Y) = \int_0^1 y \cdot 12y^2(1-y) dy = \frac{3}{5}$$

$$(3) E(XY) = \int_0^1 \int_0^x xy \cdot 12y^2 dy dx = \frac{1}{2}$$

$$\begin{aligned} (4) E(X^2 + Y^2) &= E(X^2) + E(Y^2) \\ &= \int_0^1 x^2 \cdot 4x^3 dx + \int_0^1 y^2 \cdot 12y^2(1-y) dy \\ &= \frac{2}{3} + \frac{2}{5} = \frac{16}{15} \end{aligned}$$

$$(5) D(X) = E(X^2) - [E(X)]^2 = \frac{2}{3} - \left(\frac{4}{5}\right)^2 = \frac{2}{75}$$

$$(6) D(Y) = E(Y^2) - [E(Y)]^2 = \frac{2}{5} - \left(\frac{3}{5}\right)^2 = \frac{1}{25}$$

6 7-6.pdf

我的答案:

$$\begin{aligned} 6. (1) D(X+Y) &= D(X) + D(Y) + 2\rho_{X,Y}\sqrt{D(X)} \cdot \sqrt{D(Y)} \\ &= 25 + 36 + 2 \times 0.4 \times \sqrt{25} \times \sqrt{36} = 85 \\ (2) D(X-Y) &= D(X) + D(Y) - 2\rho_{X,Y}\sqrt{D(X)} \cdot \sqrt{D(Y)} \\ &= 25 + 36 - 2 \times 0.4 \times \sqrt{25} \times \sqrt{36} = 37 \end{aligned}$$

7 7-7.pdf

我的答案:

7. 设 X_i 表示每个加数的取整误差, $X_i \sim U[-0.5, 0.5]$.

$$E(X_i) = \frac{-0.5+0.5}{2} = 0, D(X_i) = \frac{(0.5+0.5)^2}{12} = \frac{1}{12}$$

(1) 由中心极限定理,

$$\begin{aligned} P\left\{\left|\sum_{i=1}^{300} X_i\right| > 15\right\} &= 1 - P\left\{-15 < \sum_{i=1}^{300} X_i < 15\right\} \\ &= 1 - P\left\{-\frac{15}{\sqrt{300 \times \frac{1}{12}}} < \frac{\sum_{i=1}^{300} X_i}{\sqrt{300 \times \frac{1}{12}}} < \frac{15}{\sqrt{300 \times \frac{1}{12}}}\right\} \\ &= 1 - [\Phi(3) - \Phi(-3)] \\ &= 2[1 - \Phi(3)] = 0.0027. \end{aligned}$$

\therefore 将300个数相加, 误差总和绝对值超过15的概率为0.0027.

(2) 由中心极限定理,

$$\begin{aligned} 0.9 &= P\left\{\left|\sum_{i=1}^n X_i\right| < 10\right\} \\ &= P\left\{-10 < \sum_{i=1}^n X_i < 10\right\} \\ &= P\left\{-\frac{10}{\sqrt{\frac{n}{12}}} < \frac{\sum_{i=1}^n X_i}{\sqrt{\frac{n}{12}}} < \frac{10}{\sqrt{\frac{n}{12}}}\right\} = \Phi\left(\frac{10}{\sqrt{\frac{n}{12}}}\right) - \Phi\left(-\frac{10}{\sqrt{\frac{n}{12}}}\right) = 2\Phi\left(\frac{10}{\sqrt{\frac{n}{12}}} - 1\right) \\ \therefore \Phi\left(\frac{10}{\sqrt{\frac{n}{12}}}\right) &= 0.95 \Rightarrow \frac{10}{\sqrt{\frac{n}{12}}} = 1.645 \Rightarrow n \approx 443 \end{aligned}$$

设随机变量 X 的分布律为,

X	-1	0	$\frac{1}{2}$	1	2
1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$

求

(1) $E(X)$;

(2) $E(-X + 1)$;

(3) $E(X^2)$;

(4) $D(X)$.

解:

(1) $\frac{1}{3}$;

(2) $\frac{2}{3}$;

(3) $\frac{35}{24}$;

(4) $\frac{97}{72}$;

国际市场每年对我国某种出口商品的需求是一个随机变量, 它在 $[2000, 4000]$ (单位: 吨) 上服从均匀分布. 若每售出一吨, 可得外汇 3 万美元. 若销售不出而积压, 则每吨需保养费 1 万美元. 何应组织多少货源, 才能使平均收益最大?

解: 3500 吨.

设随机变量 X 的概率密度为

$$f(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$

求

(1) $E(X)$;

(2) $E(2X)$;

(3) $E(X + e^{-2X})$;

(4) $D(X)$.

解:

(1) 1;

(2) 2;

(3) $\frac{4}{3}$;

(4) 1;

有 3000 个同龄的人参加某保险公司的人寿保险, 保险期限为 1 年. 假设在 1 年内每人的死亡率为 0.1%, 参加保险的人在投保日须交付保费 10 元, 被保险人在保险期间死亡时家属可以从保险公司领取 2000 元. 试用中心极限定理求保险公司亏本的概率.

解: 0.

设随机变量 (X, Y) 的联合密度函数为

$$f(x, y) = \begin{cases} 12y^2, & 0 \leq y \leq x \leq 1, \\ 0, & \text{其他.} \end{cases}$$

求

(1) $E(X)$;

(2) $E(Y)$;

(3) $E(XY)$;

(4) $E(X^2 + Y^2)$;

(5) $D(X)$;

(6) $D(Y)$;

解:

$$(1) E(X) = \frac{4}{5};$$

$$(2) E(Y) = \frac{3}{5};$$

$$(3) E(XY) = \frac{1}{2};$$

$$(4) E(X^2 + Y^2) = \frac{16}{15};$$

$$(5) D(X) = \frac{2}{75};$$

$$(6) D(Y) = \frac{1}{25}.$$

设 $D(X) = 25$, $D(Y) = 36$, $\rho_{X,Y} = 0.4$,

求:

(1) $D(X + Y)$;

(2) $D(X - Y)$.

解:

$$(1) D(X + Y) = 85;$$

$$(2) D(X - Y) = 37.$$

计算机在进行加法时, 每个加数取整数 (按四舍五入取最为接近的数), 设所有加数的取整误差是相互独立的, 且它们服从 $[-0.5, 0.5]$ 上的均匀分布.

- (1) 若将 300 个数相加, 求误差总和绝对值超过 15 的概率;
- (2) 至多 n 个数加在一起, 其误差总和的绝对值小于 10 的概率为 0.9, 求 n 的值.

解:

(1) 0.0027 ;

(2) $n \approx 443$.