Coursework 1

Formal Languages and Finite Automata (COMP2321)

You should prepare solutions to the following questions. Your solutions should be clearly presented on A4 paper. Any work that is not clearly presented will not be marked. Late submissions are accepted up to 7 days late. Each day, or part of a day, will incur a 5% penalty. Feedback on late submissions may not be provided within 3 weeks of submission.

Submission You **must** submit your work via Minerva and as a physical submission.

1. **Minerva:** You should submit a .pdf file to the Minerva submission point.
2. **Physical copy:** Your submission should be neatly presented on A4 paper. Ensure you have attached a completed coursework header sheet.

Deadline **TBC**.

Weighting This piece of summative coursework is worth 20% of the module grade.

1. Let L_1 and L_2 be two regular languages over the alphabet $\{0, 1\}$. Let L_1 be the language containing all strings that start with 1, end with 0 and alternate between symbols in the alphabet, that is $10101010 \in L_1$ but $10011010 \notin L_1$. Let $L_2 = L(M_2)$ where M_2 is shown in Figure 1.

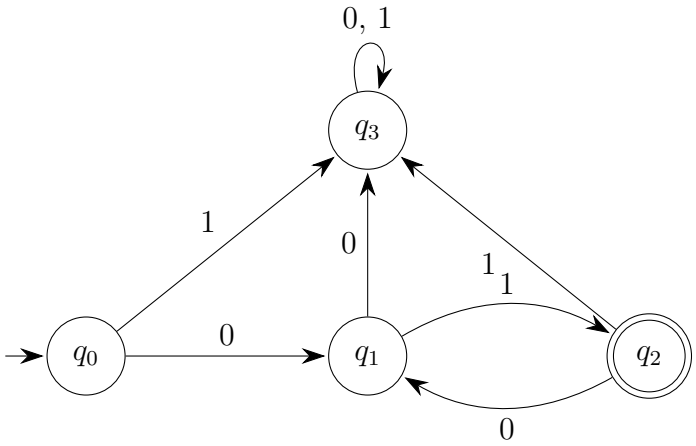

Figure 1: Finite automaton M_2.

(a) Construct a finite automata that accepts L_1.

(b) Show that $L(M_2) \neq \varnothing$.

(c) Construct a finite state automata for $L_1 \cup L_2$ and hence show that $L_1 \cup L_2$ is regular.

[12 marks]

2. For any string $w = w_1 \dots w_n$ the reverse of the string denoted $w^{\mathcal{R}}$ is the string $w' = w_n \dots w_1$. The reverse of a language L, denoted $L^{\mathcal{R}} = \{w^{\mathcal{R}} \mid w \in L\}$. Show that if L is regular then $L^{\mathcal{R}}$ is regular.

[12 marks]

The follow question is very challenging and requires a little insight. You have been taught all of the techniques required to prove it.

3. Let Σ_3 be the set of binary column vectors of size 3, that is

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \cdots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

A string over Σ_3 contains 3 binary strings r_1, r_2, r_3, the first is the string containing the binary digits contained in the first row of the column vectors, denoted r_1, the second is the string containing the binary digits contained in the second row of the column vectors and the third is the string containing the binary digits contained in the third row of the column vector, respectively. Each of these strings can be seen as representing a binary number, where the most significant bit is the left most bit in the string and the least significant bit is the right most bit. Let us define a language,

$$B_{\text{add}} = \{w \in \Sigma_3^* \mid r_3 = r_1 + r_2\}$$

Prove that B_{add} is regular.

Hint: You can use your answer from Question 2.

[24 marks]