Coursework 3

COMP2721 Algorithms and Data Structures II

1. The context-free grammar $\langle V, T, P, S\rangle$ with variables $V = \{S, A, B, C\}$, terminals $T = \{a, b\}$ and the rules

$$\begin{aligned} S &\to AB \mid BC & \qquad A &\to BA \mid a \\ B &\to CC \mid b & \qquad C &\to AB \mid a \end{aligned}$$

   generates non-empty strings in T^*. Execute the Cocke-Younger-Kasami algorithm for this grammar on the input $s = ababbab$. Give a table of all sets $V(i, k)$ computed and a parse tree for s. [0:30h expected time] [5 marks]

2. Design a dynamic programming algorithm that computes the maximum weight of a matching in an edge-weighted tree.

   Definitions: A set $M \subseteq E$ is a *matching* in the graph $G = (V, E)$ if no vertex $x \in V$ is incident to two edges in M. A vertex $x \in V$ is M-*saturated* (or *matched*) if x is incident with an edge in M, and x is M-*unsaturated* (or *unmatched*) if x is not an endpoint of any edge in M. If $w : E \to \mathbb{N}$ are weights on the edges then the *weight* of the matching M is $\sum_{e \in M} w(e)$. A *tree* is a undirected, connected and acyclic graph.

   Hint: Given a tree $T = (V, E)$ with weight function $w : E \to \mathbb{N}$, choose a root $r \in V$ and define, for each $x \in V$, the subtree T_x of T rooted at x. Derive a recurrence for the maximum weight of a matching of T_x that saturates x, and for the maximum weight of a matching of T_x that does not saturate x, such that the maximum weight of a matching of T is the maximum of these two values for T_r.

   [1:00h expected time] [5 marks]

Submission: Work out and present your solution on paper. Stitch together all your sheets and a filled header form and submit via SSO. Indicate date and time of your tutorial, that is, one of the following:

- Tuesday 12–1
- Tuesday 4–5
- Friday 1–2
- Friday 2–3

For a proof of submission, convert your solution into portable document format (via `pdflatex` if you use LaTeX or scan your manuscript) and submit it in Minerva.

Deadline: Monday 16 March 2020, 10am.

Credits: This piece of summative coursework is worth 5% of your grade.