

1. Prove that two eigenvectors of a Hermitian operator with different eigenvalues are necessarily orthogonal.
2. Suppose a composite of systems  $A$  and  $B$  is in the state  $|a\rangle|b\rangle$ , where  $|a\rangle$  is a pure state of system  $A$ , and  $|b\rangle$  is a pure state of system  $B$ . Show that the reduced density operator of system  $A$  alone is a pure state.