# 시계열자료분석

## Ch07 ARIMA

In [1]:
```
##############  경로지정
#setwd("C:\\R-Project\\DAT\\Time Series Data")

options(repr.plot.width = 12, repr.plot.height = 6)
```

### 분산이 일정하지 않은 경우

In [2]:
```
z <- AirPassengers
```

In [3]:
```
z
```

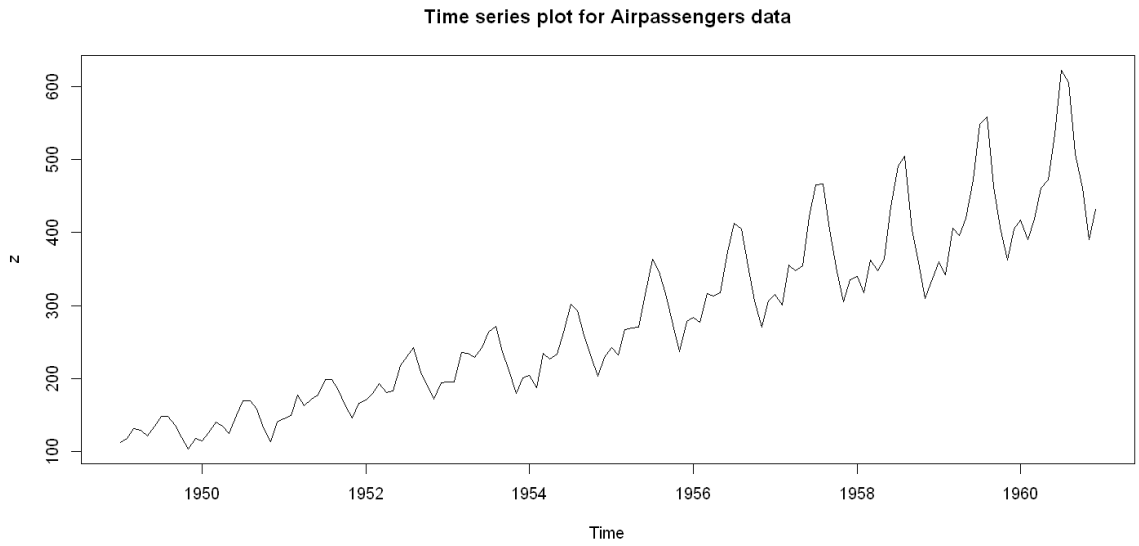
A Time Series: 12 × 12

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1949** | 112 | 118 | 132 | 129 | 121 | 135 | 148 | 148 | 136 | 119 | 104 | 118 |
| **1950** | 115 | 126 | 141 | 135 | 125 | 149 | 170 | 170 | 158 | 133 | 114 | 140 |
| **1951** | 145 | 150 | 178 | 163 | 172 | 178 | 199 | 199 | 184 | 162 | 146 | 166 |
| **1952** | 171 | 180 | 193 | 181 | 183 | 218 | 230 | 242 | 209 | 191 | 172 | 194 |
| **1953** | 196 | 196 | 236 | 235 | 229 | 243 | 264 | 272 | 237 | 211 | 180 | 201 |
| **1954** | 204 | 188 | 235 | 227 | 234 | 264 | 302 | 293 | 259 | 229 | 203 | 229 |
| **1955** | 242 | 233 | 267 | 269 | 270 | 315 | 364 | 347 | 312 | 274 | 237 | 278 |
| **1956** | 284 | 277 | 317 | 313 | 318 | 374 | 413 | 405 | 355 | 306 | 271 | 306 |
| **1957** | 315 | 301 | 356 | 348 | 355 | 422 | 465 | 467 | 404 | 347 | 305 | 336 |
| **1958** | 340 | 318 | 362 | 348 | 363 | 435 | 491 | 505 | 404 | 359 | 310 | 337 |
| **1959** | 360 | 342 | 406 | 396 | 420 | 472 | 548 | 559 | 463 | 407 | 362 | 405 |
| **1960** | 417 | 391 | 419 | 461 | 472 | 535 | 622 | 606 | 508 | 461 | 390 | 432 |

In [4]:
```
class(z)
```

'ts'

In [5]:
```
plot.ts(z, main = "Time series plot for Airpassengers data")
```




- Box-Cox transformation

$$f_\lambda(Z_t) = \begin{cases} \dfrac{Z_t^\lambda - 1}{\lambda}, & Z_t \geq 0, \lambda > 0 \\ \log(Z_t), & \lambda = 0 \end{cases}$$

In [6]:
```
forecast::BoxCox.lambda(z, method='loglik')
```

```
Registered S3 method overwritten by 'quantmod':
  method            from
  as.zoo.data.frame zoo
```

0.2

In [7]:
```
forecast::BoxCox.lambda(z, method = "guerrero")
```

-0.294715585559316

In [8]:
```
forecast::BoxCox(z,lambda= forecast::BoxCox.lambda(z, method='loglik'))
```

A Time Series: 12 × 12

| | Jan | Feb | Mar | Apr | May | Jun | Jul | |
|---|---|---|---|---|---|---|---|---|
| **1949** | 7.847352 | 7.982144 | 8.276536 | 8.215632 | 8.047493 | 8.336343 | 8.583833 | 8.5 |
| **1950** | 7.915451 | 8.153584 | 8.452835 | 8.336343 | 8.132639 | 8.602141 | 8.965606 | 8.9 |
| **1951** | 8.528312 | 8.620350 | 9.094641 | 8.848653 | 8.998313 | 9.094641 | 9.412543 | 9.4 |
| **1952** | 8.981998 | 9.126173 | 9.324566 | 9.141833 | 9.172949 | 9.677811 | 9.835957 | 9.9 |
| **1953** | 9.368824 | 9.368824 | 9.912567 | 9.899908 | 9.823034 | 10.000000 | 10.250736 | 10.3 |
| **1954** | 9.484251 | 9.249564 | 9.899908 | 9.797051 | 9.887205 | 10.250736 | 10.666479 | 10.5 |
| **1955** | 9.987634 | 9.874459 | 10.285240 | 10.308071 | 10.319435 | 10.799092 | 11.262611 | 11.1 |
| **1956** | 10.475107 | 10.398058 | 10.819103 | 10.778978 | 10.829071 | 11.351000 | 11.678616 | 11.6 |
| **1957** | 10.799092 | 10.656090 | 11.190490 | 11.117061 | 11.181384 | 11.750682 | 12.078927 | 12.0 |
| **1958** | 11.042269 | 10.829071 | 11.244701 | 11.117061 | 11.253666 | 11.852637 | 12.265784 | 12.3 |
| **1959** | 11.226711 | 11.061098 | 11.621691 | 11.538992 | 11.734774 | 12.130041 | 12.649244 | 12.7 |
| **1960** | 11.710799 | 11.497014 | 11.726798 | 12.049443 | 12.130041 | 12.564701 | 13.102063 | 13.0 |

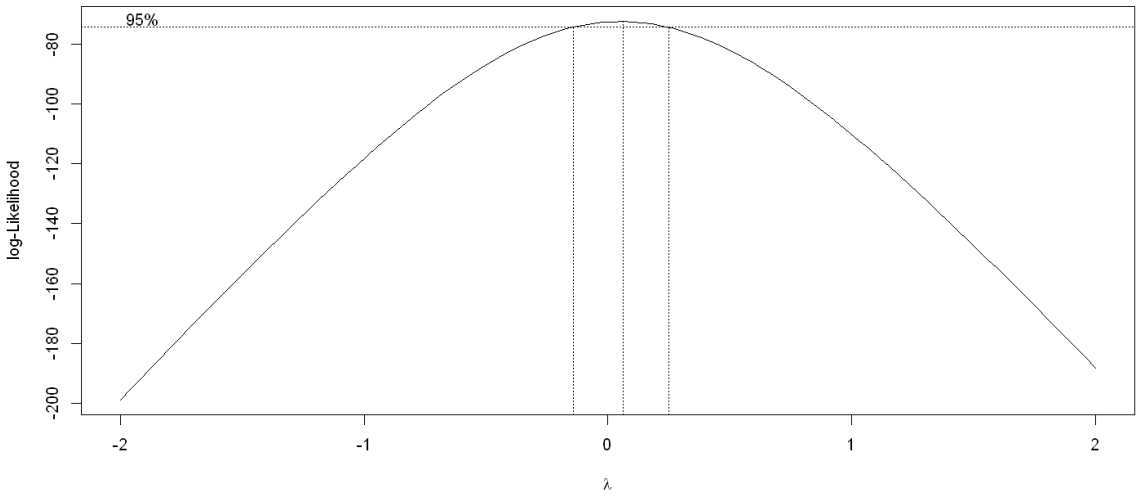
In [9]:
```
forecast::BoxCox(z,lambda= 'auto')
```

A Time Series: 12 × 12

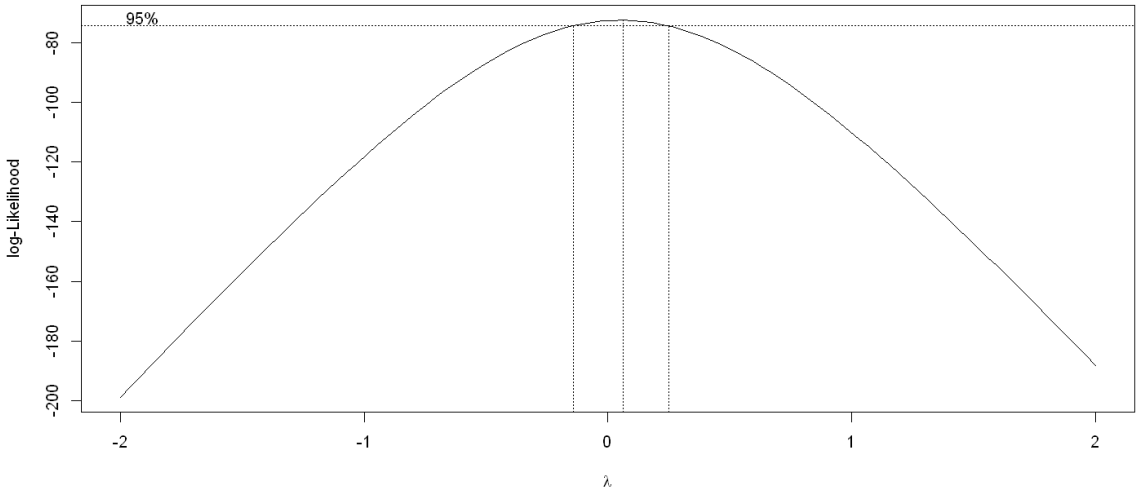
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | |
|---|---|---|---|---|---|---|---|---|---|
| **1949** | 2.548535 | 2.561426 | 2.588461 | 2.582990 | 2.567558 | 2.593773 | 2.615143 | 2.615143 | 2. |
| **1950** | 2.555089 | 2.577352 | 2.603953 | 2.593773 | 2.575434 | 2.616686 | 2.646282 | 2.646282 | 2. |
| **1951** | 2.610433 | 2.618215 | 2.656336 | 2.636968 | 2.648852 | 2.656336 | 2.680161 | 2.680161 | 2. |
| **1952** | 2.647572 | 2.658759 | 2.673698 | 2.659957 | 2.662328 | 2.699069 | 2.709945 | 2.720109 | 2. |
| **1953** | 2.676962 | 2.676962 | 2.715111 | 2.714262 | 2.709067 | 2.720927 | 2.737151 | 2.742897 | 2. |
| **1954** | 2.685357 | 2.668111 | 2.714262 | 2.707296 | 2.713408 | 2.737151 | 2.762643 | 2.756996 | 2. |
| **1955** | 2.720109 | 2.712549 | 2.739332 | 2.740768 | 2.741481 | 2.770427 | 2.796406 | 2.787934 | 2. |
| **1956** | 2.751119 | 2.746379 | 2.771588 | 2.769257 | 2.772164 | 2.801154 | 2.818211 | 2.814887 | 2. |
| **1957** | 2.770427 | 2.762027 | 2.792485 | 2.788447 | 2.791987 | 2.821853 | 2.837961 | 2.838662 | 2. |
| **1958** | 2.784288 | 2.772164 | 2.795437 | 2.788447 | 2.795922 | 2.826939 | 2.846793 | 2.851301 | 2. |
| **1959** | 2.794460 | 2.785340 | 2.815307 | 2.811044 | 2.821052 | 2.840400 | 2.864196 | 2.867286 | 2. |
| **1960** | 2.819842 | 2.808860 | 2.820650 | 2.836545 | 2.840400 | 2.860440 | 2.883580 | 2.879651 | 2. |

In [10]:
```
t <- 1:length(z)
```

In [11]:
```
MASS::boxcox(z~t)
```




In [12]:
```
bc <- MASS::boxcox(z~t)
```




In [13]:
```
#bc
```

In [14]:
```
which.max(bc$y)
```

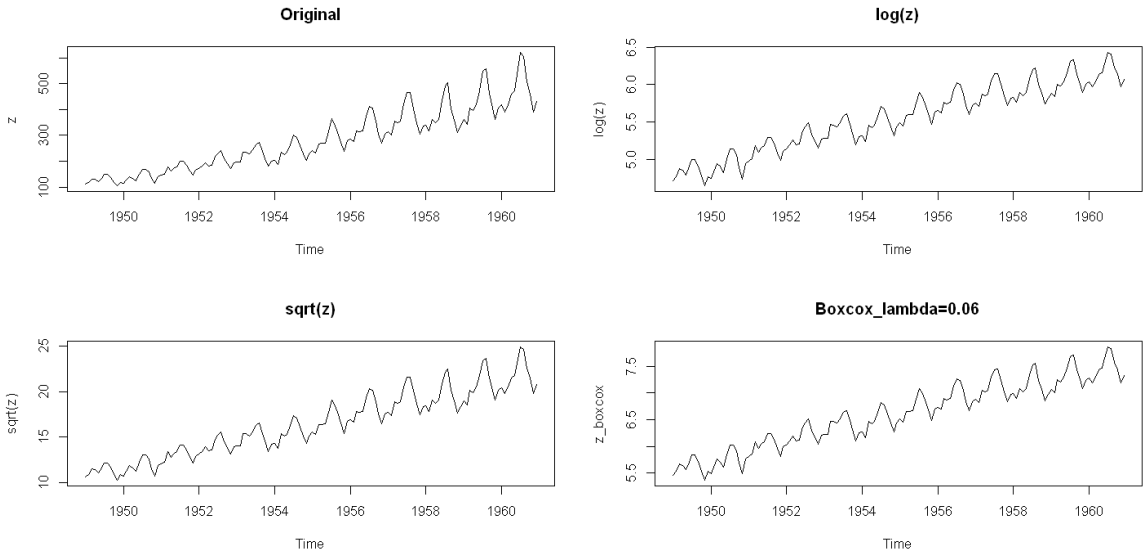
52

In [15]:
```
bc$x[which.max(bc$y)]
```

0.060606060606061

In [16]:
```
z_boxcox <- forecast::BoxCox(z, lambda=bc$x[which.max(bc$y)])
```

In [17]:
```
par(mfrow=c(2,2))
plot.ts(z, main = "Original")
plot.ts(log(z), main = "log(z)")
plot.ts(sqrt(z),, main = "sqrt(z)")
plot.ts(z_boxcox, main = "Boxcox_lambda=0.06")
graphics.off()
```




로그변환을 수행한다.

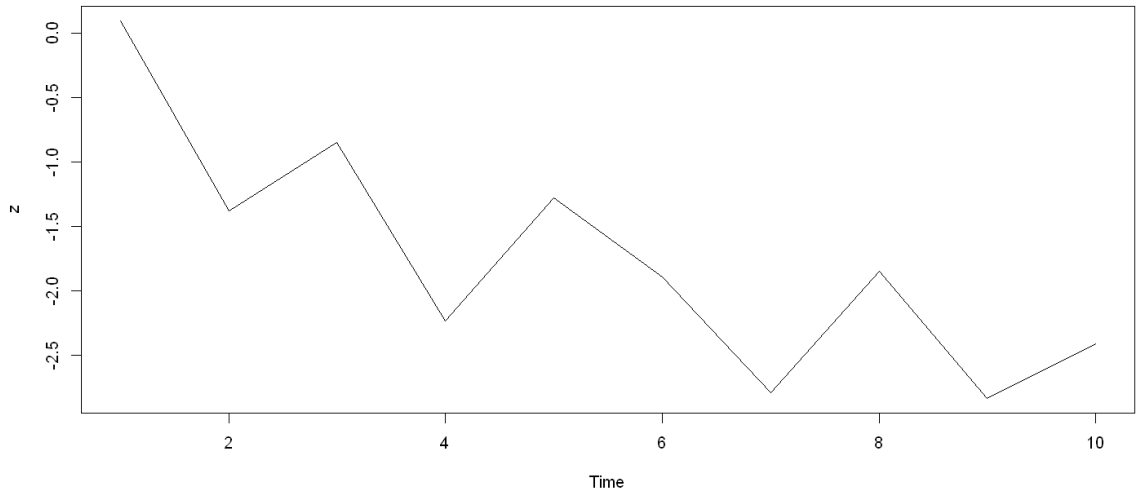
## 확률적 추세 제거

- 확률보행과정의 정상화

$Z_t = Z_{t-1} + \epsilon_t, \;\; \epsilon_t \sim WN(0, \sigma^2)$ - AR(1) with $\phi = 1$

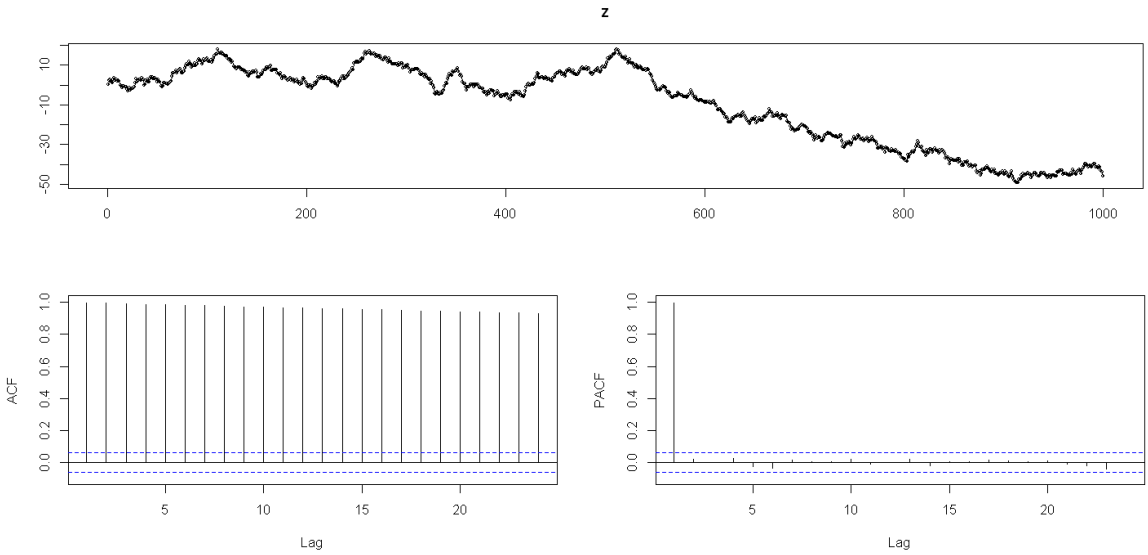
In [18]:
```
e <- round(rnorm(10),2)
z <- cumsum(e)

plot.ts(z)
```




In [19]:
```
e <- round(rnorm(1000),2)
z <- cumsum(e)

forecast::tsdisplay(z, lag.max=24)
```



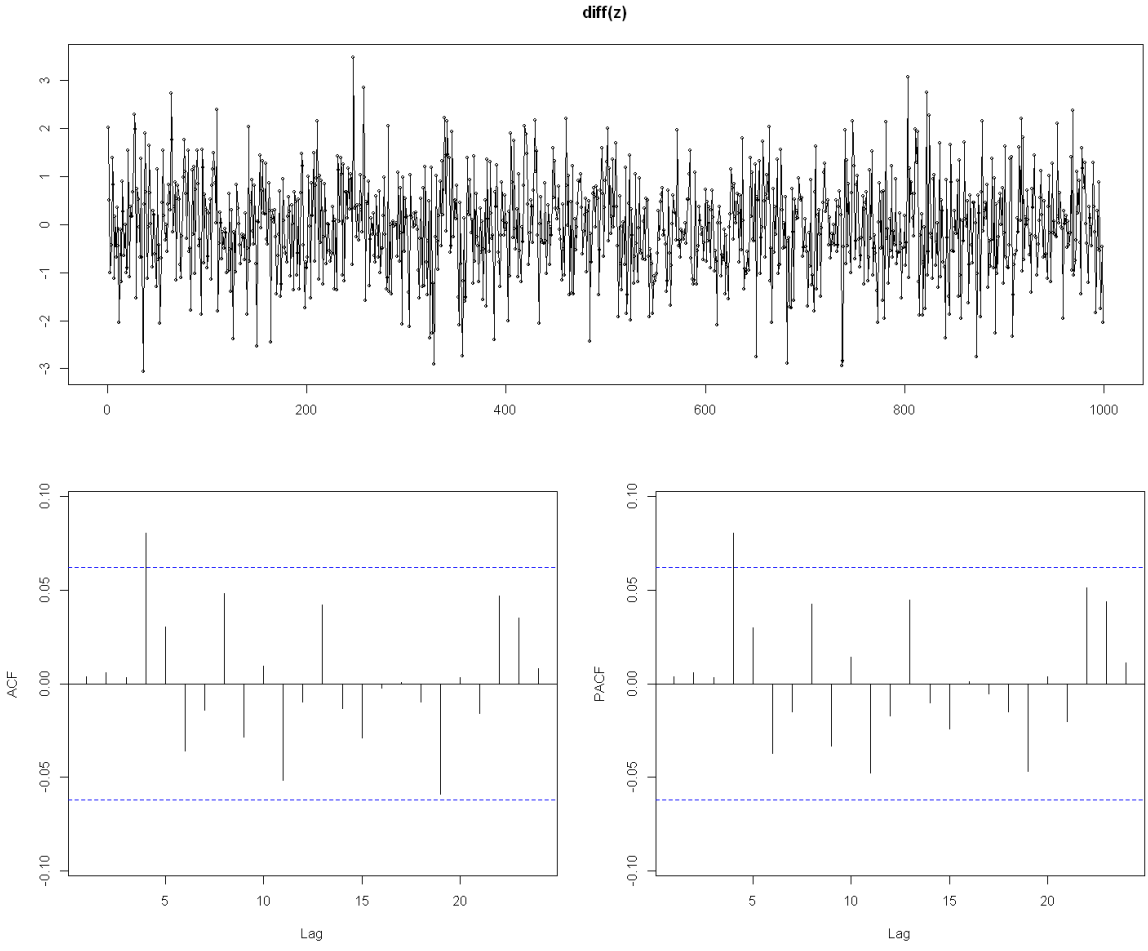

ACF가 아주 천천히 감소하므로 확률적 추세가 있다고 할 수 있다.

1차 차분을 수행한다.

$\Delta Z_t = Z_t - Z_{t-1} = \epsilon_t$

차분한 확률보행과정의 시도표 및 ACF/PACF 그림

In [20]:
```
options(repr.plot.width = 12, repr.plot.height = 10)
forecast::tsdisplay(diff(z), lag.max=24)
```

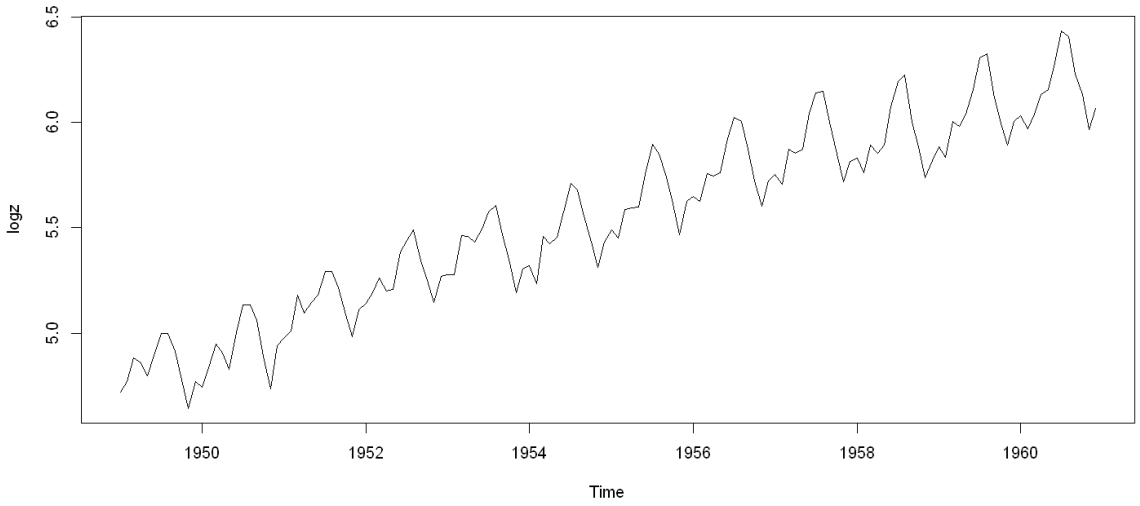


차분을 했더니 정상 시계열이 됨.
$\Delta Z_t \sim WN = ARMA(0,0) \;\Rightarrow\; Z_t \sim ARIMA(0,1,0)$

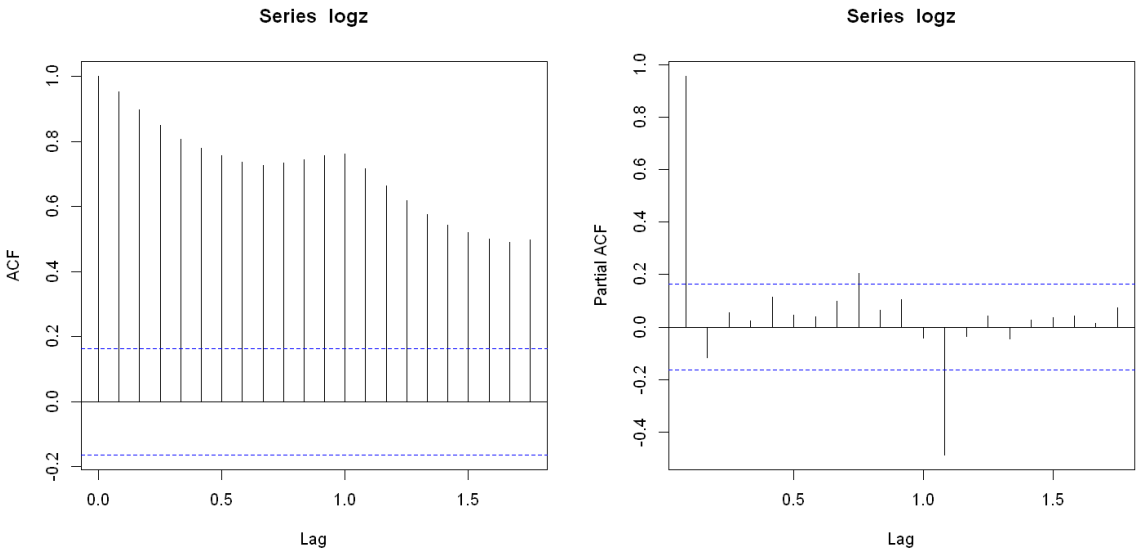
- 로그변환한 Airpassengers data 에 대한 확률적 추세 확인

In [21]:
```
logz <- log(AirPassengers)

options(repr.plot.width = 12, repr.plot.height = 6)
plot.ts(logz)
```

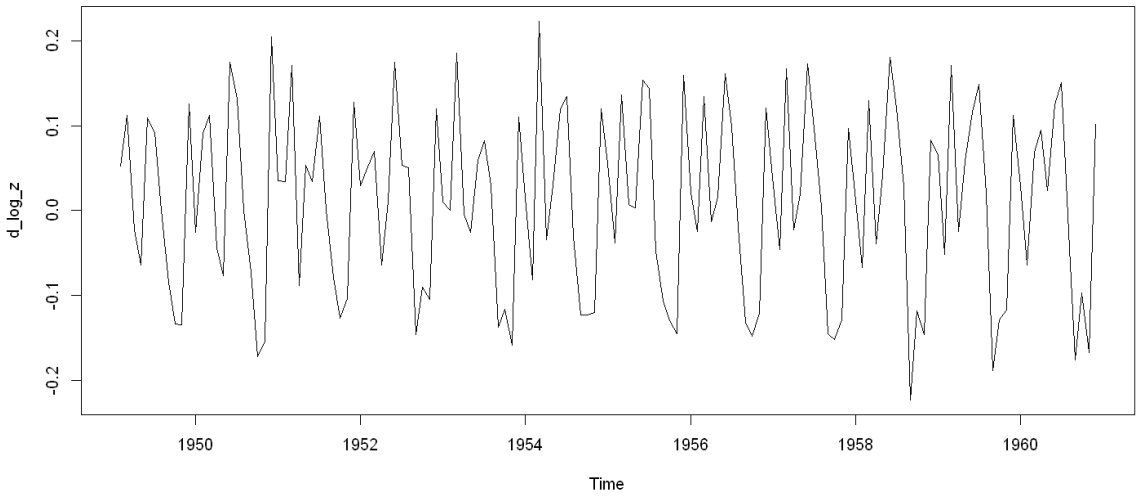



In [22]:
```
par(mfrow=c(1,2))
acf(logz)
pacf(logz)
```

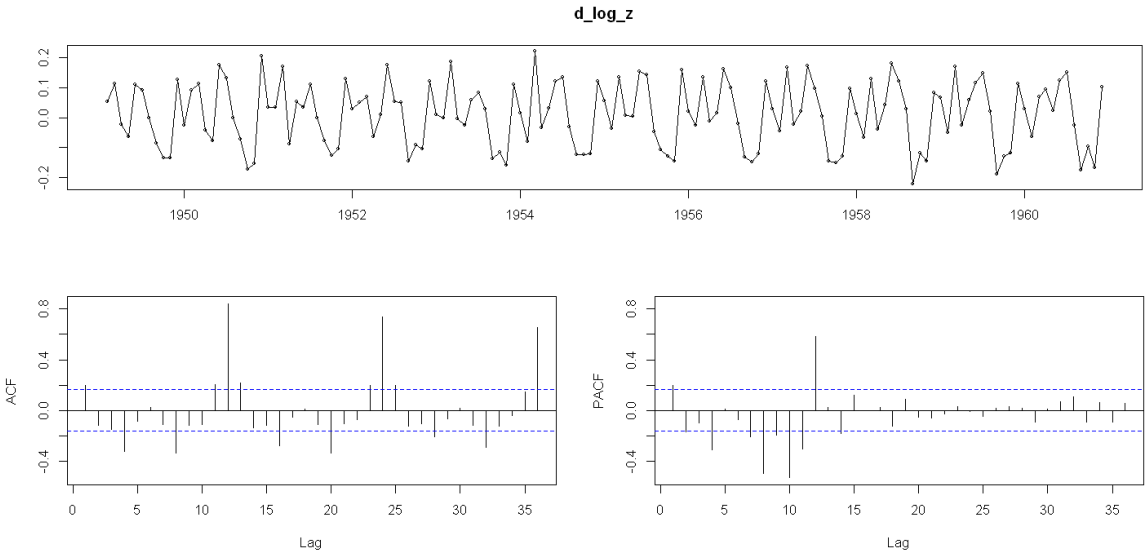



차분을 진행한다. $\Delta Z_t = (1-B)Z_t = Z_t - Z_{t-1}$

In [23]:
```
d_log_z = diff(logz)
plot.ts(d_log_z)
```



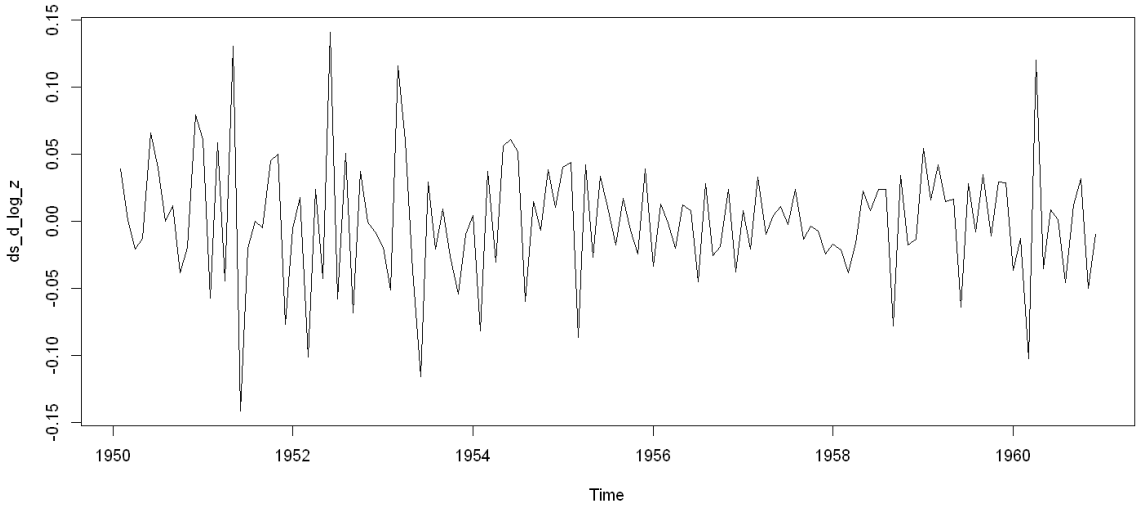

In [24]:
```
forecast::tsdisplay(d_log_z, lag.max=36)
```



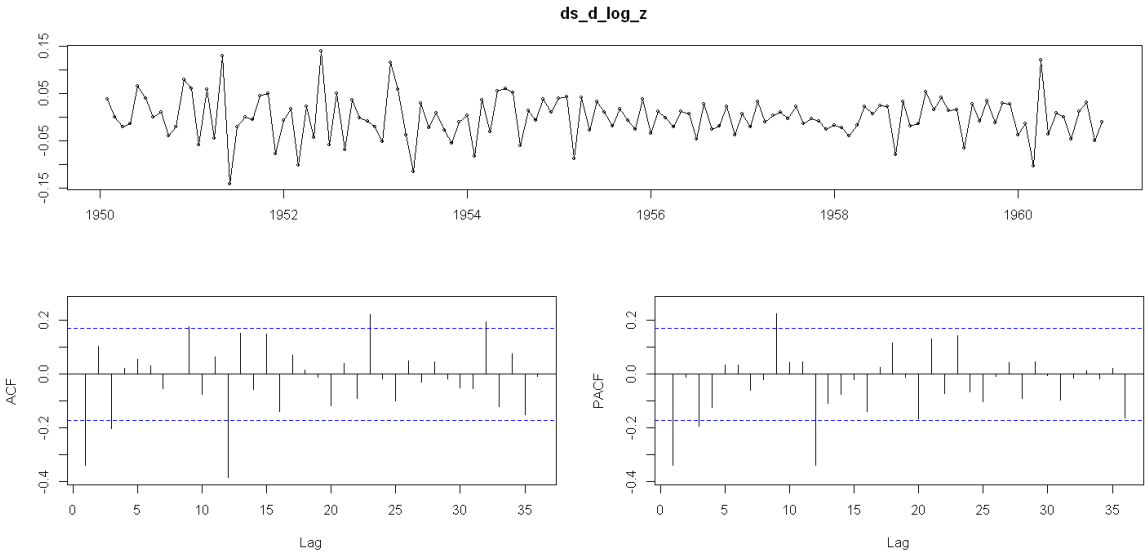

여전히 계절성분이 남아있으므로, 계절차분 진행

$$\Delta_{12}\Delta Z_t = (1-B^{12})(1-B)Z_t = (1-B^{12})(Z_t - Z_{t-1}) = Z_t - Z_{t-1} - Z_{t-12} - Z_{t-1}$$

In [25]:
```
ds_d_log_z <- diff(d_log_z, 12)
plot.ts(ds_d_log_z)
```



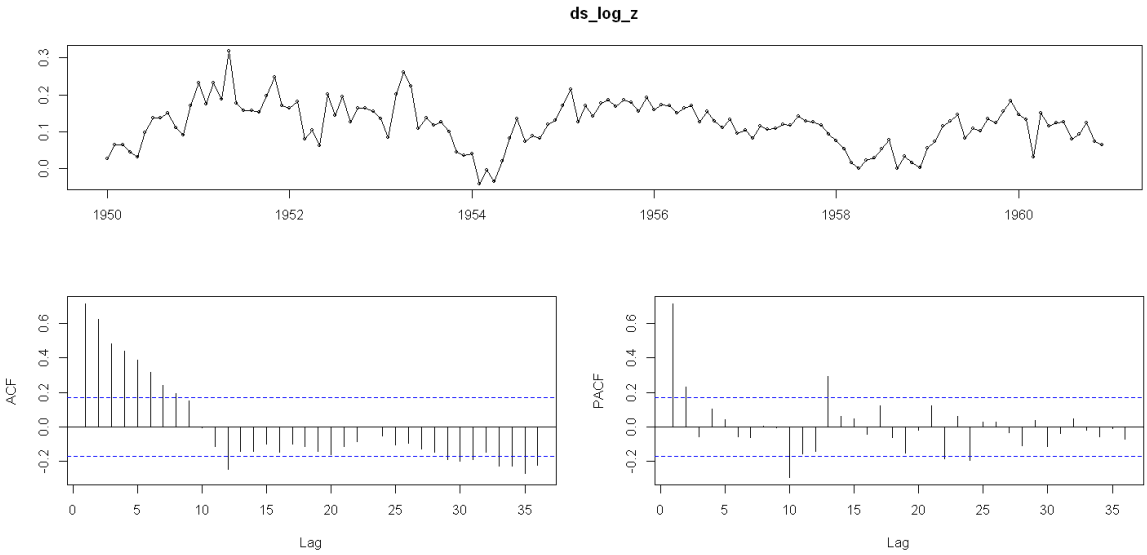

In [26]:
```
forecast::tsdisplay(ds_d_log_z, lag.max=36)
```




하지만, 일반적으로 계절성분과 추세가 동시에 있는 경우 계절차분을 먼저 진행한다.

In [27]:
```
ds_log_z <- diff(logz, 12)
ts.plot(ds_log_z)
```

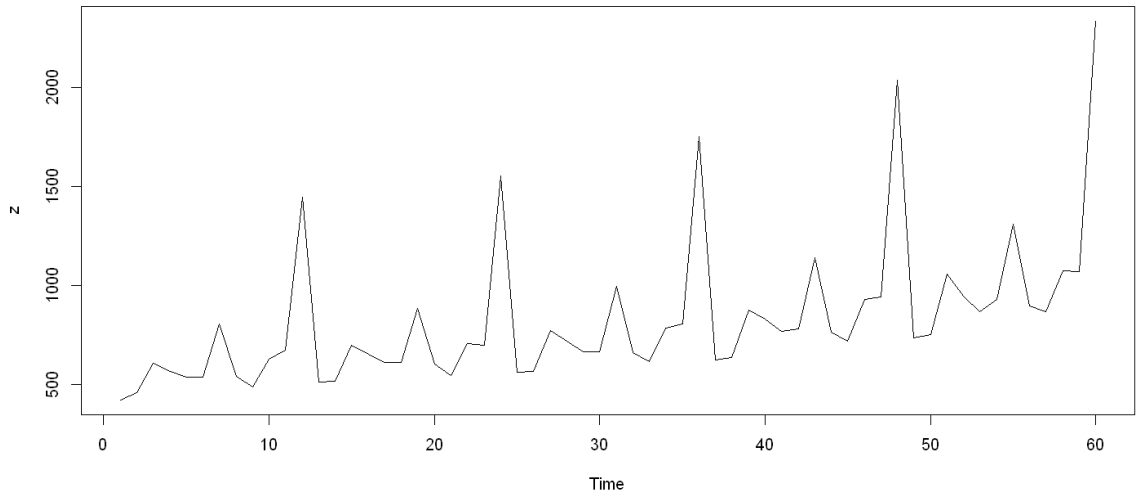

In [28]:
```
forecast::tsdisplay(ds_log_z, lag.max=36)
```



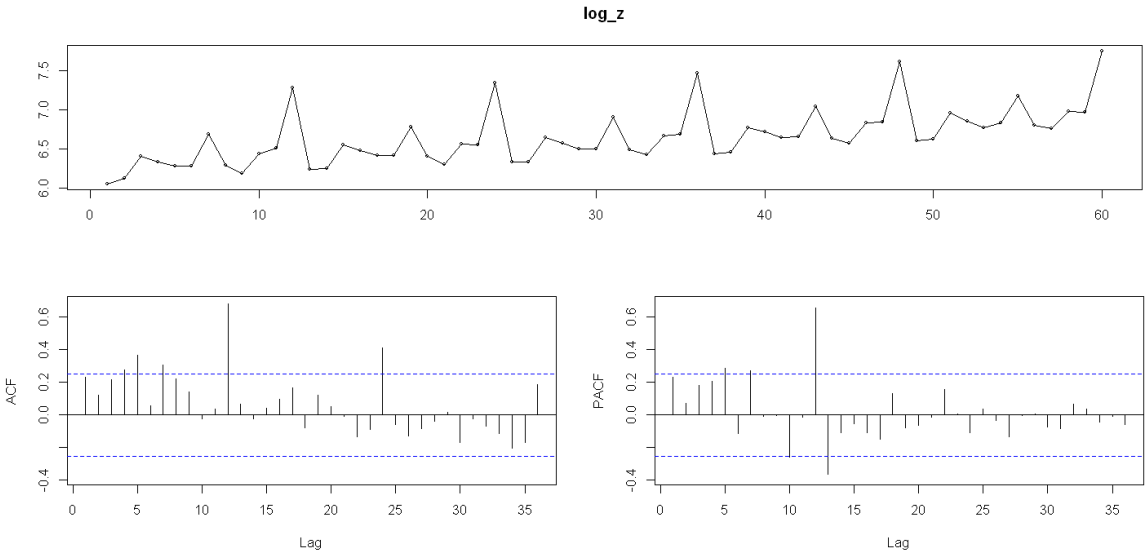

확률적추세가 있어 보이지만, AR 모형에서 ACF가 감소하는 형태라고 할 수도 있다. 이 경우에는 추세를 제거하기 위한 차분을 더 진행하는 것을 결정하는 것이 어렵다.

## depart data

In [29]:
```
z <-scan("depart.txt")
plot.ts(z)
```

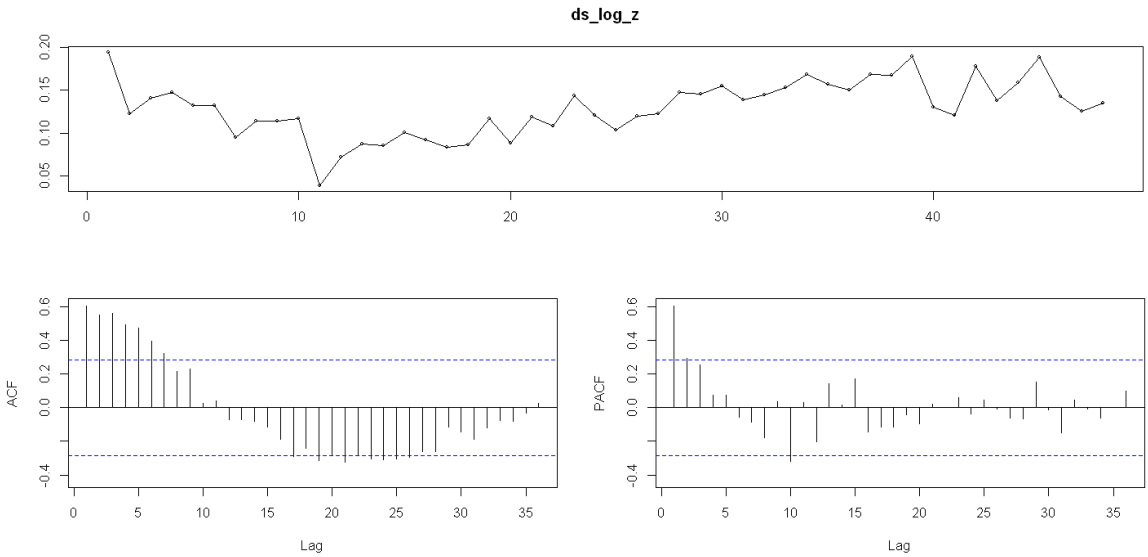



In [30]:
```
log_z <- log(z)
forecast::tsdisplay(log_z, lag.max=36)
```




계절차분을 먼저 진행

In [31]:
```
ds_log_z <- diff(log_z, 12)
forecast::tsdisplay(ds_log_z, lag.max=36)
```



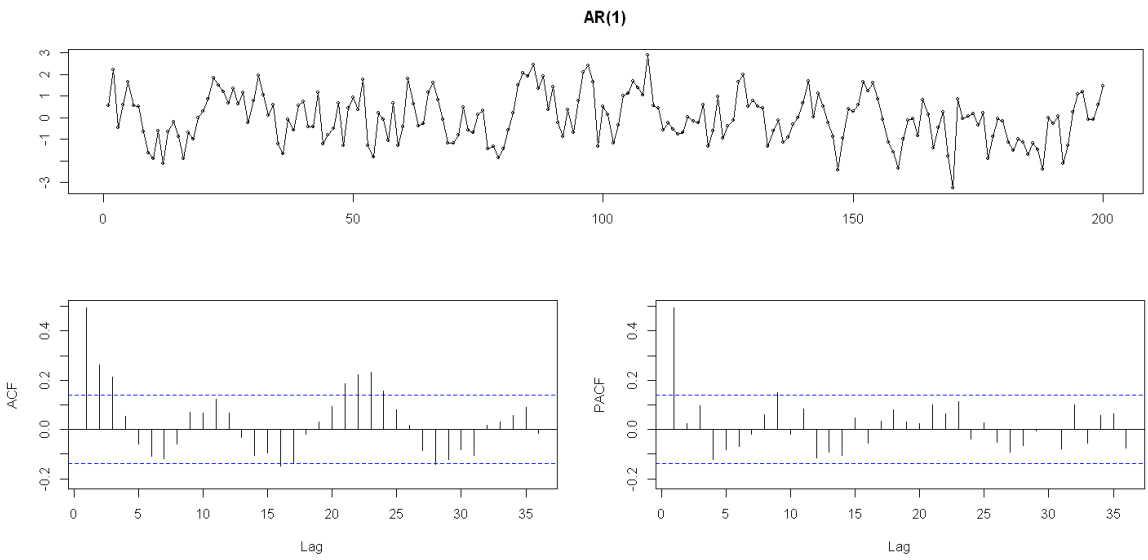

이 데이터 시도표 상으로는 확률적인 추세가 있어보이지만, 역시 ACF만 보고 차분이 필요한 가에 대한 결정을 하는 것이 쉽지 않다.

## 여러가지 Simulation

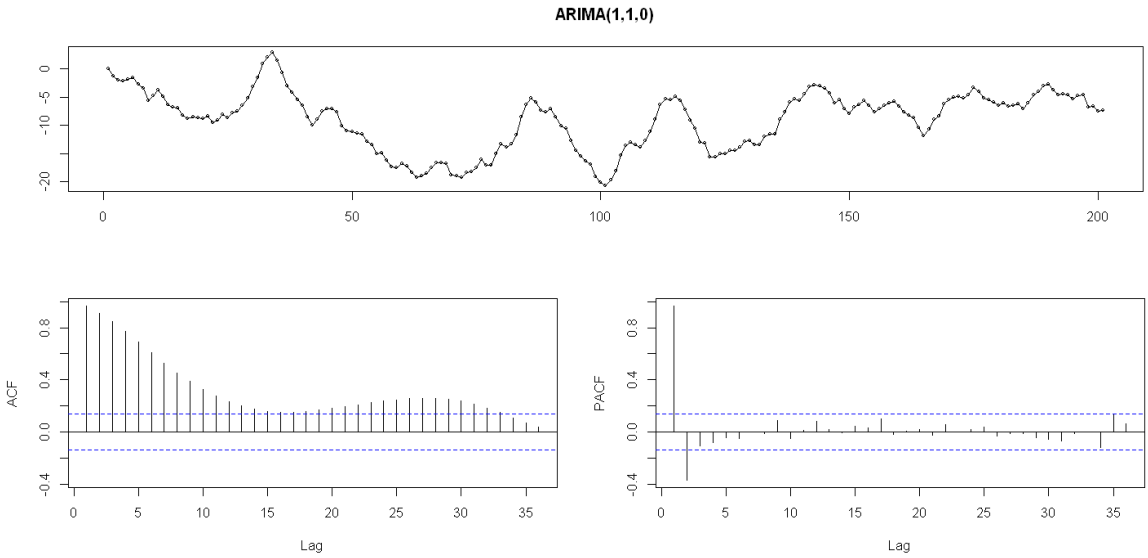
In [32]:
```
n <- 200

x = arima.sim(n=n, list(order=c(1,0,0), ar=0.5)) #AR(1)
z = arima.sim(n=n, list(order=c(1,1,0), ar=0.5)) #ARIMA(1,1,0)
```

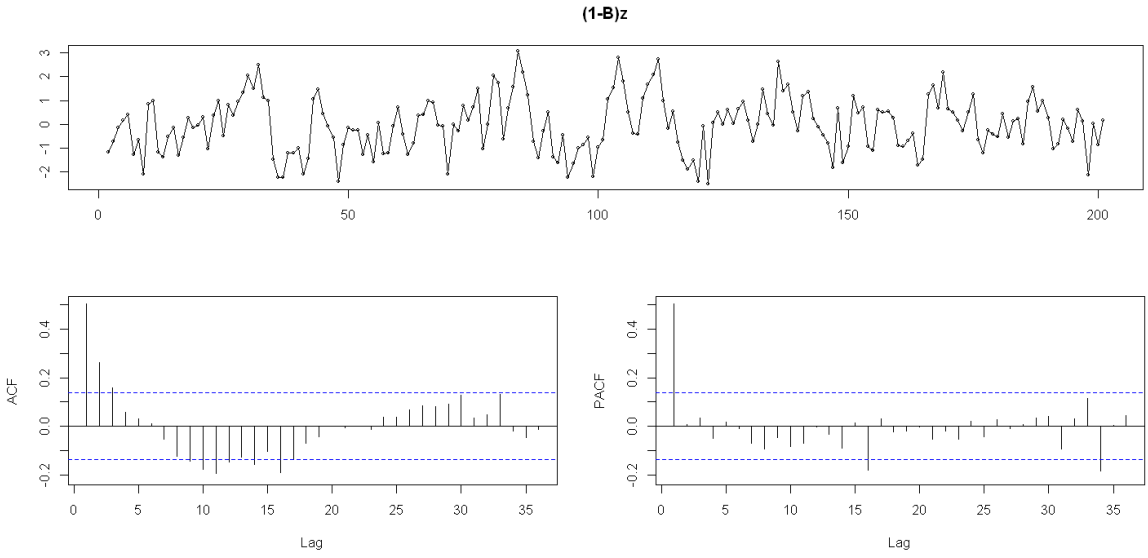
In [33]:
```
forecast::tsdisplay(x, lag.max=36, main = "AR(1)")
```




In [34]:
```
forecast::tsdisplay(z, lag.max=36, main = "ARIMA(1,1,0)")
```



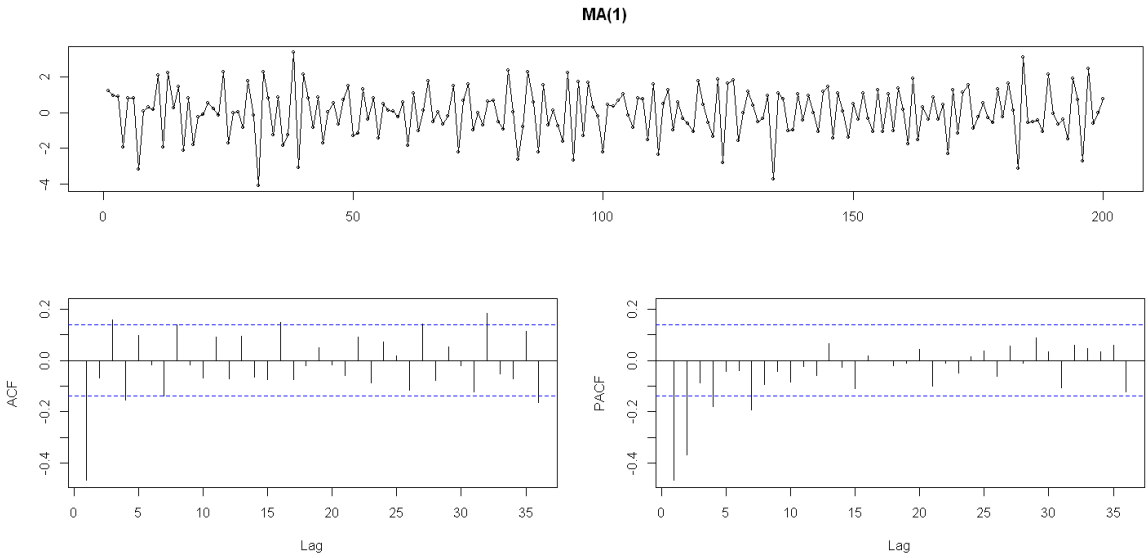

In [35]:
```
forecast::tsdisplay(diff(z), lag.max=36, main = "(1-B)z")
```



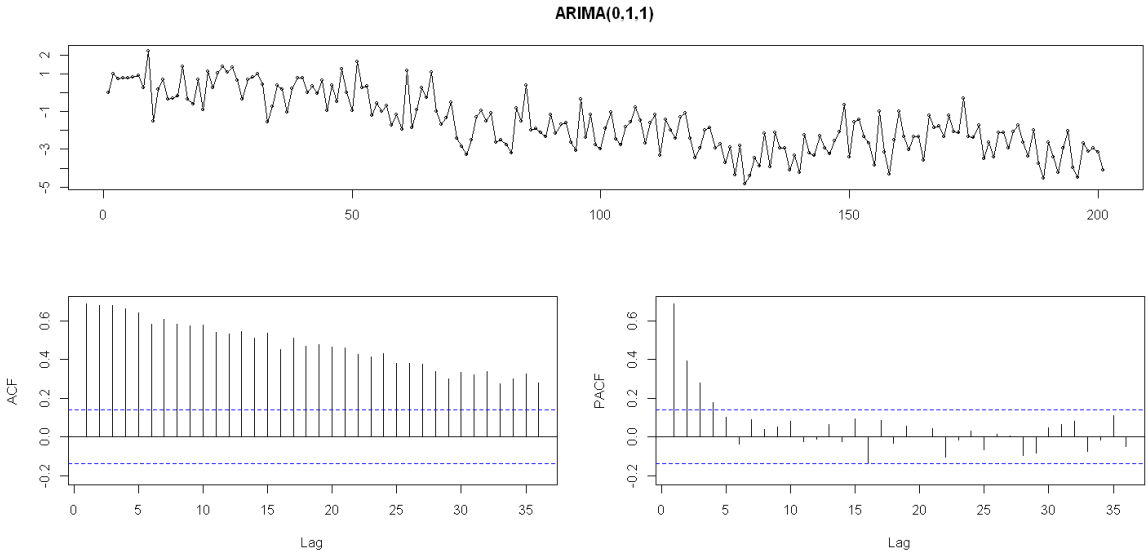

In [36]:
```
n <- 200

x = arima.sim(n=n, list(order=c(0,0,1), ma=-0.8)) #MA(1)
z = arima.sim(n=n, list(order=c(0,1,1), ma=-0.8)) #ARIMA(1,1,0)
```

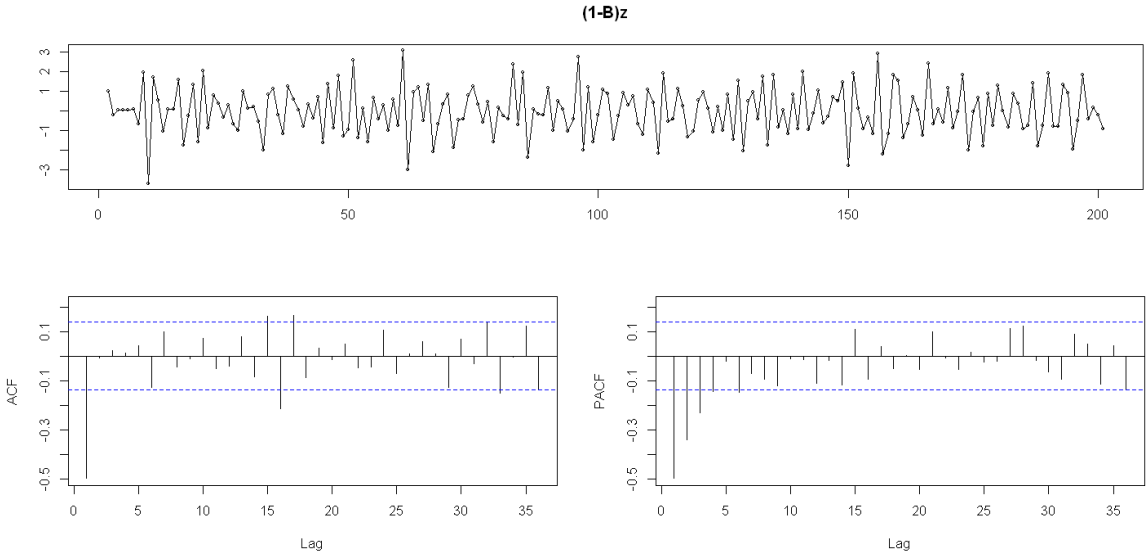
In [37]:
```
forecast::tsdisplay(x, lag.max=36, main = "MA(1)")
```




In [38]:
```
forecast::tsdisplay(z, lag.max=36, main = "ARIMA(0,1,1)")
```



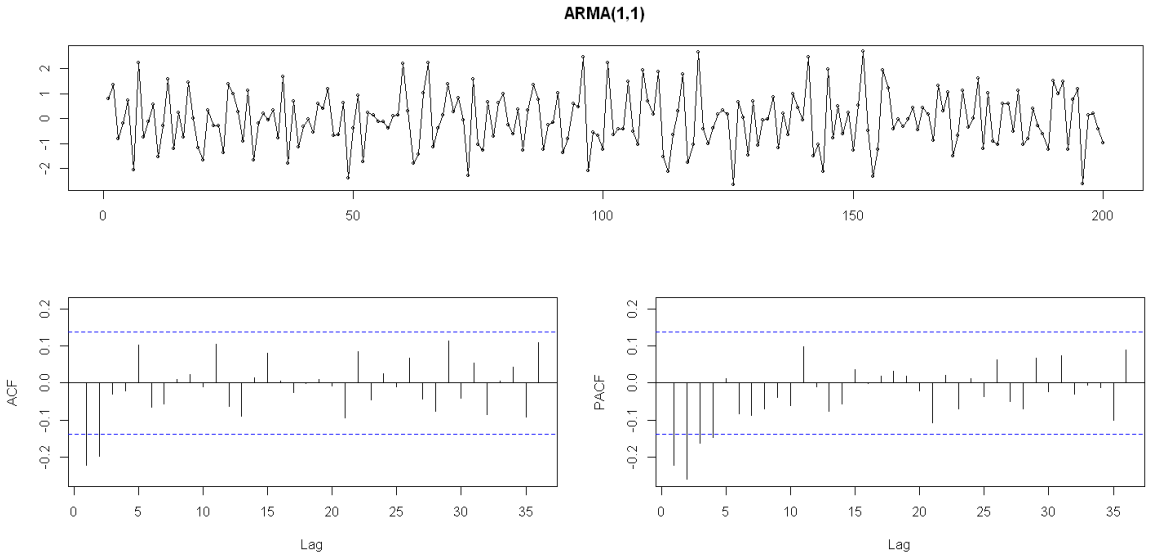

In [39]:
```
forecast::tsdisplay(diff(z), lag.max=36, main = "(1-B)z")
```



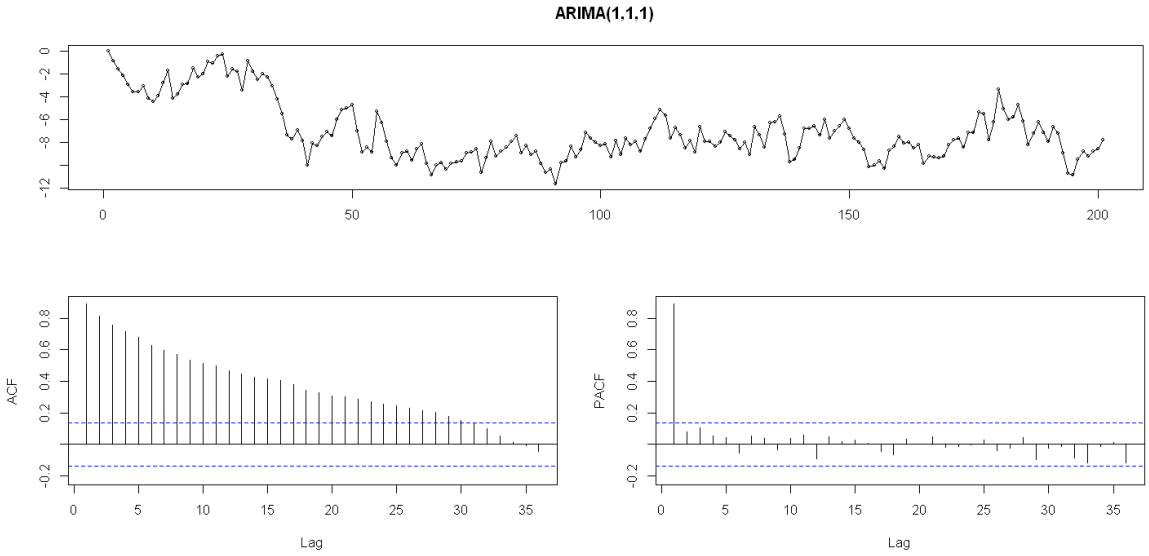
In [40]:
```
n <- 200

x = arima.sim(n=n, list(order=c(1,0,1), ar=0.5, ma=-0.8)) #ARMA(1,1)
z = arima.sim(n=n, list(order=c(1,1,1), ar=0.5, ma=-0.8)) #ARIMA(1,1,1)
```

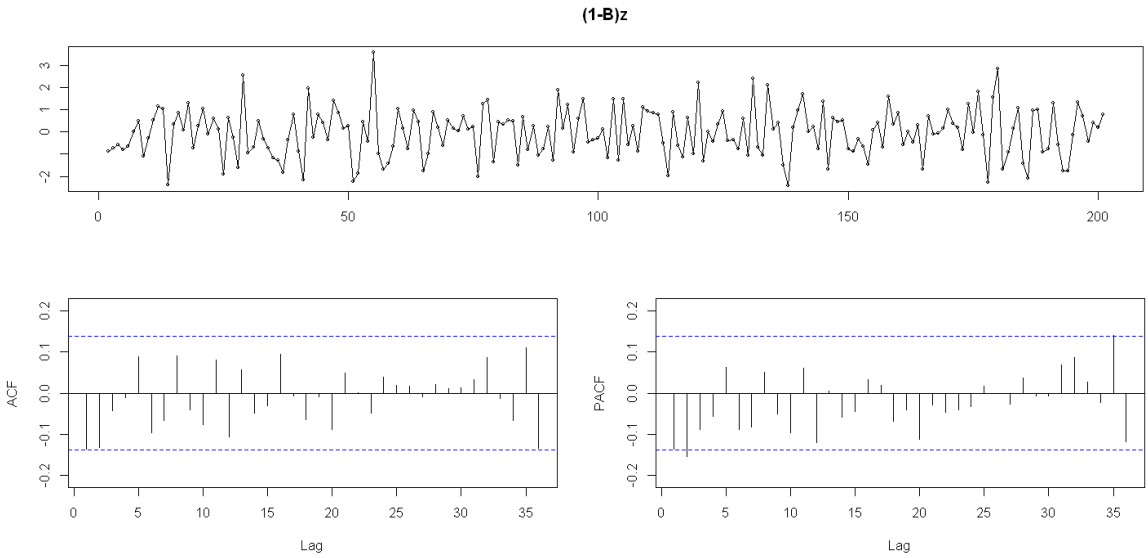
In [41]:
```
forecast::tsdisplay(x, lag.max=36, main = "ARMA(1,1)")
```




In [42]:
```
forecast::tsdisplay(z, lag.max=36, main = "ARIMA(1,1,1)")
```



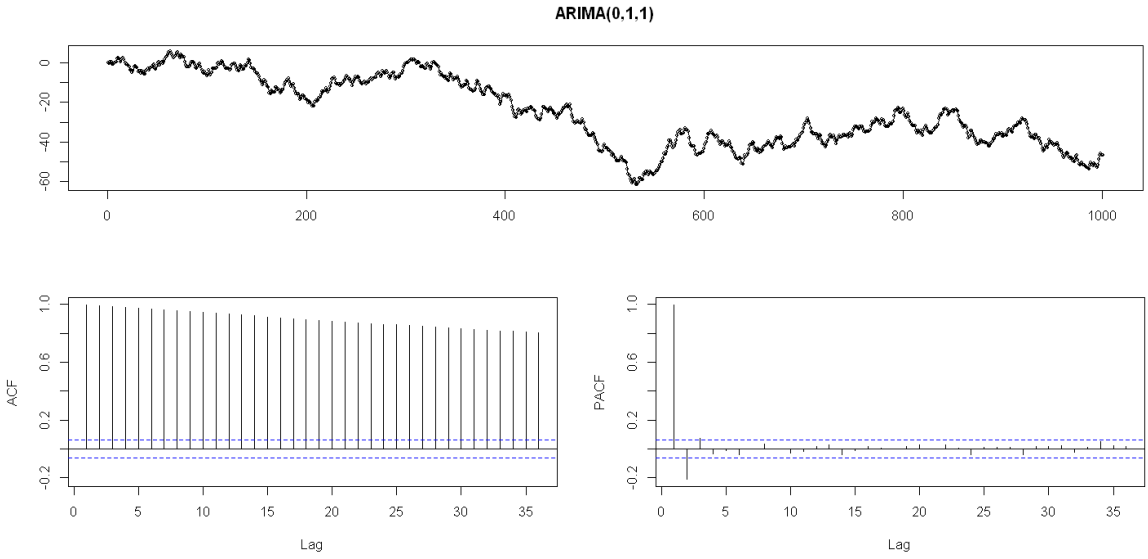

In [43]:
```
forecast::tsdisplay(diff(z), lag.max=36, main = "(1-B)z")
```



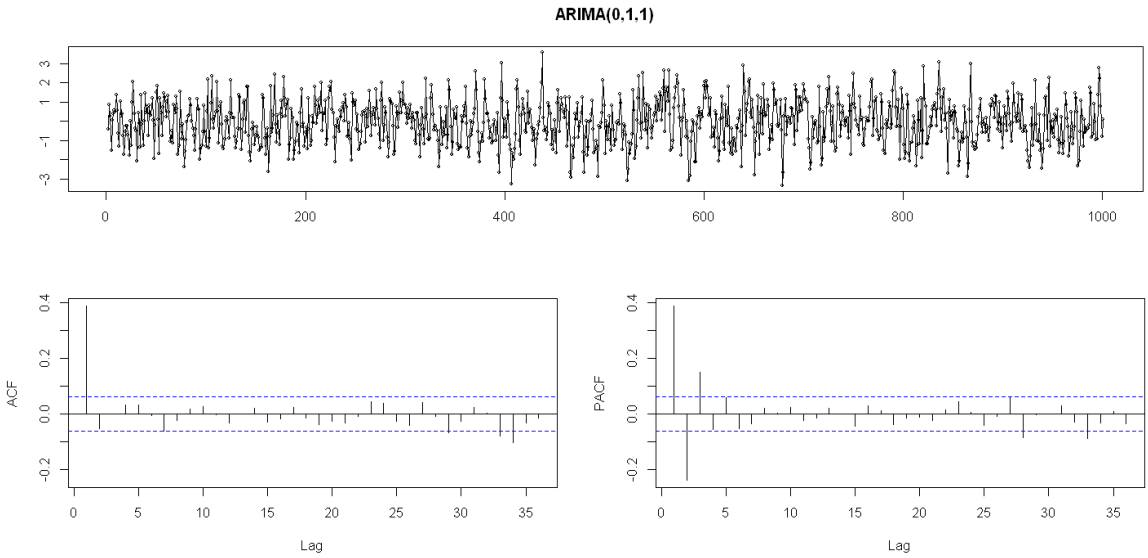

## 과대차분

In [44]:
```
## ARIMA(0,1,1)
z = arima.sim(n=1000, list(order=c(0,1,1), ma=0.5))
```

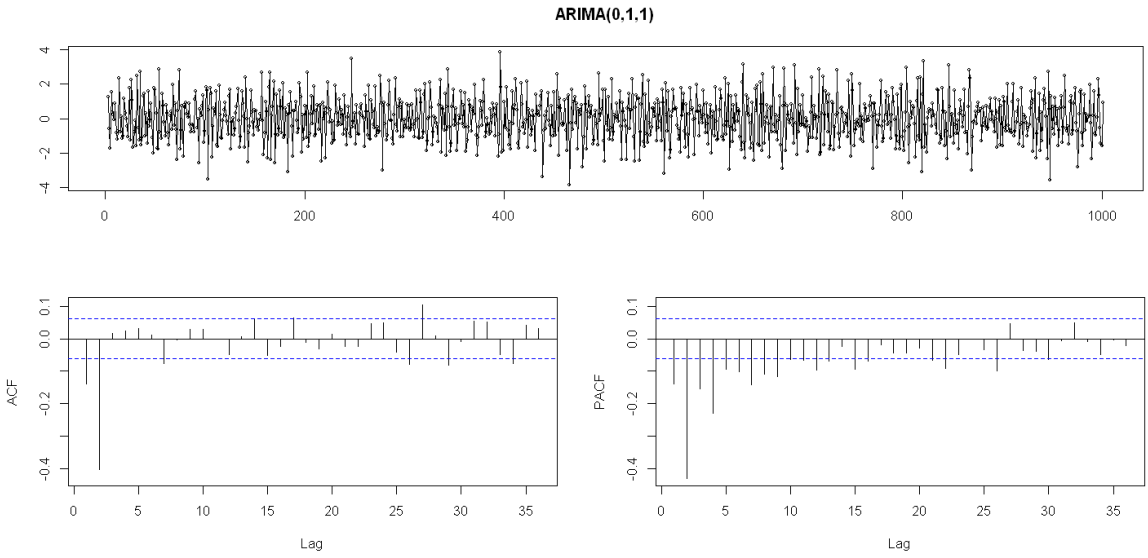
In [45]:
```
forecast::tsdisplay(z, lag.max=36, main = "ARIMA(0,1,1)")
```




In [46]:
```
#한 번 차분
d_z <- diff(z)
forecast::tsdisplay(d_z, lag.max=36, main = "ARIMA(0,1,1)")
```




In [47]:
```
# 한 번 더 차분
d2_z <- diff(d_z)
forecast::tsdisplay(d2_z, lag.max=36, main = "ARIMA(0,1,1)")
```




한 번 차분을 하면 정상시계열이 된다. 그리고 한 번 더 차분해도 마찬가지로 정상시계열이다.

In [48]:
```
sd(z)
sd(diff(z))
sd(diff(diff(z)))
```

17.3192131234587

1.14928524536831

1.27185972921689

차분을 두번 하고 나면 분산이 커진다. 이를 과대차분이라고 한다.

In [ ]:
```

```