Positive and Negative Relationships

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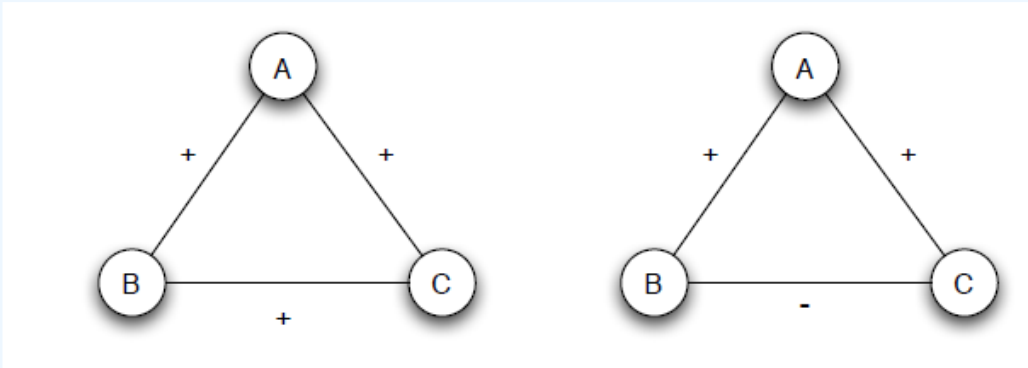
Relationships in the network

- Positive → Sharing on information, Friendship, Work Collaboration, etc.
- Negative → Disagreement, Controversy, etc.
- Annotate links → positive and negative links in the network → Aim : understand the tension between these two forces
- The notion of Structural Balance

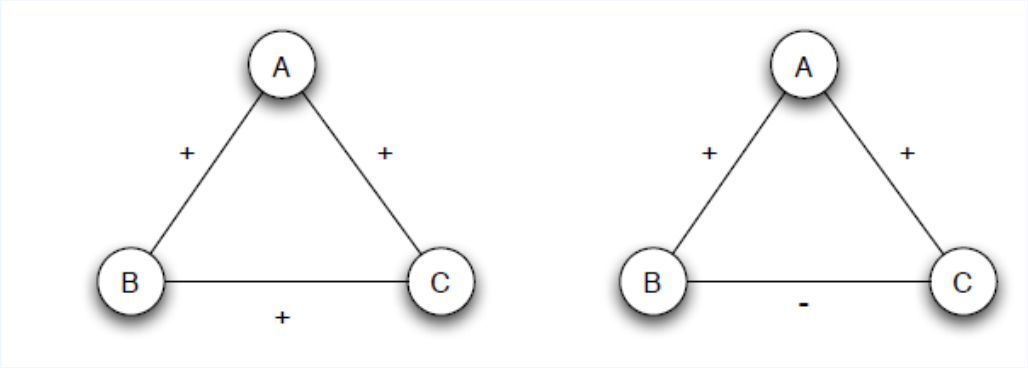
Positive and Negative Relationships

- A social network on a set of people:
  - Everyone knows everyone else in the set (a clique)
  - Edges are labeled with either + or -, where + indicates friends, and - indicates enemies
  - Example : Team of classmates, A sports team, etc.

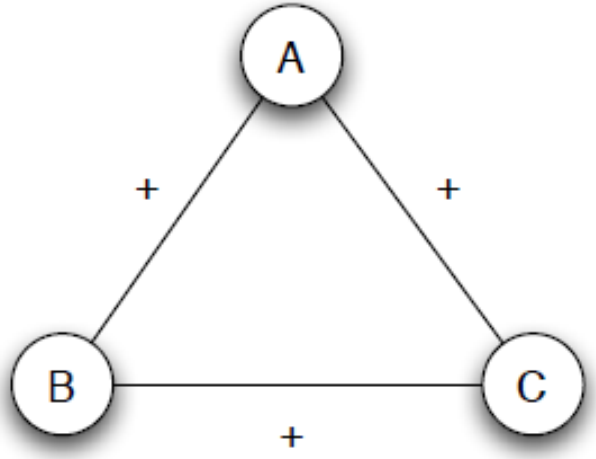


Structural Balance

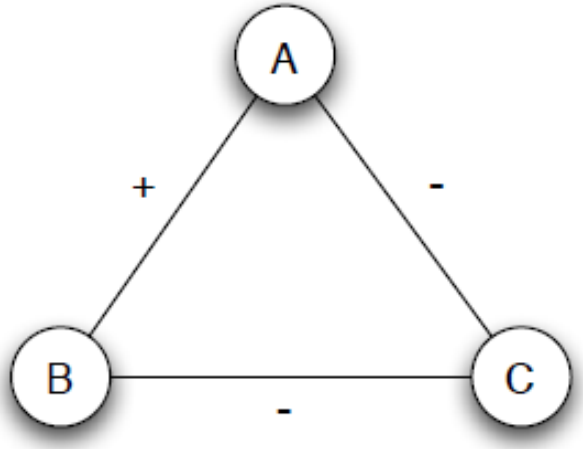
- Finding the balanced and unbalanced triangles
- What happens if network has positive/negative edges?
- What local configurations do we expect to see?
- How does this impact the global structure of the network?



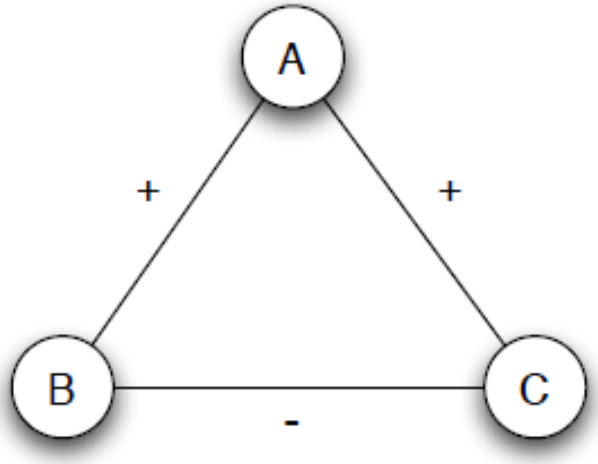
When looking a set of three people at a time in the network, how structural balance is exhibited?



(a) *A, B, and C are mutual friends: balanced.*

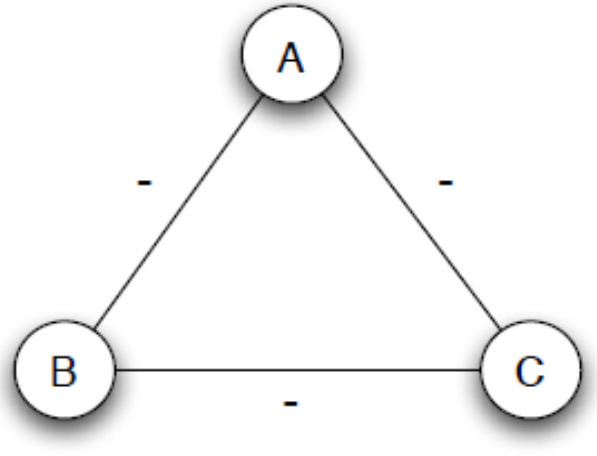


(c) *A and B are friends with C as a mutual enemy: balanced.*



(b) *A is friends with B and C, but they don't get along with each other: not balanced.*

Psychological "stress" or "instability" into the relationships



(d) *A, B, and C are mutual enemies: not balanced.*

Sources of instability in a configuration

Balanced Triangles

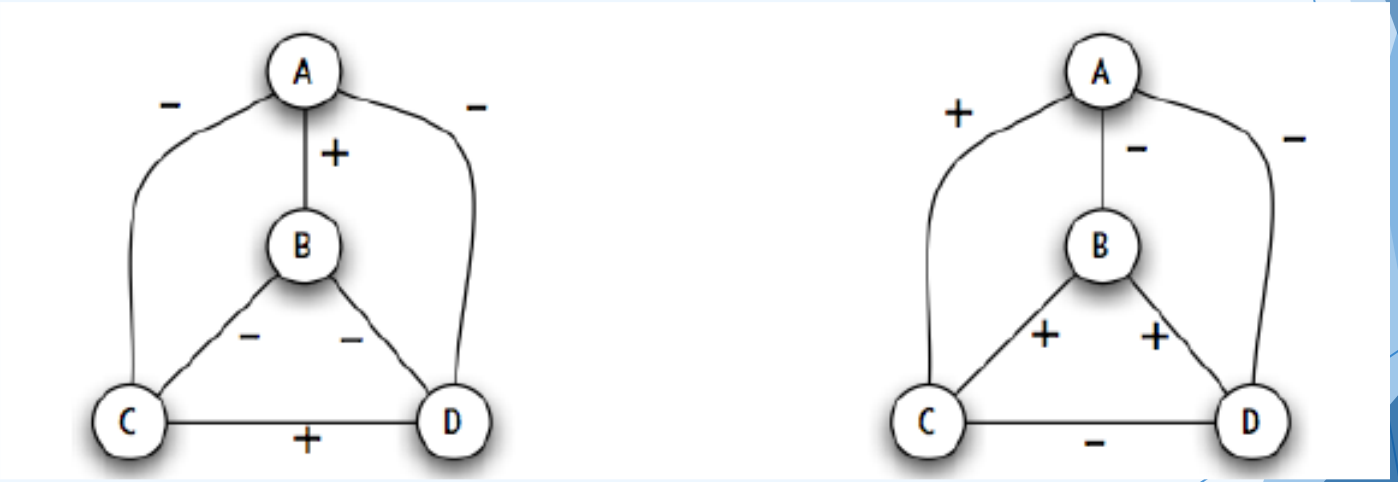
- A triangle with 1 or 3 ‘+’ edges → **Balanced triangle**
- A triangle with 0 or 2 ‘+’ edges → **Unbalanced triangle**
- Argument by structural balance theorists → As unbalanced triangles are sources of stress or psychological difference, people try to minimize them in their personal relationships, and hence they will be less abundant in real social settings than balanced triangles.

(Refer the figures in the previous two slides)

Structural Balance for Arbitrary Networks

- How to extend structural balanced to networks of arbitrary number of nodes (i.e., N > 3)?
- A labeled complete graph is balanced if every one of its triangles is balanced – that is, if it obeys the following:
- **Structural Balance Property** : **For *every* set of three nodes, if we consider the three edges connecting them, either all three of these edges are labeled +, or exactly one of them is labeled +.**
- It is a local property

Which one of the following networks satisfy the Structural Balance Property?



Characterizing the Structure of Balanced Networks

- What does a balanced network (i.e., a balanced labeled complete graph) look like?
  - Check all triangles → they each obey the balance conditions

- Consider A, B and C, D are two group of friends.
- -ve relationship exists between them.
- When is a network balanced?
  - One way → if everyone likes each other i.e. all triangles have three +
  - Two group of friends → how the groups are connected?

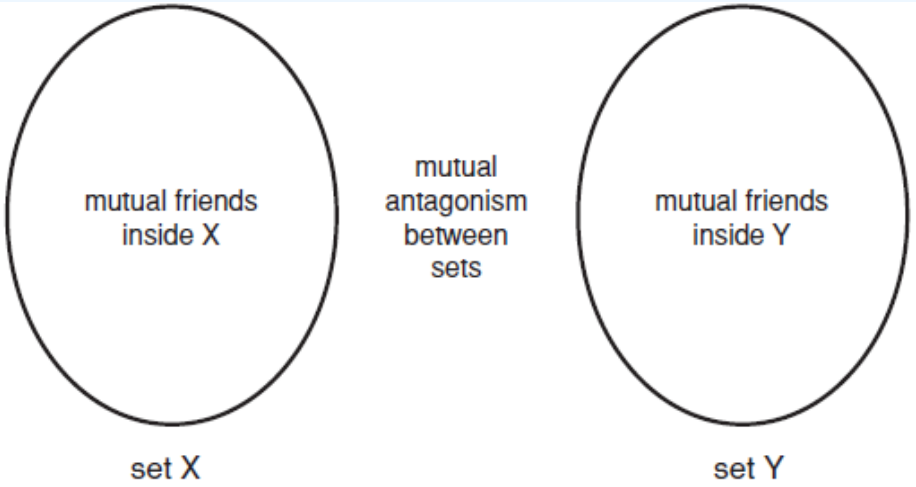


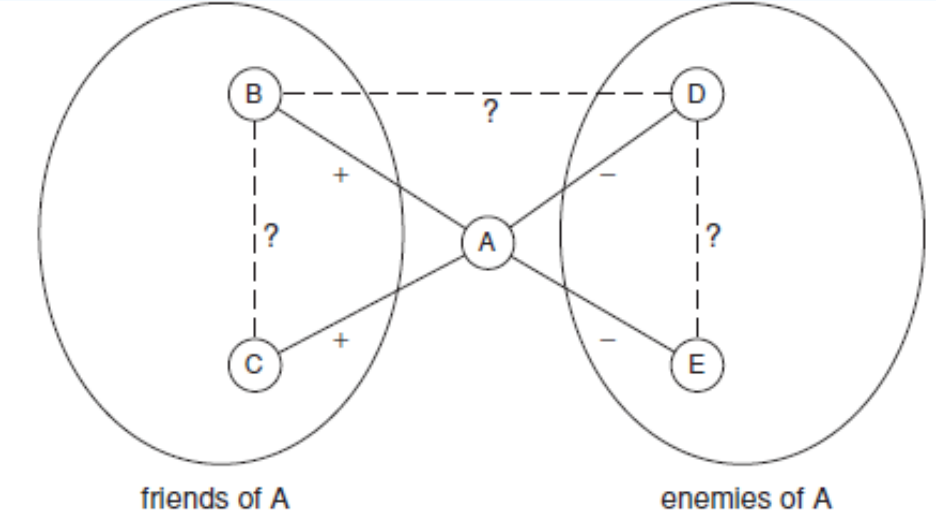
Figure 5.3. If a complete graph can be divided into two sets of mutual friends, with complete mutual antagonism between the two sets, then it is balanced. Furthermore, this is the only way for a complete graph to be balanced (other than graphs with all edges positive).

The Balance Theorem – Frank Harary (1953)

- If a labeled complete graph is balanced, then either all pairs of nodes are friends, or else the nodes can be divided into two groups, *X* and *Y* , such that each pair of people in *X* likes each other, each pair of people in *Y* likes each other, and everyone in *X* is the enemy of everyone in *Y* .

Proof

- If no negative edges, then the network is balanced.
- Assume if there is atleast one negative edge, to show that:
  i. Every two nodes in X are friends
  ii. Every two nodes in Y are friends
  iii. Every node in X is an enemy of every node in Y



Refer Textbook by Easley and Kleinberg for complete proof

Applications of Structural Balance

- International Relations
- Trust, Distrust, and Online Ratings

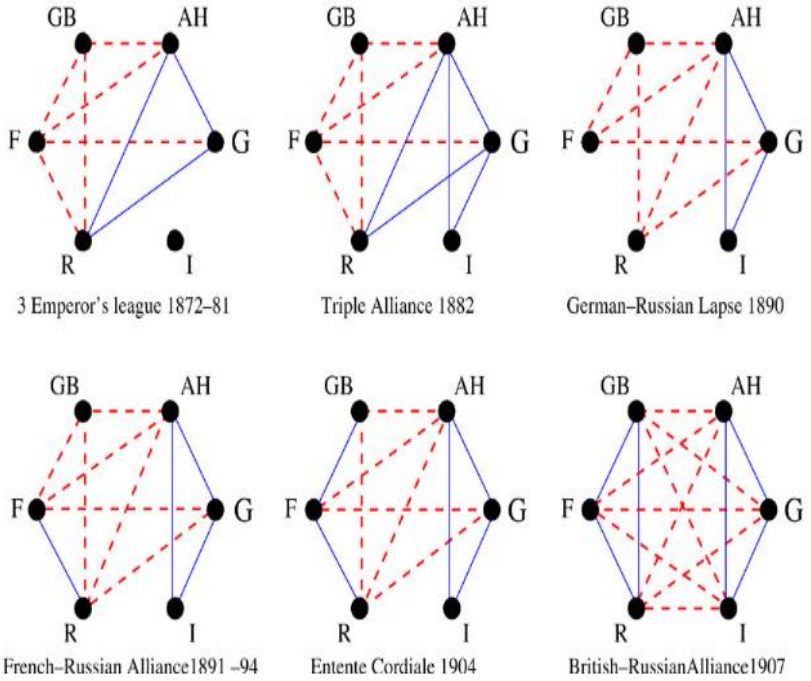
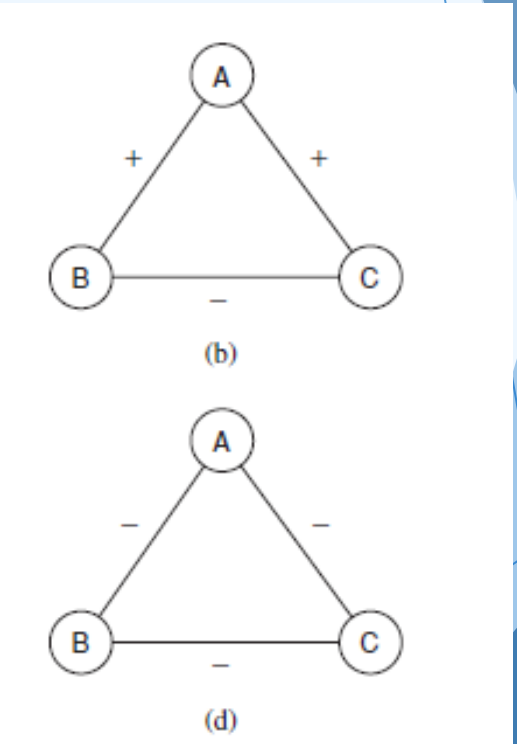


Fig. 10. Evolution of the major relationship changes between the protagonists of World War I from 1872–1907. Here GB = Great Britain, AH = Austria–Hungary, G = Germany, I = Italy,R = Russia, and F = France.

A weaker form of structural balance

- In a triangle with two positive edges, we have the problem of a person whose two friends don’t get along
- In a triangle with three negative edges, there is possibility that two of the nodes will ally themselves against the third
- Which of these factors are significantly stronger than the other?



Characterizing Weakly Balanced Network

- A complete labeled graph is weakly balanced if the following property holds:
- Weak Structural Balance Property: There is no set of three nodes such that the edges among them consist of exactly two positive edges and one negative edge.

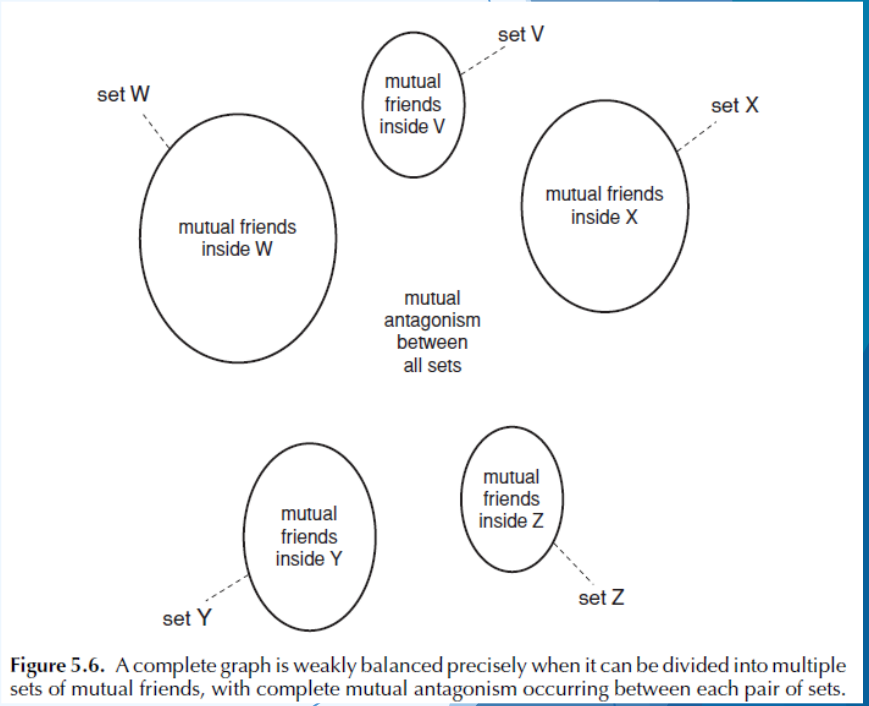
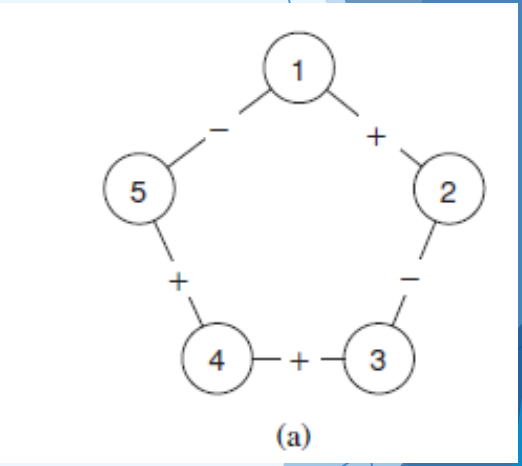
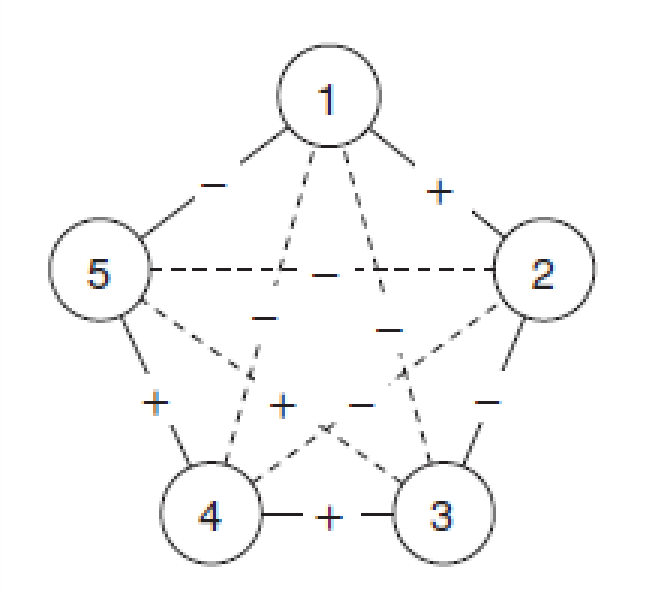


Figure 5.6. A complete graph is weakly balanced precisely when it can be divided into multiple sets of mutual friends, with complete mutual antagonism occurring between each pair of sets.

Structural Balance in Arbitrary (Non-complete) Networks

- How to define structural balance for the arbitrary network which is not complete?
- Two ways:
  - Fill remaining edges with label to produce a signed complete graph which is balanced?
  - The other way → whether it is possible to divide the nodes into two sets, *X* and *Y* , so that all edges inside *X* and inside *Y* are positive, and all edges between *X* and *Y* are negative.

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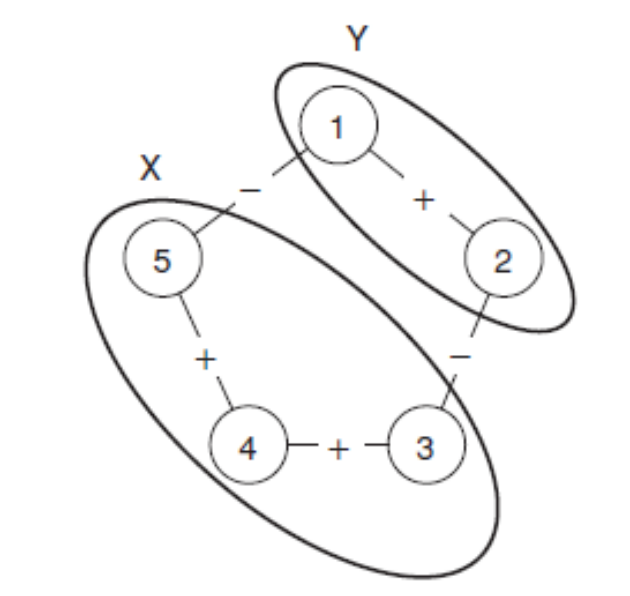
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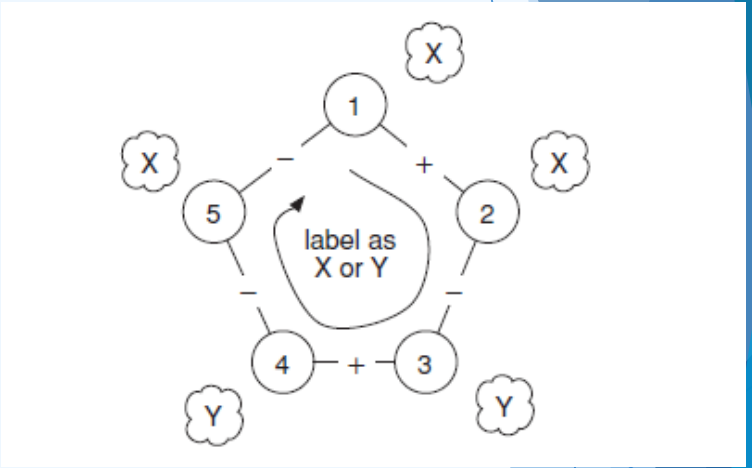
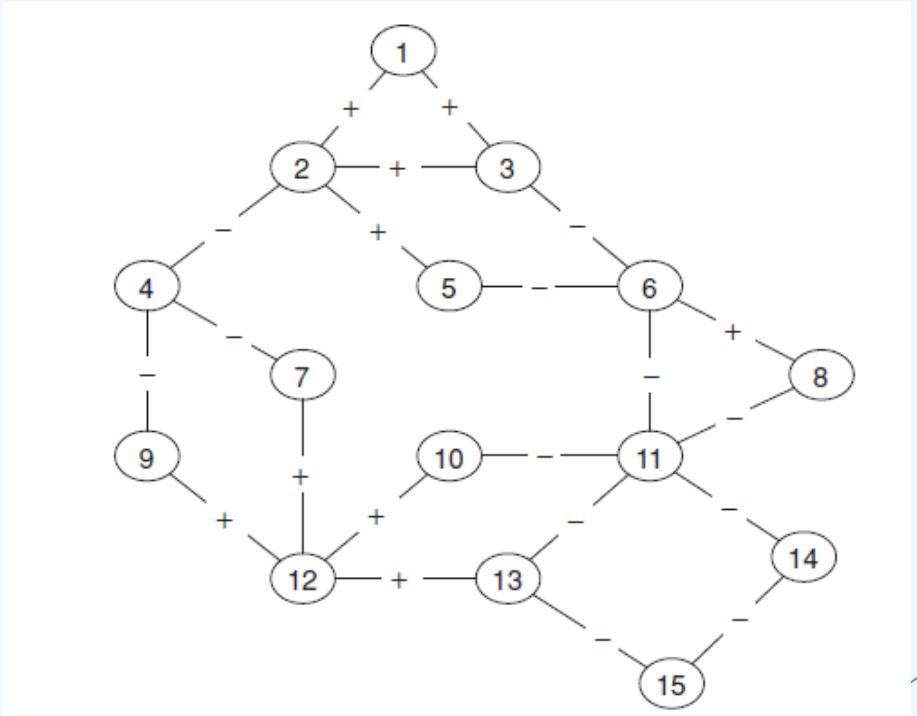
Characterizing Balance for General Networks.

- If a signed graph contains a cycle with an odd number of negative edges, then it is not balanced.
- If one of the nodes is placed in *X* , then following the set of friend/enemy relations around the cycle produces a conflict by the time we get to the starting node.



How to determine if a signed graph is balanced?



Claim: A signed graph is balanced if and only if it contains no cycle with an odd number of negative edges.

- **Proving the Characterization by identifying Supernodes**
- Find a division of the nodes into sets *X* and *Y* so that all edges inside *X* and *Y* are positive, and all edges crossing between *X* and *Y* are negative.
- Producing such a partition into sets *X* and *Y* with these properties it is called a ***balanced division***.

- The procedure works in two main steps:
- First step → to convert the graph to a reduced form that only contains negative edges
- Second step → to solve the problem on this reduced graph.
- Blobs - supernodes