Test : - $\boxed{\text{assumptions}}$

- Hypo :
- test statistic & its null distn.

$\bar{\bar{Y}}$ $\qquad$ $\bar{Y}_i$ $\qquad$ Y_{ij} $\qquad$ $\underbrace{Y_{i1}, \ldots Y_{iJ}}_{i\text{th}}$

var within a group : i : $\boxed{(Y_{ij} - \bar{Y}_i)^2}$

$X_1 \ldots X_n \to \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$

var between groups : $\bar{Y}_1, \bar{Y}_2, \ldots \bar{Y}_I$

$\bar{\bar{Y}}$

1^{th}	2^{nd}	I^{th}
Y_{11}		Y_{I1}
Y_{12}		
Y_{1J}		Y_{IJ}

$(Y_{ij} - \bar{\bar{Y}})^2 \to$ var of whole

Anova :

- Hypo ?
- How to form test statistic $\left.\begin{matrix} SS_B \\ SS_W \end{matrix}\right] \to F$ | slide 15-18
- null distn of $F \to$ slide 19.
- search , F distn & def of F-distn.

Anova : test to compare means ⊛

H_0 : all means are equal *

$\Rightarrow$ all α_i are the same, $i = 1, I$.

(I groups.

$i = 1, \ldots I$

$j = 1, \ldots, J$ //

$I = 7$;

$J = 10$.

$Y_{ij} = \mu + \alpha_i + \underbrace{e_{ij}}_{\text{random error}} \sim N(0; \sigma^2)$.

~~I~~ group 1: $\mu + \alpha_1$

2: $\mu + \alpha_2$

⋮

I: $\mu + \alpha_I$

$\boxed{\alpha_i}$ same

$$\boxed{\sum_{i=1}^{I} \alpha_i = 0}$$

Test statistic $\Rightarrow$ variance analysis :

within group var $\leftarrow SS_W$

between group var $\leftarrow SS_B$

$\Rightarrow$ F = formular of SS_B & SS_W.

F_ distn ? 2 df : df_1 & df_2.

$\chi^2_{df_1}$ has df = df_1

$\chi^2_{df_2}$ has df = df_2

are indpdn.

$\boxed{t_{df}} \sim Z$.

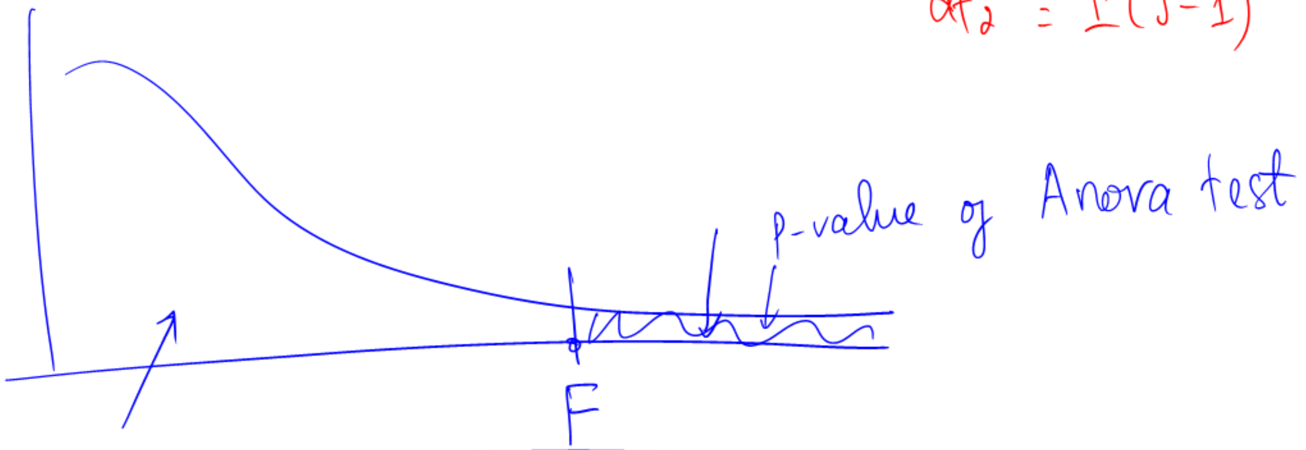
$\dfrac{\chi^2_{df_1} / df_1}{\chi^2_{df_2} / df_2}$ has F distn with df_1 and df_2.

Using the definition of F distn, we can get the distn of the null distn test statistic

$$F = \frac{SS_B/(I-1)}{SS_W/[I(J-1)]} \overset{H_0}{\sim} \underline{F_{df_1, df_2}}$$ where

$df_1 = I - 1$ &

$df_2 = I(J-1)$



Anova assumptions checking:

- equal var of I groups. ← Bar or Levene
- normality of I groups → Bar or Levene
- normality of residuals, indpd. hist, QQ plot, Shapiro test.

if each group is not normal but has large size → can accept.

21 pairwise tests ⇒ prob of having at least 1 type 1 error: 66%

⇒ too large!

⇒ need correct it. to reduce from 66% ↓ 10% only (for eg)

⇒ instead of controlling $\alpha = 0.05$ (5%) for each test, we'll control $\alpha = 0.1$ for all 21 tests. → goal.

→ we are using "correction"! ⇒ Bonferroni ←

⇒ Tukey ←

Dunnet ...

For Bonf: have total k pairwise tests ($k = 21$) $\Rightarrow$ we allow
$a = \underline{0.1}$ type 1 error for all k tests, then
for each pairwise test, we use a sig. level $\alpha = \frac{a}{k}$.

$$\alpha = \boxed{\frac{0.1}{21}}$$

$\Rightarrow$ this idea works quite well when k is small.

For Tukey correction:

for each pair of 2 groups, they form a CI for the diffe of the means. 0.1

$(\mu_1 - \mu_2)$
$(\mu_1 - \mu_3)$
$\mu_1 - \mu_7$
⋮
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Recal: CI and 2-sided test: 95%

$H_0: \mu_1 - \mu_2 = 0$
$\Downarrow$
p-value $<$ 0.05

get CI for $(\mu_1 - \mu_2)$: $(-0.5; -0.1)$

$0 \neq \mu_1 - \mu_2$

not conf. level for each interval but a conf. level of $(1 - a)$ for all the k intervals.

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in general: CI for $\mu_1 - \mu_2$:
$\Downarrow$

$$(\bar{Y}_1 - \bar{Y}_2) \pm \underbrace{\boxed{} * SE(\bar{Y}_1 - \bar{Y}_2)}$$

$\boxed{\text{point estimate} \pm \text{margin of error}}$.

For each pairwise comparison test: lab 1 vs lab 2 : μ_1 vs μ_2.
$\Rightarrow$ alpha $\alpha = 0.05$ $\Rightarrow$ compare the real p-value vs α.

For the correction (Bonf or Tukey), we donot use α for each test, but we use a (FE Rate) $\Rightarrow$ compare the adjusted p-value of each test vs a.

For eg: Bonf: $k = 21$ tests
lab1 vs lab2 → p-value: 0.01 ←
With Bonf correction, $a = 0.1$ then → adjusted p-value vs $a = 0.1$
or : p-value vs $\frac{a}{k} = \frac{0.1}{21}$
0.01 vs $\frac{0.1}{21}$

Usually, output of Bonf correction or Tukey ~~corre~~ provide
adjusted-p-value → vs a (FE rate).

$\boxed{\text{Comparing b/w Bonf vs Tukey}}$
if the samples are under Anova ⇒ Tukey > Bonf.
if the samples are not under Anova (under Kruskal Wallis)
then Tukey cannot be used → only use Bonf.