Summary: Composite Integration (Chapter 6)

Nodes: except for Gaussian and discrete formulas, $x_i = a + (i-1)h$, for $h = (b-a)/n$ and $i = 1, 2, \cdots, n+1$

Midpoint

$$I_M = h\Big[f\big(x_1 + \frac{h}{2}\big) + f\big(x_2 + \frac{h}{2}\big) + \cdots + f\big(x_n + \frac{h}{2}\big)\Big]$$

where $E_M = \frac{b-a}{24} h^2 f''(\eta)$

Trapezoidal

$$I_T = h\left(\frac{1}{2}f_1 + f_2 + f_3 + \cdots + f_n + \frac{1}{2}f_{n+1}\right)$$

where $E_T = -\frac{b-a}{12} h^2 f''(\eta)$

Simpson

$$I_S = \frac{h}{3}(f_1 + 4f_2 + 2f_3 + 4f_4 + 2f_5 + \cdots + 4f_n + f_{n+1})$$

where $E_S = -\frac{b-a}{180} h^4 f''''(\eta)$

Hermite

$$I_H = I_T + \frac{1}{12}h^2\big(f_1' - f_{n+1}'\big)$$

where $E_H = \frac{b-a}{720} h^4 f''''(\eta)$

Gaussian

$$I_G = w_1 f(z_1) + w_2 f(z_2) + \cdots + w_m f(z_m)$$

where $|E_G| \leq \frac{\alpha}{\sqrt{m}} R^{2m} ||f^{(2m)}||_\infty$ for $R = (b-a)e/(8m)$ and $\alpha = (b-a)\sqrt{\pi}/4$

k	$f(x)$	$\int_a^b f(x)dx$
0	1	ℓ
1	x	ℓx_m
2	x^2	$\ell\left(x_m^2 + \frac{1}{12}\ell^2\right)$
3	x^3	$\ell x_m\left(x_m^2 + \frac{1}{4}\ell^2\right)$
4	x^4	$\ell\left(x_m^4 + \frac{1}{2}\ell^2 x_m^2 + \frac{1}{80}\ell^4\right)$
5	x^5	$\ell x_m\left(x_m^4 + \frac{5}{6}\ell^2 x_m^2 + \frac{1}{16}\ell^4\right)$

Table 1: Values for $\int_a^b x^k dx$, where $\ell = b - a$ and $x_m = (b + a)/2$.

Discrete Data

Trapezoidal:

$$I_T = \frac{1}{2}\Big[h_1 f_1 + (h_1 + h_2)\, f_2 + (h_2 + h_3)\, f_3 + \cdots + (h_{n-1} + h_n)\, f_n + h_n f_{n+1}\Big]$$

Cubic Spline:

$$\int_a^b f(x)dx \approx \frac{1}{3}I_T + \frac{2}{3}\sum_{i=1}^{n} h_i s\left(x_i + \frac{1}{2}h_i\right)$$