Summary: Linear Least Squares (Chapter 8)

Basic: $y = v_1 + v_2 x$

Error Function

$$E(v_1, v_2) = \sum_{i=1}^{n} (v_1 + v_2 x_i - y_i)^2$$

Solution

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{1}{n\sum x_i^2 - \left(\sum x_i\right)^2} \begin{pmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{pmatrix} \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix}$$

Error Function

$$E(v_1, v_2) = \sum_{i=1}^{n} \left(\frac{v_1 + v_2 x_i - y_i}{y_i} \right)^2$$

Solution

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{1}{ad - b^2} \begin{pmatrix} a & -b \\ -b & d \end{pmatrix} \begin{pmatrix} \sum 1/y_i \\ \sum x_i/y_i \end{pmatrix}$$

where $a = \sum x_i^2/y_i^2$, $b = \sum x_i/y_i^2$, and $d = \sum 1/y_i^2$

General:

Error Function

$$E(\mathbf{v}) = ||\mathbf{A}\mathbf{v} - \mathbf{y}||_2^2$$

Moore-Penrose Solution

$$\mathbf{v} = \mathbf{A}^+ \mathbf{y}$$

where

$$\mathbf{A}^+ \equiv (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$$

QR Solution (GS version)

$$\mathbf{R}_m \mathbf{v} = \mathbf{y}_m$$

where $\mathbf{R}_m = \mathbf{G}^T \mathbf{A}$, $\mathbf{y}_m = \mathbf{G}^T \mathbf{y}$, and $\mathbf{G} = \mathrm{GS}(\mathbf{A})$

SVD Solution: $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$

$$\mathbf{A}^+ = \mathbf{V}\boldsymbol{\Sigma}^{-T}\mathbf{U}^T$$