Numerical Computing F'22
Exam 3 Name: _____

There are 8 questions. You can use a crib sheet but no calculators or other notes are to be used. Make sure to show your work, any answer without supporting work will receive no credit.

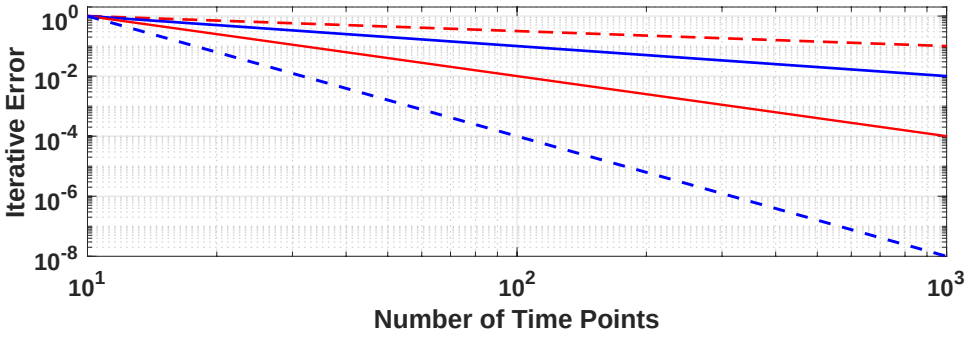
1. a) Find an approximation of $y''(t_j)$ that uses $y(t_j)$, $y(t_{j-1})$, and $y(t_{j-2})$.

b) Is the truncation error $O(k)$, $O(k^2)$ or something else?

2. Each line in the figure below represents the iterative error when solving an IVP as a function of M (i.e., the number of time points).

(a) Which line, if any, should correspond to the trapezoidal method?

(b) What would the truncation error for the method have to be to obtain the blue dashed line?



3. To solve the differential equation $y' = f(t, y)$ one can use the finite-difference equation

$$y_{j+1} = y_j + k\big[\theta f_j + (1-\theta) f_{j+1}\big],$$

where θ is a number that satisfies $0 \le \theta \le 1$ (one first picks θ from this interval and then uses the above formula to calculate the solution).

(a) For what value(s) of θ is the method explicit, and for which value(s) of θ is it implicit?

(b) For what value(s) of θ is the method A-stable?

4. a) What does the 4 indicate in RK4?
b) What two methods were used to derive the velocity verlet method?

c) In two dimensions, when solving $\mathbf{Ax} = \mathbf{b}$, what does the SDM do that causes it to take so many steps to converge?

5. Find the least squares solution of the following:

$$\begin{aligned} x - 2y &= 0 \\ x + y &= 3 \\ -2x + y &= 1 \end{aligned}$$

6. Suppose the data below is to be fit using the model function

$$y = \frac{v_1}{1 + v_2 e^x}\,.$$

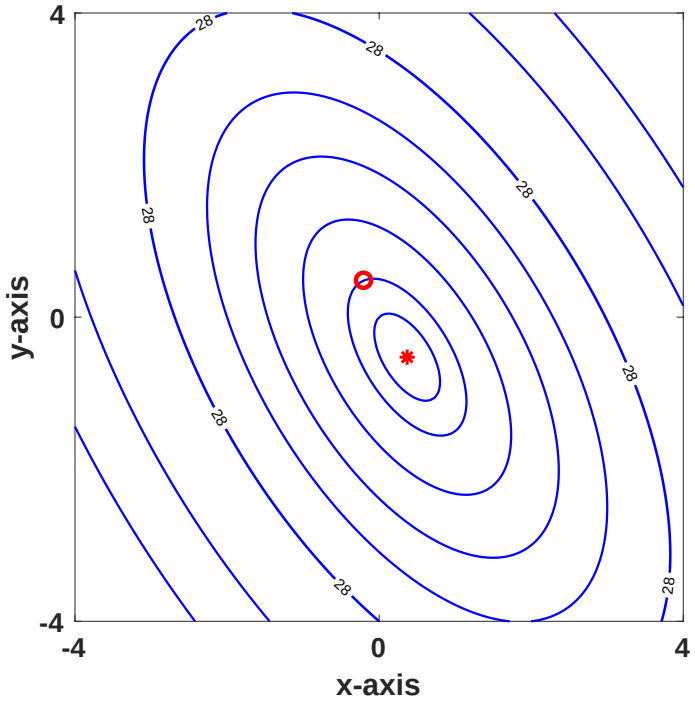
Explain how to transform the problem into a linear regression problem of the form $Y = V_1 + V_2 X$, where V_1 and V_2 are the fitting parameters. Also, rewrite the data table in terms of Y and X. You do not need to find V_1 and V_2.

x	0	1	3
y	1	3	2

7. The SDM is used to find the minimum of a function $F(\mathbf{x})$, and the contours for this function are shown in the figure below.

(a) Shown in the plot is the second point $\mathbf{x}_2$ calculated using the SDM. If the first point $\mathbf{x}_1$ is located on the contour labeled with 28, where is $\mathbf{x}_1$ located? Make sure to explain how you arrive at your answer. Also, you only need to find one location for $\mathbf{x}_1$.

(b) Suppose the starting point $\mathbf{x}_1$ is located on the contour labeled with 28. Where should it be so $\mathbf{x}_2$ is the exact solution? Make sure to explain how you arrive at your answer.



8. The equation $\mathbf{Ax} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is going to be solved using one of the following matrices:

$$\mathbf{A}_1 = \begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix} \quad \mathbf{A}_2 = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \quad \mathbf{A}_3 = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \quad \mathbf{A}_4 = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix} \quad \mathbf{A}_5 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

If the CGM is going to be used to solve the equation, which matrix can be used? Make sure to explain why. Picking this one, and assuming that $\mathbf{x}_1 = \mathbf{0}$, find $\mathbf{x}_2$ and $\mathbf{x}_3$.

Worksheet