

Numerical Computing F'22

Exam 1

Name: _____

There are 7 questions. Make sure to show your work (any answer without supporting work will receive no credit). Do not use a crib sheet, calculator, phone, etc.

1. a) What is the floating point number just to the right of $x = 8$?

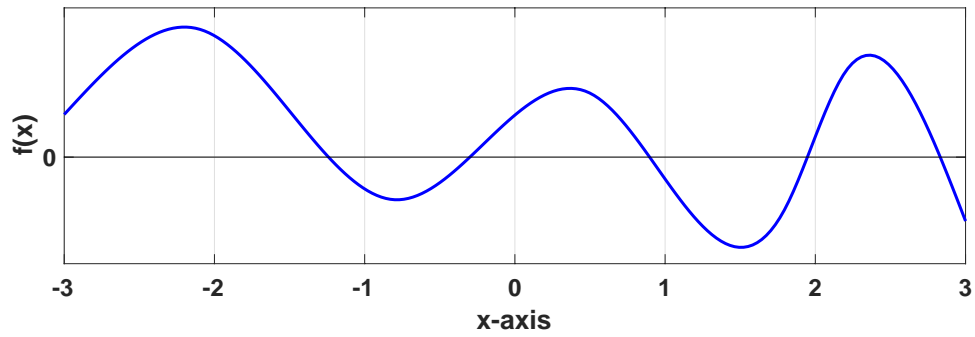
b) What is the floating point number just to the left of $x = 8$?

c) What value will MATLAB compute for $(1 + \frac{1}{4}\varepsilon)(1 - \frac{1}{3}\varepsilon)$?

d) Give an example when MATLAB will claim $x/y = NaN$ even though y is nonzero. You should also provide a short explanation of why your example works.

2. In Newton's method, given x_{k-1} then there is a formula to find x_k . This formula is derived using an approximation for $f(x)$. What is this approximation? Use this approximation to derive the formula for x_k .

3. The equation $f(x) = 0$ is to be solved, where $f(x)$ is shown in the figure below.



a) What solution will the bisection converge to if $a_0 = -3$, $b_0 = 3$?

b) If $x_0 = -2$ and $x_1 = 1$, where is, approximately, x_2 using the secant method?

4. The relative iterative error when solving $f(x) = 0$ is given in the table.

k	Bisection	Newton	Secant
1	5.00e-01	5.00e-01	
2	2.50e-01	2.50e-01	1.00e-02
3	1.25e-01	6.25e-02	1.00e-03
4	6.25e-02	3.91e-03	1.00e-05
5	3.12e-02	1.53e-05	1.00e-08

Table 1: The relative iterative error for the respective method.

a) For the bisection method, what is the relative iterative error at $k = 6$? Why?

b) For Newton's method, what is the relative iterative error at $k = 6$? Why?

c) For the secant method, what is the relative iterative error at $k = 6$? Why?

5. Find the Doolittle factorization of

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 6 & 8 \\ 1 & 6 & 14 \end{pmatrix}.$$

6. Below is a matrix and its inverse

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 1 \\ -2 & -2 & -3 \end{pmatrix} \quad \text{and} \quad \mathbf{A}^{-1} = \begin{pmatrix} -5 & 1 & -3 \\ 2 & -1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

a) Find $\kappa_{\infty}(\mathbf{A})$.

b) When solving $\mathbf{Ax} = \mathbf{b}$, if the residual is $\mathbf{r} = (-1, 0, 2)^T$, then what is the error \mathbf{e} ?

7. Here are some matrices:

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 4 & 2 \\ 2 & 5 \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

Pick one that has a Cholesky factorization and find the factorization (you can ignore the other matrices).

Worksheet