## Appendix C

## Answers

# Introduction to Scientific Computing and Data Analysis, 2nd Ed, by M. H. Holmes

## Chapter 1 Introduction to Scientific Computing

### 1.14

(c) 101.0050175993734, (e) 100

## 1.16

- a) 1000.03583648424; b) 1000.38056571994; c) 1.92929676762289; d) 0.00100058231560099;
- e) 500501.210707288; f)1000.45867514539; g) 4.70173884586451

### Extras:

1.16(a) Prove that 
$$\frac{1001}{2} < \sum_{k=0}^{1000} \frac{e^k}{1+e^k} < 1001$$
  
1.16(f) Prove that  $1000 + \ln\left(\frac{e+1}{e}\right) < \ln\left(\sum_{k=0}^{1000} e^k\right) < 1000 + \ln\left(\frac{e}{e-1}\right)$ 

### 1.17

(a) 2.5e-09, (b) 0.25

### 1.18

(b) 7.07106781186547e + 209, (c) 17.07106781186547e - 201

## 1.27

(b) 8 digits

## Chapter 2 Solving A Nonlinear Equation

### 2.2

- (f) a = 1, b = e (or similar values)
- (g) a = 2,  $b = \pi$  (or similar values)

### **2.11** (e)

 $\begin{array}{l} A = -0.808730600479392; \ B = 2.02097993808977; \ C = 0.569840290998053; \ D \\ = 0.77288295914921; \ E = 1.31409680433497; \ F = 1.19912073071795 \end{array}$ 

### 2.12

partial solution: 0.463647609000806, 3.6052402625906

### 2.13

c) 1.16556118520721

### 2.14

c)  $\theta = \pi/6$ ,  $\varphi = 5.874070380211116$ 

### 2.15

e) w = 16.212125896691699

#### 2.17

f)  $t=10 \min_{S} S = 93.518993960281492 \text{ mM}$ 

### 2.18

e) f = 0.063569292626255

## 2.19

e)  $v = 35.3425966571947 \,\mathrm{m/s}$ 

### 2 20

e)  $v = 24.4408608474134 \, m^3$ 

### 2.23

(e) A: 1; B: 0.246266172167723; C: -0.662512986344964; D: 0.759404549907607; E: 0.75250555469344

### 2.24

(d.1) 0.47693627620447, (e.1) 0.643856219147755

## Chapter 3 Matrix Equations

Note: The computed value for some of the answers involving very ill-conditioned matrices can depend on which version of MATLAB you are using. The reason is that MATLAB uses the LAPACK library (what is considered to be the best algorithms currently available) and they are often upgraded. So, for example, R2015a uses Intel MKL 11.1.1, R2016a uses Intel MKL 11.2.3, and R2016b uses Intel MKL 11.3.1. Those interested in MKL (also known as the Intel Math Kernel Library) should consult Intel's web-site for this.

3.43

$$n = 3$$
,  $||\mathbf{x} - \mathbf{x}_M||/||\mathbf{x}|| = 2.22e - 16$ ,  $\kappa(\mathbf{A}) = 5.33$   
 $n = 9$ ,  $||\mathbf{x} - \mathbf{x}_I||/||\mathbf{x}|| = 7.77e - 16$ ,  $||\mathbf{r}|| = 5.68e - 14$ 

3.44

$$n = 8$$
,  $||\mathbf{x} - \mathbf{x}_M||/||\mathbf{x}|| = 2.1647e - 10$ ,  $\kappa(\mathbf{A}) = 3.96e + 07$ 

- 3.50
- (d) x=1.17422174231682
- 3.51
- (d) x=1.9953731700632

## Chapter 4

## Eigenvalue Problems

- 4.24
- (c) -5.000049348445e 01
- 4.27
- (d) 3141592.65
- 4.33

(a) 
$$\mathbf{q}_3 = (0, -1, 0)^T$$
, (c)  $\mathbf{q}_3 = (0, -1/\sqrt{2}, -1/\sqrt{2})^T$ 

- 4.41
- (d) 109, 145, 109, 145, 182, 145, 109, 145, 109
- (e) 25
- $(f)\ 133,\ 133,\ 133,\ 133,\ 89,\ 178,\ 133,\ 133,\ 133$
- 4.43
- (b) 0.091083209233362

## Chapter 5 Interpolation

5.13

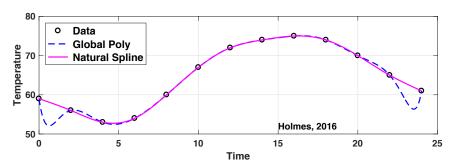
(a) 
$$s_2 = x^3 - 6x^2 + 8x - 1$$

5 14

(a) 
$$y_1 = y_2 = y_3 = y_4 = 0$$

5.23

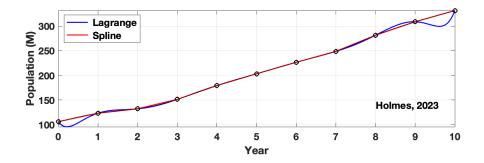
(a) see plot



(c) GP: 152.0489, NS: 58.0296

## 5.24

(a) see plot



(b) GP: 3.003643524200622e+02, NS: 3.202884797377673e+02

## 5.27

(a) n > 55, b) n > 5

## 5.33

(c)  $\delta = 1.6 \times 10^{-11}$ 

## Chapter 6 Numerical Integration

## 6.3

W(8): b) 83, c) 72, d) 238/3

## 6.12

a) 238/3

6.17

(b) hc \* 4.058674557357435e + 03

6.19

(c) 
$$nt = 40$$
;  $y(3) - y_{21} = -0.2242$ 

6.20

(c) 
$$nt = 40$$
;  $y(3) - y_{21} = -0.2505$ 

6.21

3.118169429070039

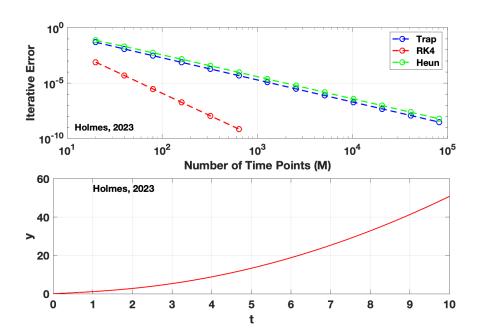
6.29

(b) 
$$K = 1/216$$

## Chapter 7 Initial Value Problems

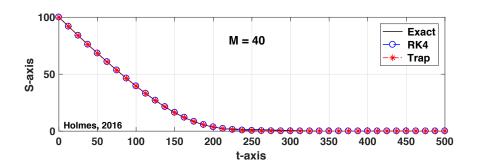
7.28

(II) see plots



7.29

(d)



e)  $e_{\infty} \approx 0.1458$  when M=20 for the trapezoidal method, and  $e_{\infty} \approx 0.0014$  when M=40 for RK4.

### 7.35

(c) Taking R to be the volumetric mean radius,  $v(0) = 8.89619614e + 03 \,\mathrm{m/s}$ 

## 7.36

(c) r(1) = 1.26775487 au

## Chapter 8

## **Optimization: Regression**

### 8.3

### 8.4

**8.5** a) 
$$\begin{pmatrix} \frac{2}{3} - \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$
, b)  $\begin{pmatrix} \frac{1}{14} & \frac{5}{14} & \frac{2}{7} \\ \frac{2}{7} - \frac{4}{7} & \frac{1}{7} \end{pmatrix}$ , c)  $\begin{pmatrix} \frac{1}{6} - \frac{1}{3} - \frac{1}{6} \end{pmatrix}$ 

8.19
a) 
$$\frac{1}{13}$$
  $\begin{pmatrix} 12 & 3 & -4 \\ 3 & 4 & 12 \\ -4 & 12 & -3 \end{pmatrix}$ , b)  $\begin{pmatrix} -\frac{11}{21} & \frac{8}{21} & -\frac{16}{21} \\ \frac{8}{21} & \frac{19}{21} & \frac{4}{21} \\ -\frac{16}{21} & \frac{4}{21} & \frac{13}{21} \end{pmatrix}$ , c)  $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 

8.20 a) 
$$\mathbf{R} = \begin{pmatrix} 2 & -2 \\ 0 & 0 \end{pmatrix}$$
,  $\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 

b) 
$$\mathbf{R} = \sqrt{2} \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$$
,  $\mathbf{Q} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$   
(c)  $\mathbf{R} = \begin{pmatrix} \sqrt{3} - \frac{2\sqrt{3}}{3} & \frac{2\sqrt{3}}{3} \\ 0 & \frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} \\ 0 & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$ ,  $\mathbf{Q} = \begin{pmatrix} \frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{3} & 0 \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{2}}{2} \end{pmatrix}$   
(d)  $\mathbf{R} = \begin{bmatrix} 2 - 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ ,  $\mathbf{Q} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$   
e)  $\mathbf{R} = \begin{pmatrix} 1 & -1 \\ 0 & \sqrt{5} \\ 0 & 0 \end{pmatrix}$ ,  $\mathbf{Q} = \begin{pmatrix} 0 & 1/\sqrt{5} & -2/\sqrt{5} \\ 0 & 2/\sqrt{5} & 1/\sqrt{5} \\ 1 & 0 & 0 \end{pmatrix}$   
f)  $\mathbf{R} = \begin{bmatrix} \sqrt{3} \\ 0 \\ 0 \end{bmatrix}$ ,  $\mathbf{Q} = \begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{3} & 0 \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{2}}{2} \end{bmatrix}$ 

8.32

(c) 
$$(p, \varepsilon) = (1.2319, 0.9660),$$
 (d)  $(p, \varepsilon) = (1.2429, 0.9232)$ 

### 8.33

(a) 
$$v_1 = 22.64$$
,  $v_2 = -0.25$  (b)  $v_1 = 24.91$ ,  $v_2 = -0.33$ 

## 8.34

(a) using 
$$E$$
:  $(\alpha, \beta) = (3.8913\text{e-}09, 2.0847)$ , using  $E_S$ :  $(\alpha, \beta) = (1.6113\text{e-}09, 2.2132)$   
(d)  $\beta = 3$ 

## Chapter 9 Optimization: Descent Methods

9.25

(a) The surface plot is shown in Figure C.1.

## Chapter 10 Data Analysis

10.3

(b) 62/51, 160/51

10.2

(c) 
$$z = 2.5$$

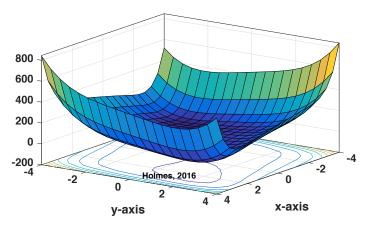


Figure C.1 Surface for Exercise 9.25(a).

(b) 
$$\mathbf{x}_1^* = (-1/2, 1/2)^T$$

10.13  
(b) 
$$\mathbf{x}_{1}^{*} = (-1/2, 1/2)^{T}$$
  
(d)  $\mathbf{y}_{i}^{*} = \frac{1}{\sqrt{2}} \begin{pmatrix} 2 & 2 \\ 4/\sqrt{5} & -4/\sqrt{5} \end{pmatrix} \mathbf{x}_{i}^{*}$