10

Introduction to Filters: The RC Circuit

10.1 INTRODUCTION

In this chapter, we introduce analog filters by exploring a simple RC-circuit consisting of a single resistor R and a single capacitor C. Because most electrophysiology labs will have some basic electronic equipment (such as multimeters and oscilloscopes) available, use this chapter as a guide for a practical exercise. If such equipment is not available, the chapter can be used as an introduction to electronic analog filters. First we will get acquainted with the basic behavior of a simple filter, and in the subsequent chapter we will worry about the mathematical analysis.

The purpose of most filters is clear-cut: to remove the part of a signal that is considered noise, and as we will see they usually do a great job. In a more general sense, the filters discussed in this text are good examples of linear time invariant (LTI) systems, and the analysis techniques we apply to these filters can also be applied to characterize physiological systems. For instance, if we know the frequency response of a sensory cell and we assume (or establish empirically) that the cell behaves in a linear fashion, then we can model the cell as a filter and predict its response to any arbitrary input.

As with LTI systems in general, filters can be studied both in the ***time*** and ***frequency*** domains and they can be implemented using either ***analog*** or ***digital*** techniques. Analog filters can be analyzed with continuous-time mathematics, whereas the digital versions are described with discrete time equations. If you want to read more about filters, see Marvin and Ewers (1996) (Introductory level) or Chirlian (1994) (Advanced level).

10.2 FILTER TYPES AND THEIR FREQUENCY DOMAIN CHARACTERISTICS

The most intuitive approach is to describe the operation of a filter in the frequency domain, where we can define the filter as an attenuator for

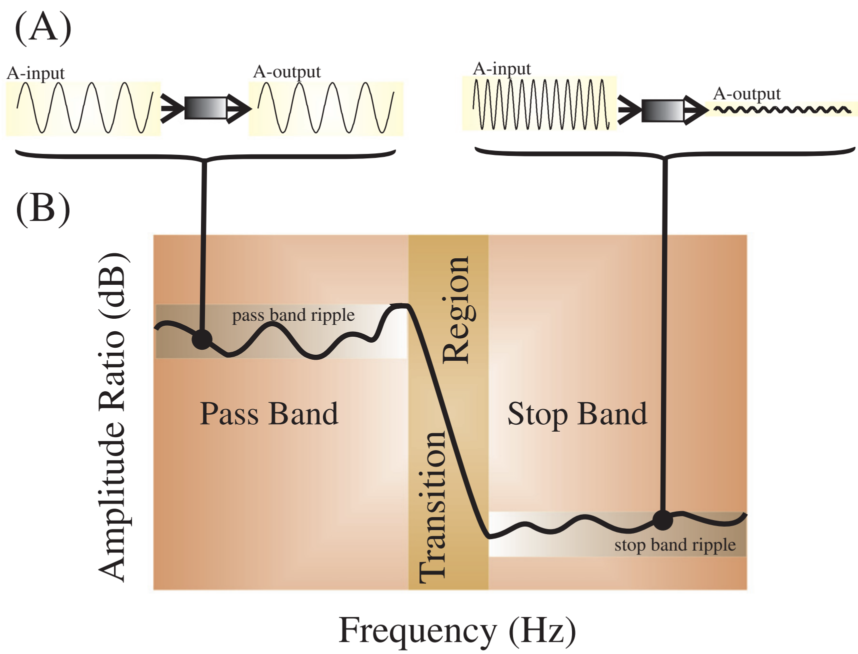

Figure 10.1 Filter characteristic determined by examining sine wave input-output relationship. (A) The two examples show that sine waves in the so-called pass band can be unattenuated (left) or significantly reduced in amplitude (right) in the stop band. The ratio between the amplitudes at the input (A-input) and the output (A-output) is used to construct the filter's frequency response characteristic. (B) The filter characteristic expressed as amplitude versus frequency of the filter input. The stop band denotes frequency regions in which amplitudes are attenuated, while the pass band indicates the range of unattenuated frequencies. In real filters, there is necessarily a transition region between these bands.

certain undesirable frequency components while passing others (Fig. 10.1); in an ideal world, a filter would completely remove all noise components. Let's focus on the behavior of a filter to a single frequency (i.e., a pure sine wave). A central characteristic of any linear system is that its response to a sine wave input $A \sin(2\pi\omega t + \phi)$ is also a sine wave in which the amplitude A or the phase ϕ may be altered. In Figure 10.1A, two examples are shown: a low-frequency sinusoid passes unattenuated while a higher-frequency sine wave is significantly reduced in amplitude at the filter output. In the examples shown in Figure 10.1A, the phase remains unaltered.

It is common practice to describe part of the frequency characteristic of a filter as the amplitude ratio between output and input for sine wave signals over a range of frequencies. In Figure 10.1B, the characteristic of

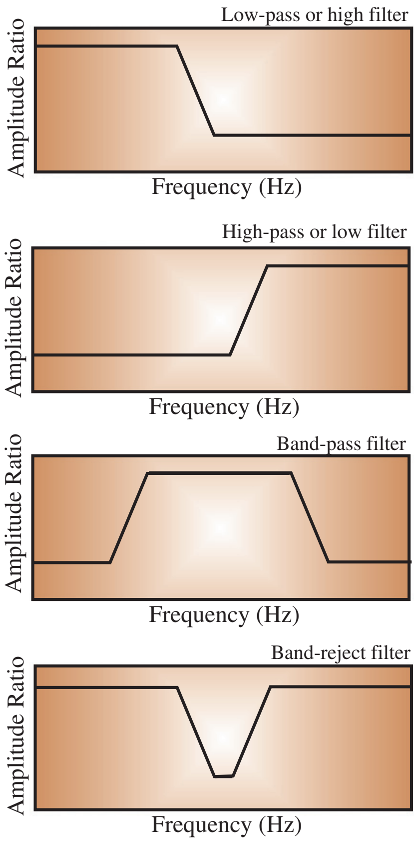

Figure 10.2 Filter types and their frequency characteristics. The plots show the amplitude ratios for positive frequencies. For digital filters, it is not uncommon to also depict the values for the negative frequencies. Because the filter characteristic is an even function, no additional information is provided in the plot for $\omega < 0$.

the filter is divided into different bands: the pass band, the stop band, and the transition between these two bands. Ideally, the frequency components in the pass band would be unattenuated (i.e., a gain of 1× or 0 dB; see Chapter 3, for a review of the dB scale), whereas the components in the stop band would be completely eliminated (i.e., a gain of 0× or $-\infty$ dB). In addition, one would like a transition region of zero width. In the real world, gains in the pass band and stop band can deviate from 1 and 0, respectively. In addition, the amplitude ratio may show ripples, and the width of the transition region is necessarily greater than 0 (Fig. 10.1).

Filters as described by their frequency response can be classified and combined into different types. The filter in Figure 10.1 passes low frequencies and attenuates the high ones. This type is referred to as a ***low-pass*** filter. The opposite type is the ***high-pass*** filter. A combination of low-pass and high-pass characteristics results in a ***band-pass*** filter and a system that attenuates a specific frequency band is a ***band-reject*** filter (Fig. 10.2).

10.3 RECIPE FOR AN EXPERIMENT WITH AN RC CIRCUIT

The simplest analog electronic filter consists of a resistor (R) and a capacitance (C). A single so-called RC circuit can either be a high-pass or low-pass filter, depending on how the components are connected. Examples of different diagrammatic representations of a low-pass filter, all denoting the same circuit, are shown in Figures 10.3A to E. If the positions of the R and C are interchanged in the low-pass circuit in Figure 10.3, we obtain a high-pass filter.

Prior to mathematical analysis, we will study input-output relationship of RC circuits with an experimental approach. We are interested in the following:

1. The ***transient response*** of the filter to a step function (a unit impulse would also be nice but is impossible to create) at the input.
2. The ***steady-state response*** to a sinusoidal input, using sine waves of different frequencies.

An example of a setup that includes a function generator to generate test signals and a dual channel oscilloscope to simultaneously measure the input and output of a filter is shown in Figure 10.4. Further, basic requirements to make testing circuitry convenient are a breadboard for mounting the circuit and a simple multimeter.

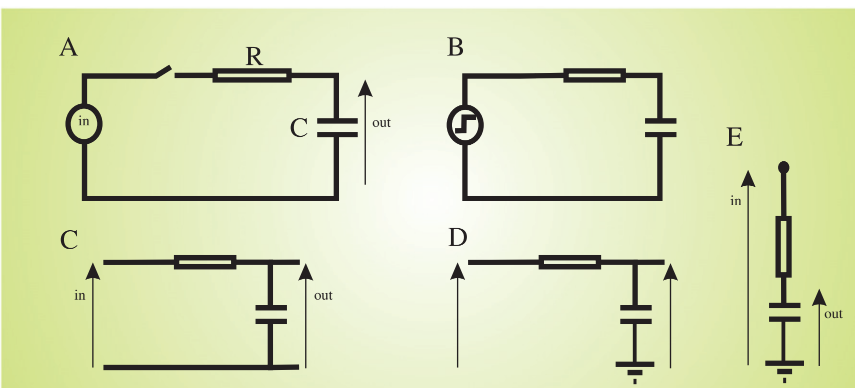

Figure 10.3 Different equivalent diagrams for an analog RC filter with low-pass characteristics. All diagrams represent a circuit with an R and C component where the input signal is supplied over both the R and C components while the output is determined over the capacitor. The diagrams in (A) and (B) show most clearly that we deal with a closed circuit. The diagrams in (C), (D), and (E) (symbolizing the same circuit) are more frequently used in electronic diagrams and engineering texts. Note that this filter is the same as the simplified ion channel model introduced in Chapter 8.

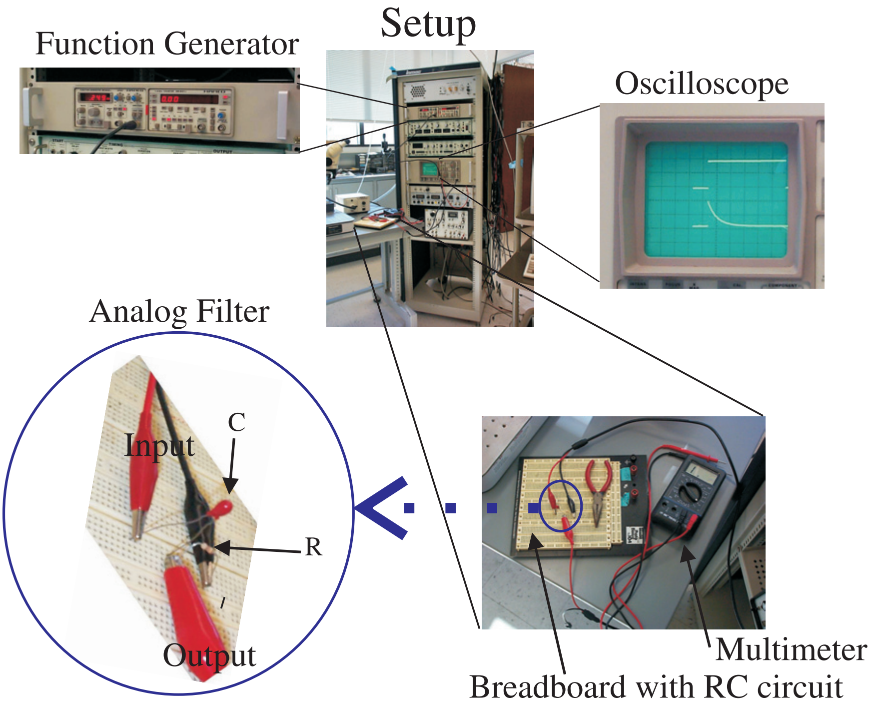

Figure 10.4 Setup used for analyzing analog filter circuits. A function generator is used to generate input signals (sine waves and step functions). The RC circuitry (a high-pass filter in this example) is built on a breadboard. The detail in the blue circle shows the R and C components plus the input (black = ground wire; red = signal wire) and output (red = signal wire) connections. The filter input comes from the output of the function generator (which also connects to the oscilloscope), while the output of the filter is connected to a second oscilloscope channel. Note that the (black) ground wire of the output can be omitted because the input and output are both measured simultaneously on the dual channel oscilloscope, and the oscilloscope only needs to be connected to the ground signal once via the input wire (an additional ground wire with the same ground signal of the output would result in a ground loop, which can add noise to a circuit).

In the first step, an analog filter is created with a 10 kΩ resistor and a 3.3 μF capacitor. Because resistors are typically specified to different levels of precision (often allowing 5% variation from the indicated value), you can use the multimeter to determine the precise resistance value; without a multimeter, you will have to believe the value indicated in the banded color code on the resistor itself (e.g., brown-black-orange for 10 kΩ).

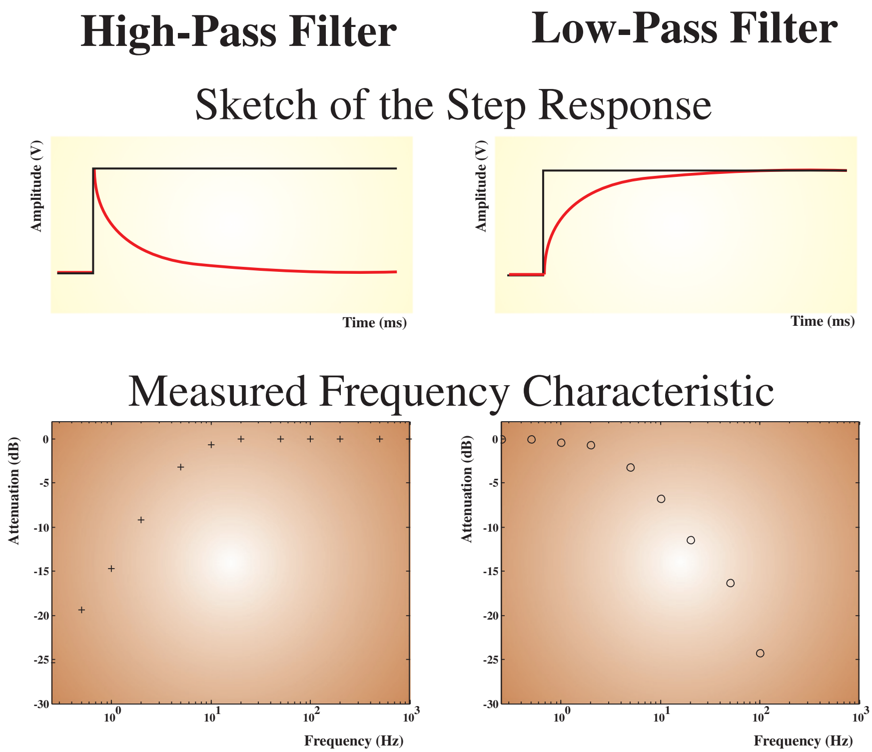

Figure 10.5 Typical result from measurements of an RC circuit either connected as a high-pass filter or low-pass filter. The response to a 1 V step in the time domain is sketched. The ratio between input-output sine wave amplitudes is compiled in a table (e.g., Table 10.1), expressed in dB, and represented in a semilog plot (using MATLAB command semilogx). These plots reflect the frequency characteristic of the particular circuit (compare these results with the plots for high-pass and low-pass filters in Fig. 10.2).

Note: A 10 kΩ resistor and a 3.3 μF capacitor would be the values I recommend for exploring a filter circuit; other values are also possible as long as we keep the resistance significantly lower than the input impedance of the oscilloscope (usually ~1 MΩ) but higher than the function generator output (usually only several Ω); also for ease of measurement with standard equipment, the product of RC should be in the range of 5 to 100 ms.

To build the test setup, construct an RC filter on the breadboard, and then do the following:

Table 10.1 Input-Output Ratios of an RC Circuit for Sine Waves at Different Frequencies

Frequency (Hz)	Low-pass AmpRatio	Low-pass (dB)	High-pass AmpRatio	High-pass (dB)
0.3	1.00	0.00	0.05	−25.38
0.5	1.00	0.00	0.11	−19.36
1	0.96	−0.34	0.18	−14.67
2	0.92	−0.70	0.35	−9.21
5	0.69	−3.19	0.69	−3.19
10	0.46	−6.72	0.92	−0.70
20	0.27	−11.40	1.00	0.00
50	0.15	−16.26	1.00	0.00
100	0.06	−24.22	1.00	0.00
200	0.03	−30.24	1.00	0.00
500	0.01	−39.36	1.00	0.00
1000	0.01	−45.38	1.00	0.00

1. Connect the output of the function generator to
   (a) the filter input and
   (b) the oscilloscope
2. Connect the filter's output to the second channel of the oscilloscope

After completing all connections we can start to characterize the circuit:

1. Determine the transient response: Measure and sketch a detailed graph of the system's response to a voltage step of 1 V. To see that transient response clearly, you can set the frequency of the signal generator to a very low value and use the trigger or storage capability of the oscilloscope to maintain the image.
2. Determine the steady-state response: Measure the system's output for sinusoidal inputs (0.2 Hz to 1000 Hz). Since we will eventually present our data in a semi-log plot, use a 1, 2, 5 sequence (i.e., 0.2, 0.5, 1, 2, 5, 10, etc.). You can also measure the phase difference between input and output signal by comparing zero-crossing times (although as compared to the amplitude ratio it is more difficult to measure this reliably).
3. Create a table and a graph of output amplitude (in dB) versus $\log_{10}$ of the frequency of each test sinusoid.
4. Now interchange the positions of R and C and redo steps 1 to 3.

Typical examples of a table and graphs for both filter types are shown in Table 10.1 and Figure 10.5. In the following chapter, we will analyze the data both in continuous time and in discrete time models of this filter.