# The self-referential problem

Holo

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Paradox 000000

Is 'Red' red?
Is 'Red' red?

Is 'Red' red? Is 'Red' red? Is 'Red' red?

Type-Token o●ooo

Is 'Red' red? Is 'Red' red? Is 'Red' red? Is 'Blue' red?

Type-Token o●ooo

- Is 'Red' red?
- Is 'Red' red?
- Is 'Red' red?
- Is 'Blue' red?

A token is an instance of word

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### Type (linguistics)

A **type** is an abstract idea that a *token* refers to

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"A rose is a rose is a rose."

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The sentence has 3 types

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"A rose is a rose is a rose."

The sentence has 3 types: "a"

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### Type (linguistics)

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"A rose is a rose is a rose."

The sentence has 3 types: "a", "is"

A token is an instance of word

### Type (linguistics)

A type is an abstract idea that a token refers to

"A rose is a rose is a rose."

The sentence has 3 types: "a", "is" and "rose"

A token is an instance of word

### Type (linguistics)

A **type** is an abstract idea that a *token* refers to

"A rose is a rose is a rose."

The sentence has 3 types: "a", "is" and "rose"

The sentence contains 3 instances of the type "a"

A token is an instance of word

### Type (linguistics)

A **type** is an abstract idea that a *token* refers to

"A rose is a rose is a rose."

The sentence has 3 types: "a", "is" and "rose"

The sentence contains 3 instances of the type "a", 2 instances of the type "is"

A token is an instance of word

### Type (linguistics)

A **type** is an abstract idea that a *token* refers to

"A rose is a rose is a rose."

The sentence has 3 types: "a", "is" and "rose"

The sentence contains 3 instances of the type "a", 2 instances of the type "is" and 3 instances of the type "rose".

Is 'Red' red?
The token 'Red' is not red

The token 'Red' is not red, the type is also not red.

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The token 'Red' is not red, the type is also not red.
Is 'Red' red?

Is 'Red' red?
The token 'Red' is not red, the type is also not red.
Is 'Red' red?
The token is red

The token 'Red' is not red, the type is also not red.

Is 'Red' red?

The token *is* red, this token refers to the same type as before, so it is also not red.

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Is 'Red' red?

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When asking "is 'A' B?" and the question doesn't make sense, the answer is "no".

The token 'Red' is not red, the type is also not red.

Is 'Red' red?

The token *is* red, this token refers to the same type as before, so it is also not red.

When asking "is 'A' B?" and the question doesn't make sense, the answer is "no".so the type of 'Red', which doesn't have any colour what so ever, is **not** red.

The '(token) palindrome' is not a palindrome.

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The '(token) civic' is a palindrome.

The '(token) palindrome' is not a palindrome.

The '(token) civic' is a palindrome.

The '(type) cat' is not "three letters long"

The '(token) palindrome' is not a palindrome. The '(token) civic' is a palindrome. The '(type) cat' is not "three letters long" The '(token) writable' is writable.

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From now on, unless specify otherwise, when asking "is 'A' B?", we will talk about the *type* of 'A' (Newhard variation).

#### The Partition Claim

Type-Token

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Every word in the English language **is not** "red". Every word in the English language **is** "word"\*.

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Every word in the English language is not "red".

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Some words in the English language are "nouns" and some are not.

Given a property "B" we can check every word in the English language and put it in one of 2 buckets: "is B", "is not B".

Every word in the English language is not "red".

Every word in the English language is "word"\*.

Some words in the English language are "nouns" and some are not.

Some words in the English language are "offensive" and some are not.

Given a property "B" we can check every word in the English language and put it in one of 2 buckets: "is B", "is not B".

Every word in the English language is not "red".

Every word in the English language is "word"\*.

Some words in the English language are "nouns" and some are not. Some words in the English language are "offensive" and some are not.

<sup>\*</sup> we can also talk about "sentences" and not only words, in which case not everything will be a "word"

Given a property "B" we can check every word in the English language and put it in one of 2 buckets: "is B", "is not B". Alternative form: Every word is either  ${\bf B}$  or not

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#### Grelling-Nelson paradox

"The Partition Claim" is false

<sup>&</sup>quot;Every word is either red or not"

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Autological words and Heterological words

#### Autological

A word is autological if it describes itself

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<sup>&</sup>quot;red" is heterological, "offensive" is heterological.

The Partition Claim for Autological and Heterological words Every word is either autological or heterological. The Partition Claim for Autological and Heterological words Every word is either autological or heterological.

We will show that the above statement is **false**, that is, there are words that are neither autological nor heterological.

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# Grelling-Nelson paradox Also known as Weyl's paradox and Grelling's paradox

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"Heterological" is not autological.

But "autological" means "describes itself", so the above is actually "Heterological" is not heterological.

But a word can't be **both** heterological and not heterological, so "heterological" cannot be heterological.

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Assume "Heterological" is autological.

Remember that autological means "describes itself', so we have "Heterological" is heterological.

But we already saw that this is impossible, so heterological can't be autological.

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Unlike heterological, where it can be neither heterological nor autological, autological can be both.

By that we mean that if we claim that "autological" is autological, we will not find any holes that will imply otherwise, but similarly if we claim that "autological" is heterological we won't find any holes in our logic.

This is the *dual* version of the paradox

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The Partition Claim

Every word is either **B** or not

Cannot hold for every "B".

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Pretty much any subject that tries to be "foundational" (be the 'building blocks' of everything else) have a variation of the paradox. In formal settings (like maths or computer science) we resolve the paradox by restricting the formal rules we are using.