Theorem(Zermelo, ZF**):** If $F : \mathcal{P}(X) \to X$ a function, then there exists unique $W \in \mathcal{P}(X)$ with unique ordering $\prec$ such that:

1. $\prec$ is well ordering of W
2. if $w \in W$ then $F(\{x \in W \mid x \prec w\}) = w$
3. $F(W) \in W$

Proof: We call $B \subseteq X$ an F-set if there exists a well ordering that witness (2).

We may notice that

- There exists an F-set: $\{F(\emptyset)\}$
- If $(B, R), (B', R')$ are F-sets with their witness then (B, R) is an initial segment of (B', R') or vice versa.

Assume that the second point is false, then both are well orders there exists order preserving injective from the smaller to the other(we know that "smaller" is well defined because they both well ordered), WOLG we may say $i : B \to B'$ be that function.

Let t be the R-minimal element such that $i(t) \neq t$, with that we get

$$t = F(\{x \in B \mid xRt\}) = F(\{x \in B' \mid xR'i(t)\}) = i(t)$$

Contradiction.

Letting W be the union of all the F-sets will be the , to show that $F(W) \in W$, suppose not, then $W \cup \{F(W)\}$ is an F-set, so $W \cup \{F(W)\} \subseteq W$, contradiction. ■