# Complexities

## Array

| | |
|---|---|
| Append | |
| Sorted Append | |
| Delete (+shift) | |
| Binary Search | |

## Linked List

| | |
|---|---|
| Insert | |
| Insert (Enhanced) | $\mathcal{O}(1)$ |
| Delete (+shift) | $\mathcal{O}(n)$ |
| Traverse(Search) | $\mathcal{O}(n)$ |

*Enhanced Linked Lists* give pointers to both head and tail

## Tree/Heap

| | |
|---|---|
| Insert | |
| Delete | |

## Heapify/Heap sort

| | |
|---|---|
| Top Down | $\mathcal{O}(n \cdot log_2 n)$ |
| Bottom up | $\mathcal{O}(n)$ |

For the $T(n)$ for the number of sinks run by Heapify, the sequence is:

$$T(n) = \frac{n}{2} \cdot 0 + \frac{n}{4} \cdot 1 + \frac{n}{8} \cdot 2 + \cdots + 1 \cdot h$$

# Big O no

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0, \infty \Rightarrow f \in \Theta(g)$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq \infty \Rightarrow f \in O(g)$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0 \Rightarrow f \in \Omega(g)$$

worst case $f(n) \in \mathcal{O}(g(n)) : f(n) \leq Cg(n)$

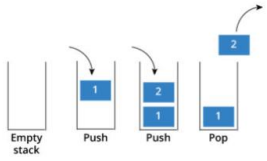
best case $f(n) \in \Omega(g(n)) : f(n) \geq Cg(n)$ $\forall n > n_0$

average case $f(n) \in \Theta(g(n)) : C_1 g(n) \leq f(n) \leq C_2 g(n)$

## Stacks

Push $\mathcal{O}(1)$

Pop $\mathcal{O}(1)$




## Queue

Enqueue $\mathcal{O}(1)$

Dequeue $\mathcal{O}(1)$

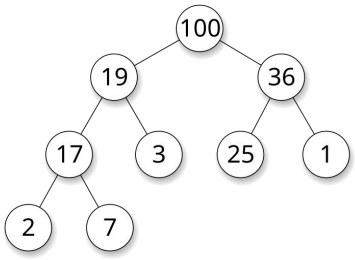

# Heaps

These are the *only* conditions:

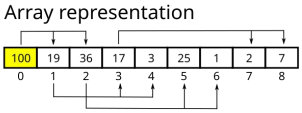
**Max Heap** $parent \geq child$

**Min Heap** $parent \leq child$

Tree representation




Array representation




**Array, node at index i:**

## Height and Size

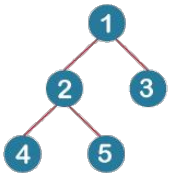
$$h(T) = \begin{cases} -1 & \text{if } T \text{ is null} \\ 1 + \max(h(T_{\text{left}}), h(T_{\text{right}})) & \text{otherwise} \end{cases}$$

$$s(T) = \begin{cases} 0 & \text{if } T \text{ is null} \\ 1 + s(T_{\text{left}}) + s(T_{\text{right}}) & \text{otherwise} \end{cases}$$

## Number of nodes

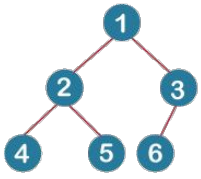
$$N = 2^{h(T)+1} - 1$$

**Strict** (or Full)



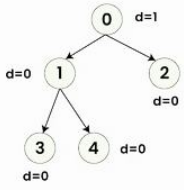

Each node has 0 or 2 children

**Complete**

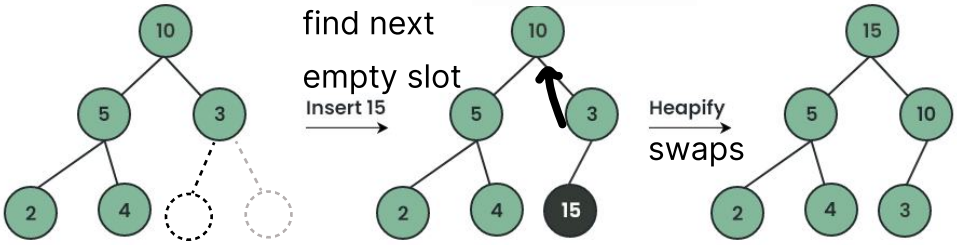



Each before the last level is filled

**Balanced Binary Tree**



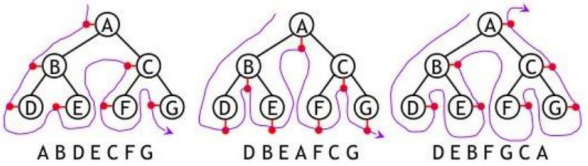

d = Depth of left subtree - Depth of right subtree = 0 or 1




# Trees

Imagine a flag attached to each node:

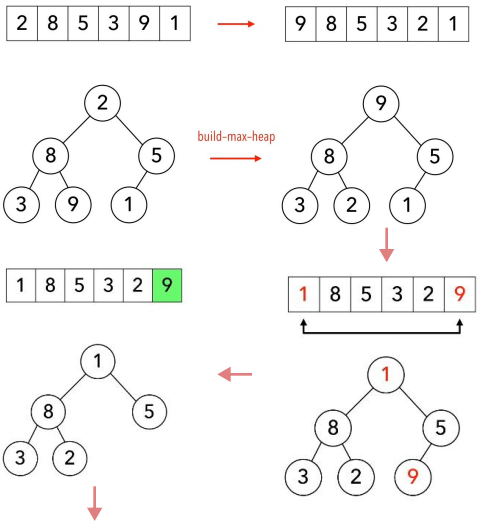

To traverse the tree, collect the flags:




LNR NLR LRN

# Heap Sort

## Top Down (siftDown) Approach:




heapify and repeat until sorted

## Bottom up (siftUp)

Pick last unsorted node, heapify up. Repeat until sorted.