15-451/651 Algorithms, Fall 2017

Homework #6 **Due: November 22, 2017**

This HW has two regular problems, and one programming problem. All problems on written HWs are to be done *individually*, no collaboration is allowed.

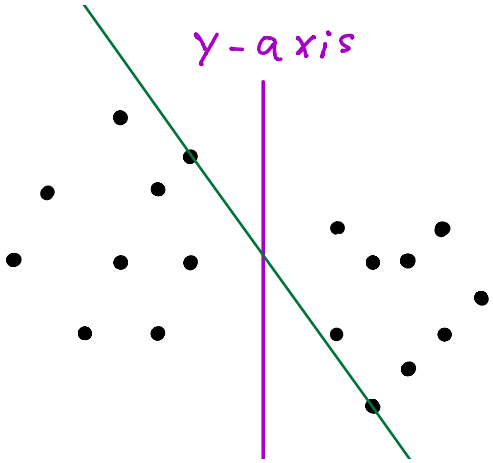
Solutions to the three written problems should be submitted as a single PDF file using `gradescope`, with the answer to each problem starting on a new page.

Submission instructions for the programming problem will be posted on the website and Piazza.

(25 pts) 1. **Lowest-Slope**

Let $p_1, p_2, \ldots, p_n$ be a set of points to the left of y-axis. (I.e. their x coordinates are all < 0.) You're also given $q_1, q_2, \ldots, q_n$ that are to the right of the y axis.

The problem is to find the pair (i, j) such that the line from p_i to q_j has the lowest slope of any such line.

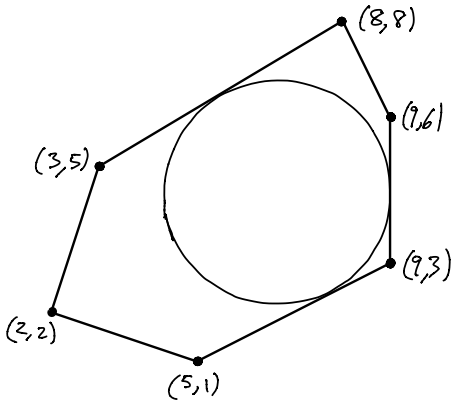


(a) Give a deterministic algorithm that solves this problem in $O(n \log n)$ time.

(b) Show how to set the problem up as a 2-dimensional linear program.

(25 pts) 2. **Circle in a Polygon**

You are given $(x_0, y_0), \ldots, (x_{n-1}, y_{n-1})$, the vertices of a convex polygon P in the plane $\mathbf{R}^2$. The vertices are distinct and in counter-clockwise order.



(a) Show how to represent this polygon as the intersection of n linear inequalities of the form $a_i x + b_i y \geq c_i$. Express the coefficients of these inequalities in terms of the input points.

(b) Given a point $(p, q) \in \mathbf{R}^2$, give an expression for the signed distance of this point from the line $a_i x + b_i y = c_i$. (It should be positive for any point strictly inside the polygon.)

(c) Write a linear program with three variables that computes the largest radius r of a circle that can be inscribed within the polygon.

(25 pts) 3. **Most Populous Circle**

Write a program that takes as input a set of points S in the plane, and a radius r. Your program will output the maximum number of points of S which can be contained inside (or on the boundary) of a circle of radius r, assuming the circle is placed where it contains the most points. (Arbitrary real coordinates for the center of the circle are allowed. To avoid requiring too much precision, the data is selected such that the answer does not change if r is increased to $r + .001$.) The time limit is 15 seconds.

Input: The first line contains two space-separated integers: n and r. The next n lines each contain a pair of integers (x, y) which define a point of S. n will be at most 2000, and $0 \leq r, |x|, |y| \leq 10000$.

Output: Just print a number, the correct answer.

```
2 2500
1000 1000
5000 4000
```
2

```
2 2499
1000 1000
4000 5000
```
1

```
8 5
4 5
-2 -3
-2 5
4 -3
5 4
-3 -2
5 -2
-3 4
```
8

```
8 4
4 5
-2 -3
-2 5
4 -3
5 4
-3 -2
5 -2
-3 4
```
4