Homework 3

October 10, 2018

1 Homework 2

Question 1 *Prove that:*

$$\frac{1}{N}\sum_{i=1}^{N} I(f(x_i) \neq y_i) \leq exp(-2\sum_{k=1}^{K}\gamma_k^2)$$

Where:

$$\gamma_k = \frac{1}{2} - \epsilon^{(k)}$$

Proof 1 *From* $f(\hat{x}) = sign(\sum_k a^{(k)} f^{(k)}(\hat{x}))$*:*

When$f(\hat{x}) \neq y_i$, $\sum_k a^{(k)} f^{(k)}(x_i)yi \leq 0$, *so* $exp(-\sum_k a^{(k)} f^{(k)}(x_i)yi) \geq 1$*. Hence we have:*

$$\frac{1}{N}\sum_{i=1}^{N} I(f(x_i) \neq y_i) \leq \frac{1}{N}\sum_{i=1}^{N} exp(-\sum_k a^{(k)} f^{(k)}(x_i)yi)$$

$f(x_i)$

From $d_{k+1} = \frac{d_i^{(k)} exp(-a^{(k)} y_n f^{(k)}(x))}{Z^{(k)}}$*, we have:*

$$d_{k+1} Z^{(k)} = d_i^{(k)} exp(-a^{(k)} y_n f^{(k)}(x))$$

$$\frac{d_{k+1} Z^{(k)}}{d_i^{(k)}} = exp(-a^{(k)} y_n f^{(k)}(x))$$

Then:

$$\frac{1}{N}\sum_{i=1}^{N} exp(-\sum_{K} a^{(k)} f^{(k)}(x_i)y_i)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\prod_{K} exp(-a^{(k)} f^{(k)}(x_i)y_i)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\prod_{K} exp(-a^{(k)} f^{(k)}(x_i)y_i)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\prod_{K-1} \frac{d_{k+1} Z^{(k)}}{d_i^{(k)}}$$

$$= \prod_{K-1} Z^{(k)}$$

And:

$$Z^k = \sum_N d^k exp(-a^{(k)} f^{(k)}(x_i) y_i)$$

$$= \sum_{right} d^k exp(-a^{(k)}) + \sum_{wrong} d^k exp(a^{(k)})$$

$$= (1 - \epsilon^{(k)}) exp(-a^{(k)}) + \epsilon^{(k)} exp(a^{(k)})$$

$$\leq \sqrt{2(1 - \epsilon^{(k)}) \epsilon^{(k)}}$$

$$\leq \sqrt{1 - 4\gamma_k^2}$$

As a result:

$$\frac{1}{N} \sum_{i=1}^{N} I(f(x_i) \neq y_i) = \prod_K Z^{(k)} \leq \prod_K \sqrt{1 - 4\gamma_k^2} \leq exp(-2 \sum_{k=1}^{K} \gamma_k^2)$$

Q.E.D.