Homework 9

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1 8.15

The two sentences can not imply the fact that $x \notin s$.

For example, prove that 1 is not in $\emptyset$. We know from the defination that 1 is in all the sets with a 1 in it, yet we will never know if 1 is in $\emptyset$ for nothing can be drived from the 2 rules.

2 8.20

Lemma:

We first define prediction "is Nature Number" N(x).

$$N(x) := (x = 0) \vee ((x = y + 1) \wedge (N(y)))$$

Also, we can generate nature number z by using function "+" for multiple times:

$$z := +(1, +(1, ...)) \ ((z - 1) functions\ in\ total)$$

For example:

$$3 := +(1, +(1, 1))$$

Based on these lemmas:

2.1 8.20.a

$$iseven(x) := \exists y \ x = +(y, y)$$

or

$$iseven(x) := \exists y(\ N(y) \wedge (< (X(2, y), x) \wedge (< (x, X(2, +(y, 1)))) \vee (x = 0)$$

2.2 8.20.b

$$isprime(x) := \neg\exists a, b \ x = X(a, b) \wedge (< (1, a)) \wedge (< (1, b)) \wedge N(a) \wedge N(b)$$

or

$$isprime(x) := \forall a, b \ x = X(a, b) \rightarrow (a = 1) \vee (b = 1)$$

2.3 8.20.c

$$Goldbach := \forall x\ \exists a, b\ \ iseven(x) \wedge isprime(x) \wedge isprime(b) \wedge (x = X(a, b))$$

~~So in fact we do not need these lemmas. 23333~~
