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INTERNSHIP REPORT

Classical Analog Model of Light–Matter Strong Coupling in an Optical Cavity

SUPERVISOR: Dr. Rémi Avriller

STUDENT: François Deberdt – Second-year undergraduate in fundamental physics–

INTERNSHIP INSTITUTE: ISIS - Institut de Science et d'Ingénierie Supramoléculaires

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François DEBERDT
francois.deberdt@etu.unistra.fr
University of Strasbourg

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Classical Analog Model of Light–Matter Strong Coupling in an Optical Cavity

Institut de Science et d'Ingénierie Supramoléculaires (ISIS)

To explain the formation of the Rabi splitting in the spectrum of a strong coupling cavity, we use a classical analog model to demonstrate that in this conditions the proprieties of matter are modify. We demonstrate the formation of two polaritonic states, the conditions for there creations, and the influence of the different parameters on the evolution, numerically and with an analytical approximation.

I. INTRODUCTION

Light–matter interaction in optical cavities has gained increasing attention over the past decades. This field is being studied for potential applications: enhanced energy transport in organic materials, non-linear optical properties, chemical properties modifications[5][2], and quantum information processing[2]. The interest in these hybrid states lies in their ability to acquire characteristics of light and matter.
The spontaneous emission of a photon from a material is influenced by the resonance between the local electromagnetic (E-M) field and the emitter. The emission rate behave differently in free space or in optical cavities. Indeed, in free space the E-M field consists of an infinite number of optical modes interacting weakly with the emitters. As shown in Figure 1.d, the finite width of an optical cavity, such as placing emitters between two metallic mirrors, limits the allowed optical modes, enabling more efficient coupling with selected emitters. The coupling between the E-M field and the materials occurs even in dark, without external driving.
In weak coupling configuration, an observation of the Purcell effect[1] is possible, which correspond to an enhancement of the spontaneous emissions of the emitters in the cavity. When the cavity frequency approaches the emitters frequencies, the emission rate decreases. This effect occurs because if the leakage rate of photons and the non-radiative losses of the emitters are lower than the criteria for strong coupling, there is an exchange of photons between the cavity E-M field and the emitters at a coupling rate called the Rabi frequency Ω. This phenomenon, called strong coupling, leads to longer emitters excited-state lifetimes in the cavity than in free space[4].
In strong coupling configuration, the coupled cavity and emitters behave like a new quasiparticle called a polariton, which has two polaritonic states generated by this continuous exchange of energy, and separated by an energy gap called Rabi splitting Ω_R. They result from the strong coupling between photons and excitons, where an exciton is an electron–hole pair[4]. As shown in the Figure 1.c, the polaritonics states alter the matter and modifies the transmission spectrum with two new peaks[3].

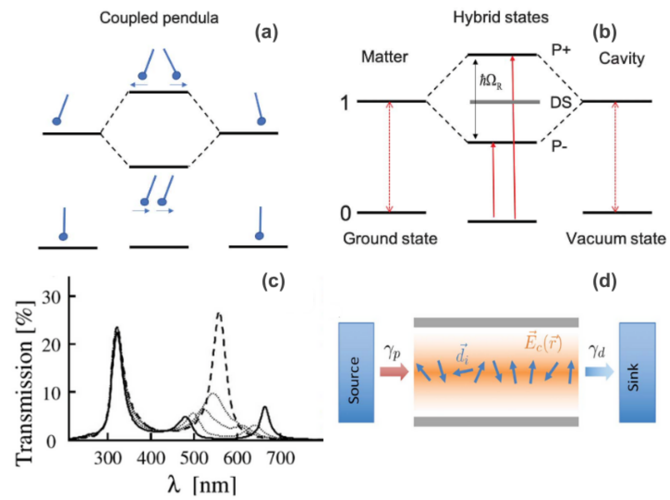

Figure 1. Figures (a),(b) and (d) taken from[2], (c) taken from[3]. (a) Two classical oscillating systems of coupled pendulums, corresponding to the two polaritonic states. (b) Two quantum states of a polariton P+ and P-, separated by the Rabi splitting energy gap. (c) Transmission spectra from the photoisomerization of molecules in strong coupling conditions, uncoupled in dash and coupled in solid line. (d) Representation of a driven microcavity of two metallic mirrors, with emitters dipoles and E-M field.

These polaritonic states can be represented as a linear combination of quantum eigen states, which can be modeled using the eigenmodes of classical coupled harmonic oscillators, as shown in Figure 1.a and b. In this analogous model, the first oscillator will represent the cavity electromagnetic field, driven by a laser beam to simulate the transmission profile. A second oscillator will represent the emitters collective degree of freedom, coupled with the cavity by the Rabi frequency, and leading to a resonance phenomenon. This model allow to understand the experimental Rabi splitting observations, observed in strong coupling emitters absorption spectrum, as the resonances of two coupled harmonic oscillators.

II. SIMPLE MODEL OF OPTICAL CAVITIES

We want a simple model of an optical cavity to study the cavity modes. To represent the driven harmonic oscillator corresponding to the cavity, we need to introduce the the loss term κ, corresponding to the cavity lifetime. The cavity frequency mode is ω_c, and the E-M field going through the cavity have an amplitude of E_L.

$$q'' + \frac{\kappa}{2}q' + {w_c}^2 q = E_L cos(\omega_L t) \tag{1}$$

The solution of this differential equation consists of a homogeneous and a particular solution: $q(t) = q_h(t) + q_p(t)$, with two initial conditions $q(0)$ and $q'(0)$. The homogeneous equation being linear, we can assume an exponential form $q_h(t) = Ae^{rt}$.

We substitute this solution into Eq.(1), giving us a quadratic equation. Its roots are complex, because the discriminant $\tilde{\omega} = (\frac{\kappa}{2}^2 - 4\omega_c)^2$ is negative. This corresponds to a low-damping oscillating system, which is not the case in critical or overdamped systems as show in appendix A. The homogeneous solution is a linear combination of complex exponential forms, taking the real part:

$$q_h(t) = e^{-\frac{\kappa}{4}t}(\alpha \cos \frac{\sqrt{-(\frac{\kappa}{2})^2 + 4{\omega_c}^2}}{2} t) + \beta \sin(\frac{\sqrt{-(\frac{\kappa}{2})^2 + 4{\omega_c}^2}}{2} t)) \tag{2}$$

The α and β constants will be determined with the initial conditions after finding the particular solution $q_p(t)$.

The particular solution is a complex exponential solution: $\tilde{q_p}(t) = \tilde{A}e^{j(\omega t - \phi)}$ corresponding to a linear equation. Substituting this solution into the differential Eq.(1) we obtain:

$$(-\omega^2 + j\omega\frac{\kappa}{2} + \omega_c^2)\tilde{A}e^{j(\omega t - \phi)} = E_L e^{j\omega t} \tag{3}$$

Dividing by $e^{j\omega t}$, we can find both the real and imaginary parts of $\tilde{A}$, and calculate the modulus and the phase of the complex number. We can write the particular solution:

$$q_p(t) = \frac{E_L}{\sqrt{(\omega_c^2 - \omega^2)^2 + (\omega\frac{\kappa}{2})^2}} \cos(\omega t - \phi)$$
$$\phi = \arctan[\frac{Im[\tilde{A}]}{Re[\tilde{A}]}] = \arctan[\frac{-\omega\frac{\kappa}{2}}{\omega_c^2 - \omega^2}] \tag{4}$$

The full solution is the sum of the homogeneous and particular solutions. With the two initial conditions $q(0) = q_0$ and $q'(0) = q_0'$, we can compute the α and β constant coefficients of the homogeneous solution. The damping decade term $e^{-\frac{\kappa}{4}t}$ in Eq.(2) depends on the cavity's quality factor Q, which is directly related to the optical cavity lifetime as $Q = \frac{\omega_c}{\kappa}$. As shown in Figure 2.b, the initial conditions lost their influence because of the damping term, and only the particular driven solution still. A resonance occurs when

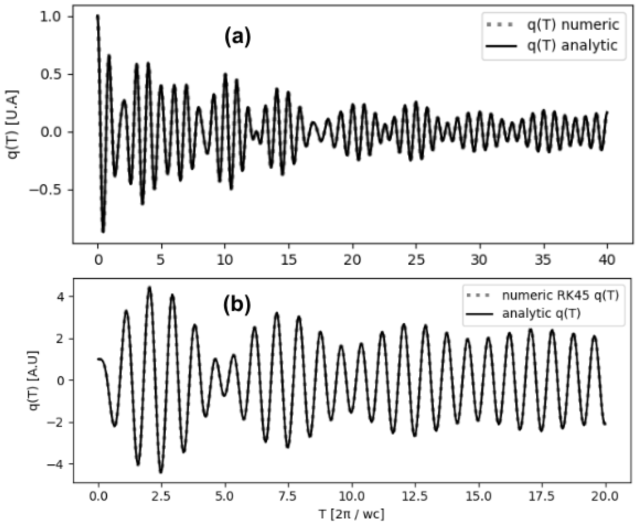

Figure 2. Numerical Runge-Kutta RK45 and analytical resolutions of the cavity periodic oscillations for a laser excitation frequency of $1.2[\omega_c]$, with the values of Table 1. (a) the cavity driven by a laser and coupled with the emitter, and (b) the cavity driven by a laser.

ω tends approximately to ω_c because the $|\tilde{q}_p(w)|$ amplitude term in Eq.4, reaches a maximum, which corresponds to the coupled long-term oscillation frequency. The cavity dissipative element $(\omega\frac{\kappa}{2})^2$ is the physical limitation that corresponds to the photon leakage rate.

III. SIMPLE MODEL OF LIGHT-MATTER INTERACTIONS IN OPTICAL CAVITIES

The light-matter coupling in cavities can be modeled by two oscillators, coupled by the Rabi frequency as show in Figure. The emitters have a frequency of Δ, corresponding to their excitation frequency, with a non-radiative losses term Γ. The cavity is driven by a laser, represented by an amplitude of E_l. The laser is only needed to visualize the transmission and so the two eigenmodes frequencies of polaritons, but they also occurs without external excitation in dark cavities. This system is composed of two differential equations coupled by the Rabi frequency as shown in Eq.5. In the strong coupling regime, the two equations solutions will become very similar because of the formation of a hybrid single quasi-particle.

$$q'' + \frac{\kappa}{2}q' + {w_c}^2 q + \omega_c\Omega(q - Q) = E_l cos(\omega t) \tag{5}$$
$$Q'' + \frac{\Gamma}{2}Q' + \Delta^2 Q + \omega_c\Omega(Q - q) = 0 \tag{6}$$

In the same way as the uncoupled case, the particular solutions of the two differential equations consists of complex exponential solutions, allowing us to find the susceptibility $\chi(\omega) = (\tilde{q}(w)e^{-j\omega t}, \tilde{Q}(w)e^{-j\omega t})$. Since these

Light-matter	Definition	Fig.2.3.4 Values
Q	Quality of the cavity	10
κ	Cavity loss coefficient	0.1 [wc]
Γ	Spontaneous emission rate	$10^{-5}[wc]$
Ω	Rabi frequency	6 fs
Δ	Emitters frequency	2.2 eV
ω_c	Cavity frequency	$\sim \Delta$
E_l	Laser E-M field amplitude	
ω_L	Laser frequency	[0.9;1.3] [wc]

Table I. Corresponding terms for the analog differential equation solution.

are two linear equations, we can find the corresponding matrix M by exploiting the exponential derivation properties, which highlight their mutual coupling.

$$\begin{pmatrix} -\omega^2 - j\omega\frac{\kappa}{2} + \omega_c^2 & -\omega_c\Omega \\ -\omega_c\Omega & -\omega^2 - j\omega\frac{\Gamma}{2} + \Delta^2 \end{pmatrix} \begin{pmatrix} \tilde{q}(\omega)e^{-j\omega t} \\ \tilde{Q}(\omega)e^{-j\omega t} \end{pmatrix} = \begin{pmatrix} E_l e^{-j\omega t} \\ 0 \end{pmatrix} \quad (7)$$

In order to find the expressions $\tilde{q}(w)$ and $\tilde{Q}(w)$, we use linear algebra, noting that if the matrix invertible, $det(M) \neq 0$, we have:

$$\text{With} \quad F = \begin{pmatrix} E_L \\ 0 \end{pmatrix} \quad \text{we have} \qquad MX(w) = F$$
$$X(w) = M^{-1}F$$

We find the real and imaginary parts of $\tilde{q}(\omega)$ and $\tilde{Q}(\omega)$, as shown in appendix B. As in the simple forced harmonic oscillator case, we obtain the modulus and phase of the complex solutions. Then we obtain the particular solutions $q_p(t) = |\tilde{q}(w)|\cos(wt + \phi_q)$ and $Q_p(t) = |\tilde{Q}(w)|\cos(wt + \phi_Q)$, as shown in appendix B.

Still using the same method as the simple oscillator case, the homogeneous solution is a linear combination of exponential forms. We can compute the following matrix by deriving:

$$\begin{pmatrix} r^2 + r\frac{\kappa}{2} + {\omega_c}^2 & -\omega_c\Omega \\ -\omega_c\Omega & r^2 + r\frac{\Gamma}{2} + \Delta^2 \end{pmatrix} \begin{pmatrix} \tilde{q}(w)e^{rt} \\ \tilde{Q}(w)e^{rt} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (8)$$

Non trivial solutions for $\chi(\omega)$ with $(\tilde{q}, \tilde{Q}) \neq 0$ are found with the condition $det(M) = 0$.

$$det(M) = (r^2 + r\frac{\kappa}{2} + {\omega_c}^2)(r^2 + r\frac{\Gamma}{2} + \Delta^2) - (\omega_c\Omega)^2 = 0 \quad (9)$$

As explain in appendix B, the solutions of this fourth polynomial equation, will be complex numbers corresponding to 4 eigen values λ. We find each corresponding eigenvectors V_λ and there coefficients determined with the initial conditions.

To find the corresponding M_n matrix, we need to find a matrix with a dimension compatible with our four initial conditions. By derivation, we can compute the M_4 matrix, multiplied by any vector V_λ, and with the $q(t), q'(t), Q(t), Q'(t)$ terms vector, corresponding to the four initial conditions.

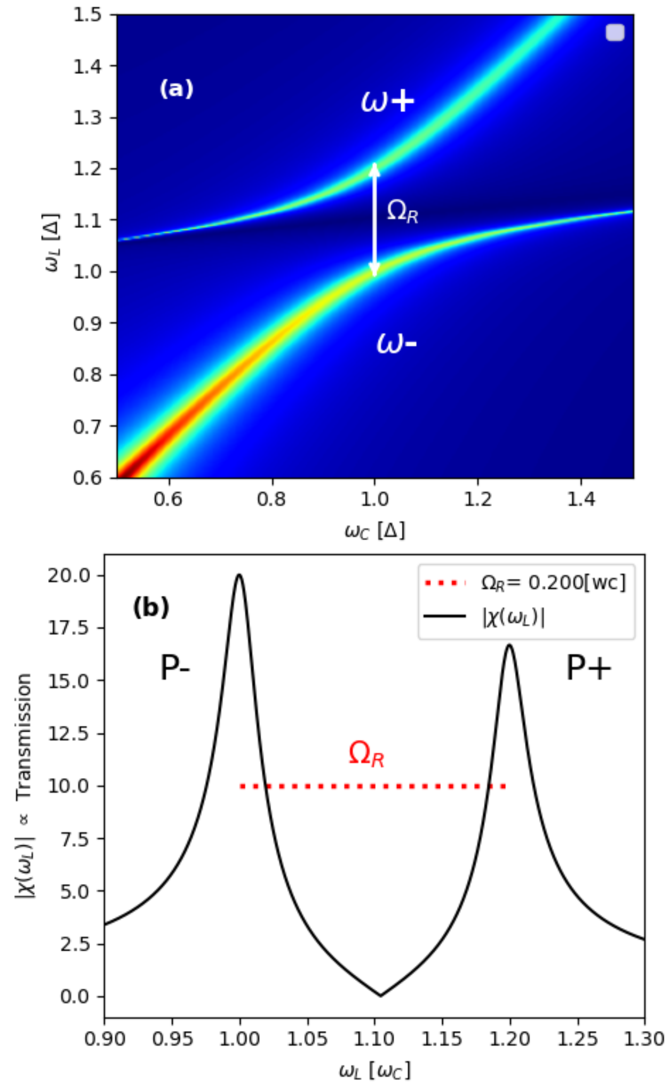

Figure 3. Diagrams with experimentals values of strong coupling cavities configurations [3]. $\Delta = 2.2$ eV, and $q_p(\omega) = \chi(w)E_l(\omega)$. Cavity lifetime of 25 fs and an excitation state lifetime $\sim$ 50 ps. The Rabi frequency is at 6 fs. (a) The two polaritonic modes with a detuning of ω_c and Δ. (b) The two polaritonic-states P- and P+ with $\Delta = 1.2[\omega_c]$.

$$\begin{pmatrix} 0-\lambda_i & 1 & 0 & 0 \\ -\omega_c^2 - \omega_c\Omega & -\frac{\kappa}{2} - \lambda_i & \omega_c\Omega & 0 \\ 0 & 0 & 0-\lambda_i & 1 \\ \omega_c\Omega & 0 & -\Delta^2 - \omega_c\Omega & -\frac{\Gamma}{2} - \lambda_i \end{pmatrix} \begin{pmatrix} V_{iq} \\ V_{iq'} \\ V_{iQ} \\ V_{iQ'} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (10)$$

Resolving this four linear systems, we can found the V_{iq} and V_{iQ} eigen-vectors for the C_1, C_2, C_3, C_4 coefficient and find the homogeneous solutions:

$$\begin{cases} q_{hom}(t) = c_1 V_{1q} e^{\lambda_1 t} + c_2 V_{2q} e^{\lambda_2 t} + c_3 V_{3q} e^{\lambda_3 t} + c_4 V_{4q} e^{\lambda_4 t} \\ Q_{hom}(t) = c_1 V_{1Q} e^{\lambda_1 t} + c_2 V_{2Q} e^{\lambda_2 t} + c_3 V_{3Q} e^{\lambda_3 t} + c_4 V_{4Q} e^{\lambda_4 t} \end{cases} \tag{11}$$

The complete solution of the coupled harmonic oscillator problem is given by the sum of the homogeneous and particular solutions. The long term particular solution describes the stationary regime, and the homogeneous describe the transitory regime, disappearing around twenty periodic oscillations and corresponding to the influence of the initial conditions, as we can see on the Figure 2.a.

By defining $q_p(\omega) = \chi(w_L)E_l(\omega_L)$, with $\chi(w_L)$ the susceptibility directly related to the transmission,we observe that the emitters and cavity states in strong coupling are modified, as shown in figure 3.a. We observe two resonances at ω_+ and ω_-, corresponding to the two energies of the polaritonic states. We can find the Rabi splitting of these polaritons states, and because $E = \hbar\omega$, the Rabi splitting will be an energy gap equal to $\hbar(\omega_+ - \omega_-)$. This clearly shows that in the strong coupling regime, light and matter excitation do hybridization resulting in mode coupling. This model give very similar result to the experimental observations of the transmission spectrum in strong coupling cavities.

As we can see on Figure 4.a, the decoupling cause one of the two resonances peaks to disappear, and shift the other to the correspondent uncoupled frequency which is not an hybrid state anymore.

We also observe that in the strong coupling configuration, the widths of the peaks are approximately equal, depending on the κ and Γ factors of the cavity and emitter. This relation could also be use, conversely, to find the cavity lifetime or the non-radiative losses from the peaks widths in the spectrum.

This analog model of strongly coupled oscillators represents well the formation of polaritonic states, however many other factors are not taken into account, for example the number of elements in the cavity, their dipoles, the direction of the E-M field, the neglect of temperature effects, the mirrors transmissions and reflection coefficients, the collective effects and a lot of other parameters. This model provides parts for building a representation of the physical mechanism responsible of the strong coupling observations.

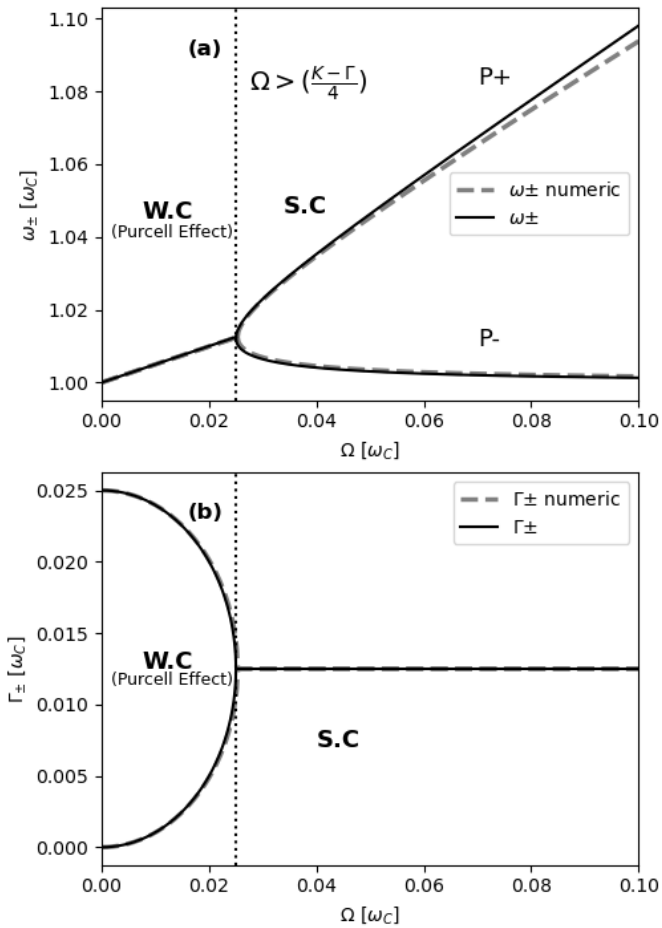

Figure 4. (a) In black, the analytical eigenvalues approximation for $\omega_L \sim \omega_c \sim \Delta$, and in a dotted gray, the numerical solutions with the fourth polynomial equations solution versus a given Rabi frequency. The vertical doted line corresponds to the delimitation between the weak coupling and the strong coupling regime. (b) The black line represent the emission frequencies with analytical approximation, while the gray dotted line corresponds to the numerical solution, versus a Rabi frequency.

IV. ANALYTICAL EXPRESSION OF POLARITONIC STATES

The formation of polaritons due to the coupled phenomena occurs only when ω_c approaches Δ as shown in Eq.4, meaning that the detuning tends to zero. We can therefore assume that an approximation the poles near $\omega_c \sim \Delta$ shows interesting analytical results for both strong and weak coupling. This approximation simplifies the fourth order polynomial equation in Eq.(9), enabling a clearer analysis with only two eigenfrequencies against four before. Starting from the Eq.(7), which described a coupled and driven oscillating system, we look for the frequencies of the two peaks. Using Eq.(8), we note by Cramer's rule that when the determinant of the matrix approaches zero, there is a maximum. We simplify the matrix by setting the resonant situatoin $\omega_c = \Delta$ and the approximation $\omega_L \sim \omega_c$. With a first-order approximation with the Taylor series expansion, we find the following matrix M:

$$\begin{pmatrix} 2\omega_c(\omega_c - \omega_L) + \omega_c\Omega - j\omega_c\frac{\kappa}{2} & -\omega_c\Omega \\ -\omega_c\Omega & 2\omega_c(\omega_c - \omega_L) + \omega_c\Omega - j\omega_c\frac{\Delta}{2} \end{pmatrix} \tag{12}$$

We now have an second-order polynomial equation resolutions of the determinant equation, as shown in appendix

C: if $\Omega < \frac{\kappa-\Gamma}{4}$, weak coupling configuration:

$$\omega_{\pm} = \omega_c + \frac{\Omega}{2} \tag{13}$$
$$\Gamma_{\pm} = (\frac{\Gamma+\kappa}{8}) \pm \frac{\sqrt{(\frac{\Gamma+\kappa}{4})^2 - \frac{\kappa\Gamma}{4} - \Omega^2}}{2}$$

if $\Omega > \frac{\kappa-\Gamma}{4}$, strong coupling configuration:

$$\omega_{\pm} = \pm\frac{\sqrt{-(\frac{\Gamma+\kappa}{4})^2 + \frac{\kappa\Gamma}{4} + \Omega^2}}{2} + \omega_c + \frac{\Omega}{2}$$
$$\Gamma_{\pm} = (\frac{\Gamma+\kappa}{8}) \tag{14}$$

The Rabi splitting value:

$$\Omega_R = \omega_+ - \omega_- \tag{15}$$

There is observable transition between the weak and strong coupling at $\Omega \approx \frac{\kappa-\Gamma}{4}$. The weak coupling regime correspond to $\Omega < \frac{\kappa-\Gamma}{4}$ as shown in Eq.14, the strong coupling regime correspond to $\Omega > \frac{\kappa-\Gamma}{4}$, for Eq.15. The imaginary parts of the solutions corresponds to the polaritons eigenfrequencies $\omega_{\pm}$, while the real parts belong to the emitters linewidth $\Gamma_{\pm}$, the lifetime.

As shown in Figure 4.a, in weak coupling regime, only one eigenfrequency is observable, corresponding to the cavity frequency. This show that the cavity keep it frequency coupled only with the laser, corresponding to the simple model of optical cavities II. The Figure 4.b shows two linewidths in weak coupling conditions, caused by the purcell effect[1] which enhance emissions. When the coupling value is null, the two lifetimes corresponds to κ and Γ, which are the two uncoupled harmonic oscillator losses. This is because the losses exceed the energy exchange rate Ω, so the emit photons escapes the cavity before getting reabsorbed.

When the Rabi frequency approaches the transition regime $\Omega \approx \frac{\kappa-\Gamma}{4}$, the linewidths converges to reach a single linewidth value. This is because of the formation of the new quasiparticle.

In strong coupling conditions, the energy exchange rate is bigger than the losses $\Omega > \frac{\kappa-\Gamma}{4}$, we observe the apparition of two eigen frequencies an Figure 4.a, corresponding to the two polaritonic states. The Rabi splitting $\Gamma \sim \frac{\kappa-\Gamma}{8}$ is the gap of exchanged energy between the two states, which grows approximately linearly with the Rabi frequency for the approximation $\omega \sim \omega_c$. The two linewidths are reunited because of the single hybrid quasiparticle lifetime. In the strong-coupling regime, the hybrid light–emitters quasiparticle emits twice less than the free-space emitter, resulting in a longer cavity excitation lifetime. The emission rate stabilizes at approximately $\Gamma \sim \frac{\kappa-\Gamma}{8}$, the rates of energy exchange and loss balance. However, once the Rabi frequency has exceeded the critic transition value, the emission rate no longer relies on it. We can also observe that the numerical result is very accurate for $\Omega < 0.1[\omega_c]$, but loss precision with the increase of Ω, which confirm that this approximation is only pertinent at the poles.

This shows that in this regime, the cavity quality becomes the most important factor determining the excitation lifetime of the polaritons. The more the photons are emitted and reabsorbed inside the cavity, the longer the emitters remains excited. This clearly shows the potential of strong coupling cavities for example in enhanced energy transport.

V. CONCLUSION

In conclusion, we demonstrated the emergence of two eigenstates under strong coupling conditions and clarified the influence of parameters such as non-radiative losses, the cavity quality, and the Rabi frequency. The Light–matter interactions in a strong coupling cavity, give rise to polaritons with a Rabi splitting $\Omega_R = (\omega_+ - \omega_-)$, which increases as the Rabi frequency increases approximately linearly. We determined that there is a transition between the weak and strong coupling regimes at $\Omega \sim \frac{\kappa-\Gamma}{4}$, because photons can be emit and reabsorbed inside the cavity. The contribution of this phenomenon to enhanced energy transport and its influence on molecular reactions is promising. The classical analog explication of the Rabi splitting apparition shows very similar relations to the experimental observations, which makes it a pertinent approach to understand the dynamics inside a strong coupling cavities. Future work could include the adding of noise to get closer results to the experimental observations.

Appendix A: Simple model of optical cavities

Differential equation of the cavity oscillator.

$$q'' + \frac{\kappa}{2}q' + {w_c}^2 q = E_L cos(\omega t) \tag{A1}$$

To resolve the homogeneous equation we pose the exponential form solution $q_h = Ae^{rt}$ and we find:

$$r^2 + \frac{\kappa}{2}r + {w_c}^2 = 0 \tag{A2}$$

We therefore observe three different solutions of Eq.(A2), depending on the damping coefficient κ. If $\kappa > 4\omega_c$, $\Delta > 0$, corresponding to heavily damped system with no observable periodic oscillations.

If $\Delta = 0$, the system is called critic and the regime will be perfectly compensated so the q(t) solution will have a single exponential shape.

If $\kappa < 4\omega_c$ the solution will be complex due to $\Delta < 0$ term in the solution, corresponding to a periodic oscillating system composed with imaginary exponential forms. This last regime is the most interesting due to its resonance behavior. In this case the solution will corresponds to a second polynomial complex form, because of low damping:

$$\tilde{r_1}, \tilde{r_2} = \frac{-\frac{\kappa}{2} \pm j\sqrt{-\Delta}}{2} \tag{A3}$$

The solution being a linear combination of exponential forms, it is possible to simplify by the following observation: $Re[\tilde{r_1}] = Re[\tilde{r_2}]$ and $Im[\tilde{r_1}] = -Im[\tilde{r_2}]$, and calculate $q_h(t)$.

$$q_h(t) = e^{Re[\tilde{r_1}]t}(Ae^{jIm[\tilde{r_1}]t} + Be^{-jIm[\tilde{r_1}]t}) \tag{A4}$$

We can take the real part:

$$q_h(t) = e^{-\frac{\kappa}{4}t}(\alpha\cos(\frac{\sqrt{-\Delta}}{2}t) + \beta\sin(\frac{\sqrt{-\Delta}}{2}t)) \tag{A5}$$

To resolve the particular equation: with a $\tilde{q_p}(t) = \tilde{A}e^{j(\omega t - \phi)}$.

$$\begin{aligned} \alpha &= q_0 - A\cos(\phi) \\ \beta &= \frac{q_0' + \omega A\sin(\phi) + \frac{\kappa}{4}\alpha}{\frac{\sqrt{-\Delta}}{2}} \end{aligned} \tag{A6}$$

Appendix B: Simple model of light-matter interactions in optical cavities

The differentials equations.

$$q'' + \frac{\kappa}{2}q' + {w_c}^2 q + \omega_c\Omega(q - Q) = E_l cos(\omega t) \tag{B1}$$

$$Q'' + \frac{\Gamma}{2}Q' + \Delta^2 Q + \omega_c\Omega(Q - q) = 0 \tag{B2}$$

With the particular solution form :$\tilde{q}(w)e^{-j\omega t}$ and $\tilde{Q}(w)e^{-j\omega t}$ because of linearity we have:

$$\begin{pmatrix} -\omega^2 - j\omega\frac{\kappa}{2} + \omega_c^2 & -\omega_c\Omega \\ -\omega_c\Omega & -\omega^2 - j\omega\frac{\Gamma}{2} + \Delta^2 \end{pmatrix} \begin{pmatrix} \tilde{q}(\omega)e^{-j\omega t} \\ \tilde{Q}(\omega)e^{-j\omega t} \end{pmatrix}$$

$$= \begin{pmatrix} E_l e^{-j\omega t} \\ 0 \end{pmatrix}$$

Both sides divided by $e^{-j\omega t}$, the linear problem simplified. Using separate imaginary and real coefficients for better visualization in computing we have:

$$M = \begin{pmatrix} a - jb & \gamma \\ \gamma & a' - jb \end{pmatrix} \tag{B3}$$

We use linear algebra properties for $det(M) \neq 0$, meaning reversible matrix. After the use of Cramer's rules and some literal calculations for simplification, we can pose this coefficients and solutions of both the real and imaginary parts of the two factors:

$$\alpha = (aa' - bb' - \gamma^2), \quad \beta = (ba' + ab')$$

$$\tilde{q}(\omega) = \begin{cases} \mathrm{Re}[\tilde{q}(\omega)] = \frac{f}{\alpha^2 + \beta^2}(a'\alpha + b'\beta) \\ \mathrm{Im}[\tilde{q}(\omega)] = \frac{f}{\alpha^2 + \beta^2}(b'\alpha - a'\beta) \end{cases}$$

$$\tilde{Q}(\omega) = \begin{cases} \mathrm{Re}[\tilde{Q}(\omega)] = \frac{f}{\alpha^2 + \beta^2}(\gamma\alpha) \\ \mathrm{Im}[\tilde{Q}(\omega)] = \frac{f}{\alpha^2 + \beta^2}(-\gamma\beta) \end{cases}$$

With the complex number proprieties we find the amplitude and the phase of each particular solutions:

$$\begin{cases} |\tilde{q}(w)| = \sqrt{\mathrm{Re}[\tilde{q}(\omega)]^2 + \mathrm{Im}[\tilde{q}(\omega)]^2} \\ \phi_q = \arctan[\frac{\mathrm{Im}[\tilde{q}(\omega)]}{\mathrm{Re}[\tilde{q}(\omega)]}] \end{cases} \tag{B4}$$

(B5)

And finally find the particular solutions:

$$q_p(t) = |\tilde{q}(w)|\cos(wt + \phi_q) \tag{B6}$$

$$Q_p(t) = |\tilde{Q}(w)|\cos(wt + \phi_Q) \tag{B7}$$

To resolve the Homogeneous equation we use the propriety of an irreversible matrix so $det(M) = 0$.

$$det(M) = (r^2 + r\frac{\kappa}{2} + {\omega_c}^2)(r^2 + r\frac{\Gamma}{2} + \Delta^2) - (\omega_c\Omega)^2 = 0 \tag{B8}$$

We have the following relations:

$$(M_n - \lambda_i In)V_i = 0 \qquad V_i = \begin{pmatrix} V_{iq} \\ V_{iq'} \\ V_{iQ} \\ V_{iQ'} \end{pmatrix} \tag{B9}$$

To find the M_n matrix, resolving the (15) equation for each eigen vector, we need to find the matrix with a dimension allowing a solution with the four initial conditions. By derivation, we can compute the matrix M_4, multiplied by any vector V_i, and with the $q(t), q'(t), Q(t), Q'(t)$ terms of initial conditions.

$$\frac{dq}{dt} = q' \qquad \frac{dq'}{dt} = (-{\omega_c}^2 - \omega_c\Omega)q + (-\frac{\kappa}{2})q' + (\omega_c\Omega)Q \tag{B10}$$

$$\frac{dQ}{dt} = Q' \quad \frac{dQ'}{dt} = (-{\omega_c}^2 - \omega_c\Omega)Q + (-\frac{\kappa}{2})Q' + (\omega_c\Omega)q \tag{B11}$$

$$\begin{pmatrix} 0 - \lambda_i & 1 & 0 & 0 \\ -\omega_c^2 - \omega_c\Omega & -\frac{\kappa}{2} - \lambda_i & \omega_c\Omega & 0 \\ 0 & 0 & 0 - \lambda_i & 1 \\ \omega_c\Omega & 0 & -\Delta^2 - \omega_c\Omega & -\frac{\Gamma}{2} - \lambda_i \end{pmatrix} \begin{pmatrix} V_{iq} \\ V_{iq'} \\ V_{iQ} \\ V_{iQ'} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Resolving this linear systems, we can found V_i for i from 1 to 4, and find the coefficients C_1, C_2, C_3, C_4 with initial conditions with the particular solution influence. Because $\forall\lambda_i$ if $t = 0, e^{\lambda_i t} = 1$, we have to resolve the corresponding system:

$$\begin{cases} q_0 - q_p(0) & = c_1V_{1q} + c_2V_{1q'} + c_3V_{1Q} + c_4V_{1Q'} \\ q'_0 - q'_p(0) & = c_1V_{2q} + c_2V_{2q'} + c_3V_{2Q} + c_4V_{2Q'} \\ Q_0 - Q_p(0) & = c_1V_{3q} + c_2V_{3q'} + c_3V_{3Q} + c_4V_{3Q'} \\ Q'_0 - Q'_p(0) & = c_1V_{4q} + c_2V_{4q'} + c_3V_{4Q} + c_4V_{4Q'} \end{cases}$$

To find the following solution:

$$\begin{cases} q_{hom}(t) = c_1V_{1q}e^{\lambda_1 t} + c_2V_{2q}e^{\lambda_2 t} + c_3V_{3q}e^{\lambda_3 t} + c_4V_{4q}e^{\lambda_4 t} \\ Q_{hom}(t) = c_1V_{1Q}e^{\lambda_1 t} + c_2V_{2Q}e^{\lambda_2 t} + c_3V_{3Q}e^{\lambda_3 t} + c_4V_{4Q}e^{\lambda_4 t} \end{cases} \tag{B12}$$

Appendix C: Analytical expression of polaritonic states

By posing $\omega_c = \Delta$ and $\omega_c \sim \Delta$:

$$\begin{aligned} (\omega_c + \omega_L)(\omega_c - \omega_L) &\sim 2\omega_c(\omega_c - \omega_L) \\ (\Delta + \omega_L)(\Delta - \omega_L) &\sim 2\omega_c(\omega_c - \omega_L) \end{aligned} \tag{C1}$$

So we have:

$$\begin{pmatrix} 2\omega_c(\omega_c - \omega_L) + \omega_c\Omega - j\omega_c\frac{\kappa}{2} & -\omega_c\Omega \\ -\omega_c\Omega & 2\omega_c(\omega_c - \omega_L) + \omega_c\Omega - j\omega_c\frac{\Delta}{2} \end{pmatrix} \tag{C2}$$

Trying to find the null determinant:

$$\begin{aligned} \tilde{\omega} &= (\omega_L - \omega_c - \frac{\Omega}{2}) \\ det(\frac{M}{2\omega_c}) &= (\tilde{\omega} - j\frac{\kappa}{4})(\tilde{\omega} - j\frac{\Delta}{2}) - \frac{\Omega^2}{4} = 0 \end{aligned} \tag{C3}$$

if $\Omega < \frac{\kappa - \Gamma}{4}$, weak coupling configuration:

$$s_\pm = -j(\frac{\Gamma+\kappa}{8}) \pm \frac{\sqrt{\mathbf{-(\frac{\Gamma+\kappa}{4})^2 + \frac{\kappa\Gamma}{4} + \Omega^2}}}{2} + \omega_c + \frac{\Omega}{2} \tag{C4}$$

if $\Omega > \frac{\kappa - \Gamma}{4}$, strong coupling configuration:

$$s_\pm = -j(\frac{\Gamma+\kappa}{8}) \pm j\frac{\sqrt{\mathbf{(\frac{\Gamma+\kappa}{4})^2 - \frac{\kappa\Gamma}{4} - \Omega^2}}}{2} + \omega_c + \frac{\Omega}{2} \tag{C5}$$

With the complex terms corresponding to the eigenfrequencies $\omega_\pm$ and the real parts to the emission rates $\Gamma_\pm$. To have identical results in both the numerical and analytical results, we need to consider the cavity frequency with cavity losses for the numerical result, which is not identical to the cavity resonance. We consider:

$$\Delta = \omega_c \tag{C6}$$

$$\omega'_c = \sqrt{\Delta^2 + (\frac{\kappa}{2})^2 - (\frac{\Gamma}{2})^2} \tag{C7}$$

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Nom : DEBERDT **Prénom :** Francois **Sexe :** M **Né(e) le :** 15/11/2003

Adresse : 2225 CHEMIN DE LA BRETONNIERE 26240 SAINT-JEAN-DE-GALAURE FRANCE

Tél : 0475684941 **Portable :** 0628220065 **Mél :** francois.deberdt@etu.unistra.fr

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GENET, Cyriaque, DR CNRS
directeur du laboratoire Interactions lumière - matière dans les systèmes complexes