Simple Model of directed free kicks in soccer matches

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Description of free kicks

Free kick is a usual penalty when a foul occurs outside the penalty area, otherwise, it is called penalty kick. It has a fixed location where the foul occurs to start a free kick. A player can either shoot the ball when the location is close to goal, or pass the ball to teammate when the location is far from goal. Unlike penalty kicks, free kicks in soccer matches usually are hard to score. Not only because of its longer distance to goal, but also because free kicks introduce a third player, the man wall, who helps to block the ball. With such difficulties mentioned above, a free kick goal would be fantastic and amazing, especially when the ball hits toward the top bin(top corner of the goal).

Why matters

Despite regular score, free kick and penalty kick play an important role to take advantages. The penalty kick has an extremely high success rate, while free kicks are usually seen as long as there is a foul. Soccer is a team work with 11 players versus 11 players. However, for a directed free kick, from the striker's perspective, it is more about an individual's ability. Unlike basketball games, which has high scores rate that one can see 120-118 when the match ends, soccer usually has an extremely low scores in one match. 1-0, 2-0, 1-1 matches are usual, and 5 to 6 goals in total can be called "crazy match". As a result, each goal in a match is crucial and that may play a decisive role. Since free kicks are common in matches, a team with relatively high success rate on free kick absolutely takes advantages in matches. So, how to convert a free kick to goal is the basic question that needs to be answered by the model.

Model Description

This model is intended to analyze and gives out suggestions/solutions of which directions to kick, what part of sphere/ball surface needed to kick based on the striker's current location in order to reach a high success rate of converting a free kick to goal. This model may or may not concern the Man Wall depending on the difficulty and the time I spent on making this model. Note Man wall can be considered as a barrier during the the process of the ball flying to goal. Basically, this is a Aerodynamic Model of directed free kicks but with some assumptions and constraints which will be mentioned below.

Basically, the model will take the inputs x and y in 2 dimensional space and output (N, α, θ) where N is the initial kinetic energy, α is the horizontal angle, and θ is the vertical angle. These three outputs decide the direction and the trajectory of the ball when flying towards goal.

Assumption and Constraints

1. This model is intended to discuss the directed free kick which a striker will shoot the ball directly to goal. It does not consider the case where a player starts the ball by passing it to his teammate and his teammate finishes the goal.

2. Historically, logically, and for the simplicity, shooting to top bins with enough kinetic energy usually has an incredible success rate. So, the end point of the ball will be either top left corner or top right corner in the three dimensional space. The model does not consider other ending locations, including but not limited to bottom left, bottom right, top mid, bottom mid, soccer not in goal range, etc.

3. For the simplicity, this model would be an ideal model where the air resistance may not be considered unless necessary.

4. The motion of the ball is a basic projectile motion.

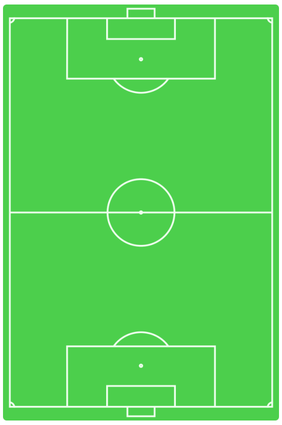
5. Vertical jump height. Assume players at defensive wall do not jump to block the ball.

6. Assume there is no wind flow.

7. Assume the defensive wall will always stand at the location 9.144 meters away from the shooting point. Note: 9.144 meters will be explained at the Fixed Variable section.

Variables

Let the soccer field be the xy-plane, and the height be the z-plane. That is, the horizontal line would the x direction, and the vertical line would be the y direction. Further details will be discussed when the actual project begins. Below is just a sample field.



Non-Fixed Variable

1. Shooting location (x,y,0).

2. The angle α in xy plane which determines the ending position of the ball, left or right.

3. The angle θ in yz plane which determines the ending position of the ball, height.

4. Distance to goal d_{right} and d_{left}.

Fixed Variable

1. Size of the soccer field. We use Camp Nou Stadium as our model. The length is 105 meters and the width is 68 meters.[1]

2. Size of the goal. According to the Federation Internationale de Football Association(FIFA), the size of goal is 2.44 meters height and 7.32 meters length.[2]

3. Width of the penalty area is 16.46 meters. This is important because free kicks usually not happen in the penalty area, so the distance to goal usually at least the width of the penalty area.[3]

4. The min distance from defensive wall to the shooting point is 10 yards(9.144 meters).[4]

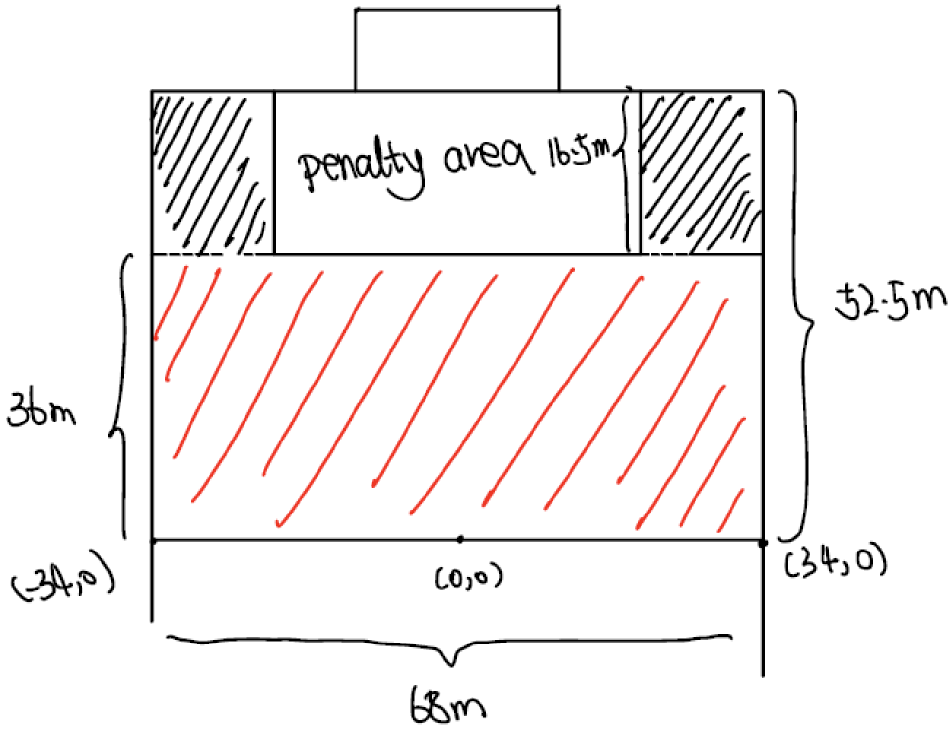
5. The average height of soccer player in 2018 World Cup was 182.4 cm.[5]

6. The circumference of the ball is between 68-70 cm. So, we use 69 cm, which means the radius of the ball is around 10.98 cm $\approx$ 11 cm. [7]

7. Gravitational acceleration constant a = -9.81 m/s^2.

8. Though free kick happens as long as not in penalty area, this model does not consider the black painted area. It only consider red painted are. So,

$-34 < x < 34, 0 < y < 36$



Mathematics and Physics

This is an ideal basic projectile model. Assuming there is no air resistance, ignoring the air pressure toward the ball. Basically, this only a hyperbola trajectory.

Here are some formulas.

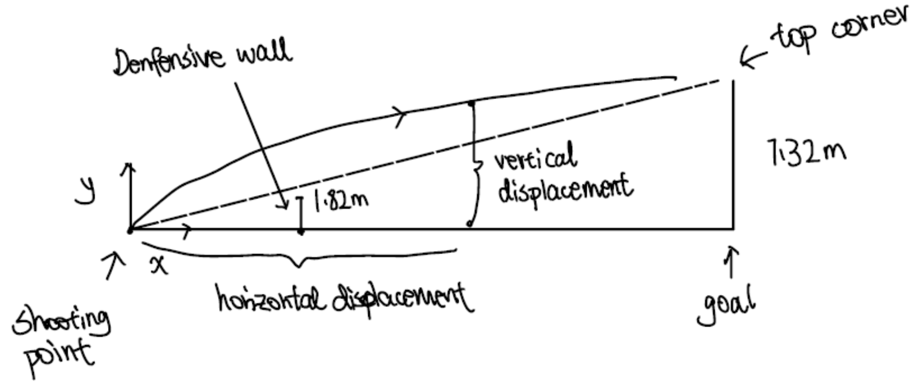
1. $\boldsymbol{V_{vertical} = V_{vi} - a * t}$ where a denotes the gravitational constant, which is $9.81m/s^2$. V_{vi} denotes the initial velocity at vertical direction. t denotes the time.

2. $\boldsymbol{V_{horizon} = V_{hi}}$ In horizontal direction, it is a constant speed. V_{hi} denotes the initial velocity at horizontal direction.

3. $\boldsymbol{d_{vertical} = V_{vi} * t - 0.5 * a * t^2}$ [6] This is the vertical displacement formula. Besides, this is also the trajectory of the ball.

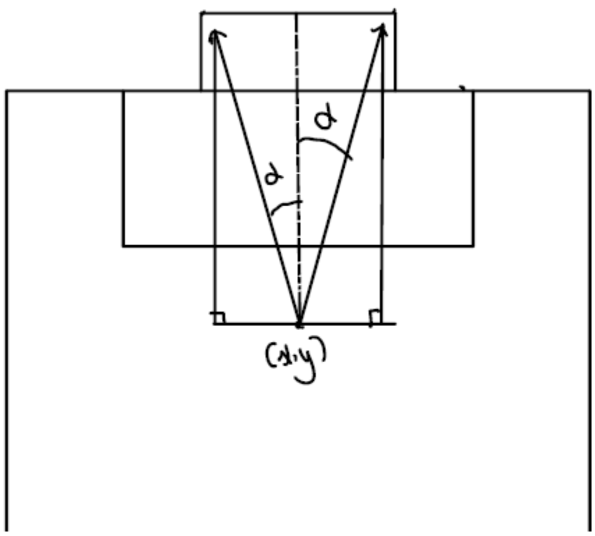
4. $\boldsymbol{d_{horizon} = V_{horizon} * t}$ This is the horizontal displacement formula.

Side View



This image represents the side view of the projectile motion. Note: the x and y in this image are not the same x y mentioned below. For clarification, the x mentioned here is the distance to goal, and the y here is the vertical displacement of the ball.

Top View

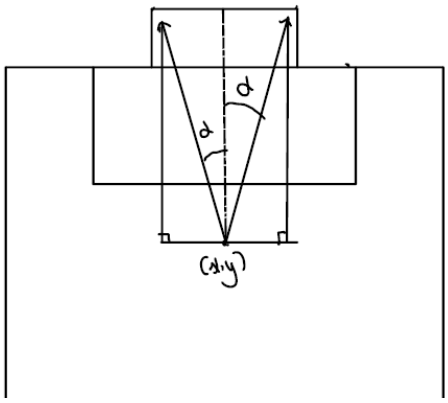


This image represents the top view of the field.

Computation

Distance to Goal: Assuming the court is an xy plane that the middle point of the field is the origin. With the fixed variable that the length of the court is 105m, the length of the goal is 7.32m and the height of the goal is 2.44m. Then the coordinates of the left corner top corner and right top corner are $right = (3.66, 52.5, 2.44)$, $left = (-3.66, 52.5, 2.44)$. Note the radius of the ball is 0.11m. In order to let the ball score, adjust the target. The new right top corner would be $right = (3.66 - 0.11, 52.5, 2.44 - 0.11) = (3.55, 52.5, 2.33)$ and $left = (-3.66 + 0.11, 52.5, 2.44 - 0.11) = (-3.55, 52.5, 2.33)$. Thus, for any given shooting point, (x,y,0). The distance to goal is $\boldsymbol{d_{right}} = \boldsymbol{\sqrt{(x - 3.55)^2 + (y - 52.5)^2}}$, and the $\boldsymbol{d_{left}} = \boldsymbol{\sqrt{(x - (-3.55))^2 + (y - 52.5)^2}}$.

Horizontal angle α:



Recall $right = (3.55, 52.5, 2.33)$, shooting point is (x,y,0), then $tan\alpha_{right} = \frac{|y-52.5|}{|x-3.55|}$, thus the $\alpha_{right} = 90 - \arctan\frac{|y-52.5|}{|x-3.55|}$, same for the $\alpha_{left} = 90 - \arctan\frac{|y-52.5|}{|x+3.55|}$. Since the actual angle we want is shown above.

Modify the formula: Recall the horizontal displacement formula: $\boldsymbol{d_{horizon} = V_{horizon} * t}$. Modify it and get $t = \frac{d_{horizon}}{V_{horizon}}$, plug it to the horizontal displacement formula, $d_{vertical} = V_{vi} * t - 0.5 * a * t^2$ and get $\boldsymbol{d_{vertical} = V_{vi} * \frac{d_{horizon}}{V_{horizon}} - 0.5 * 9.81 * (\frac{d_{horizon}}{V_{horizon}})^2}$. This formula needs to satisfy some constraints in order to have a successful free kick. The **first** constraint is, when the horizontal displacement is at 9.144 meters, which is the location of defensive wall, the vertical displacement needs to bigger than 1.824 meters, which is the average height of the soccer players in the World Cup. If the vertical displacement of the ball at 9.144 meters is smaller or equals to 1.824 meters, then the defensive wall will successfully block the ball. This means $\boldsymbol{d_{vertical} = V_{vi} * \frac{9.144}{V_{horizon}} - 0.5 * 9.81 * (\frac{9.144}{V_{horizon}})^2} >$

1.824. The **second** constraint is about the goal line. That is, when the horizontal displacement reaches d_{left} or d_{right}, the vertical displacement must be smaller than the height of goal, which is 2.44 meters. Since the ball has radius 0.11 meters, the vertical displacement of the ball center must smaller than 2.44-0.11=2.33 meters. However, since this model only targets the top corner, the vertical displacement should not be smaller than 2 meters. This means $\boldsymbol{2 < d_{vertical} = V_{vi} * \frac{d}{V_{horizon}} - 0.5 * 9.81 * (\frac{d}{V_{horizon}})^2 < 2.33}$, where d denotes d_{left} or d_{right}. The **third** constraint is the time needed to goal must smaller or equals to 2 seconds, otherwise, the motion is so slow that the goalkeeper will have the time to react and catch the ball. This means $\boldsymbol{t = \frac{d}{V_{horizon}} <= 2}$, and thus $\boldsymbol{\frac{d}{2} <= V_{horizon}}$. The **Fourth** constraint is that the 5 fastest free kicks ever recorded were 230 kmph, 188 kmph, 137 kmph, 127 kmph and 119 kmph. The model does not expect a regular free kick can reach such speed. So the velocity will be at most 119 kmph, which is around 33 m/s. This velocity will be decomposed to x, y, and z velocity vectors in 3 dimensional space, where all three velocity vectors will be smaller than 33 m/s according to the Pythagorean theorem[8]. Note the model only considers horizontal and vertical velocity.

Simulation Procedure: For any input (x,y), make sure $y > 0$. since y = 0 is the half field line. Compute the distance to goal d_{left}. Compute the Horizontal angle α. For $\boldsymbol{\frac{d}{2} <= V_{horizon} <= 33}$, each step $\boldsymbol{V_{horizon} = V_{horizon} + 0.01}$, For $\boldsymbol{0.01 <= V_{vi} <= 33}$, each step $\boldsymbol{V_{vi} = V_{vi} + 0.01}$, check
$\boldsymbol{d_{vertical} = V_{vi} * \frac{9.144}{V_{horizon}} - 0.5 * 9.81 * (\frac{9.144}{V_{horizon}})^2} > 1.824$ and
$\boldsymbol{2 < d_{vertical} = V_{vi} * \frac{d}{V_{horizon}} - 0.5 * 9.81 * (\frac{d}{V_{horizon}})^2 < 2.33}$. Record the eligible pairs $(V_{horizon}, V_{vi})$. Calculate each overall velocity and vertical angle θ, output the max overall velocity and its angle needed. Do this again for d_{right}.

File

File 1: **javaCode.java** (Click to redirect to the github for source code)
This model uses java to simulate all the possible combinations of horizontal and vertical velocity as described in the above section. Then it will output a csv file contains all the eligible combinations.

File 2: **output.csv** (Click to redirect to the github for source code)
This file is the output, which contains 2412 eligible combinations of initial velocity in both vertical and horizontal direction.

File 3: Jupyter Notebook for data visualization in python. (Still working on it)

Visualization and Analysis

Still working on it.

approximate timeline

Basically, the biggest part of the project is done, despite some language improvements and the visualization/analysis of the output. I will finish my visualization/analysis part before the Thanksgiving Day. Language improvements and Conclusion will be updated before or after talk.

Relevant work

There already exists some free kick models:
1. This is a comprehensive and complicated model which includes wall. `http://www.weizmann.ac.il/complex/falkovich/sites/complex.falkovich/files/uploads/FreeKick.pdf`
2. This is another comprehensive free kick model: `https://www.researchgate.net/publication/332464975_Simulation_and_Modeling_of_Free_Kicks_in_Football_Games_and_Analysis_on_Assisted_Training`
3. Here is a data analysis about free kick with data in American soccer league: `https://www.americansocceranalysis.com/home/2018/4/3/the-art-of-a-free-kick-and-how-to-gi`

Though similar models have been done before, my model is simpler and more ideal, and it focuses on the providing horizontal and vertical angle with some initial kinetic energy to make sure the ball ends at either side of the top corner. This will be my own model.