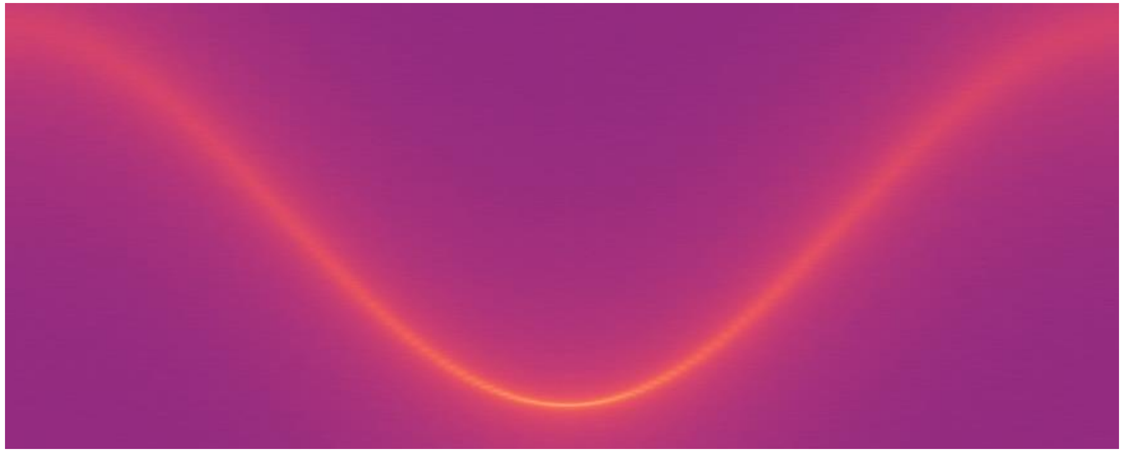
Universität Konstanz



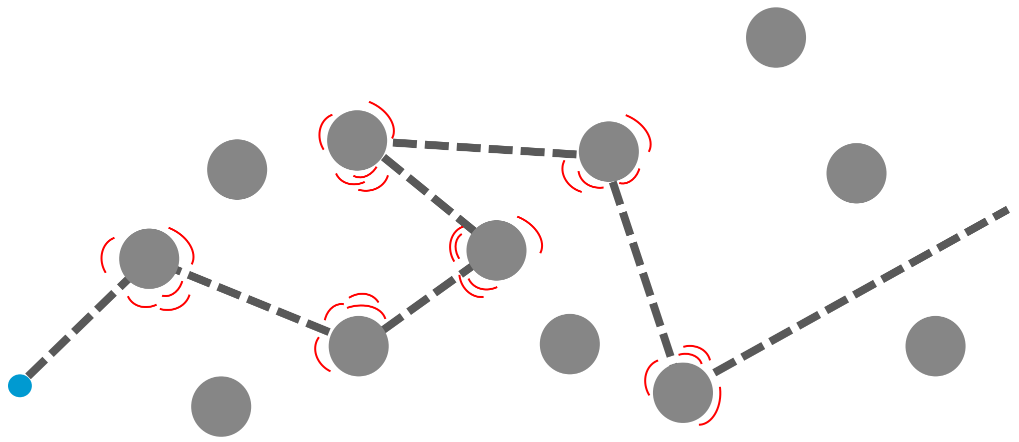
Magnons in Ferromagnets

Julian Beisch
Konstanz, 17.12.2024

Motivation

Current computing

- Electronics
- Information by moving electrons *(charge)*
  - But they scatter → Joule heating



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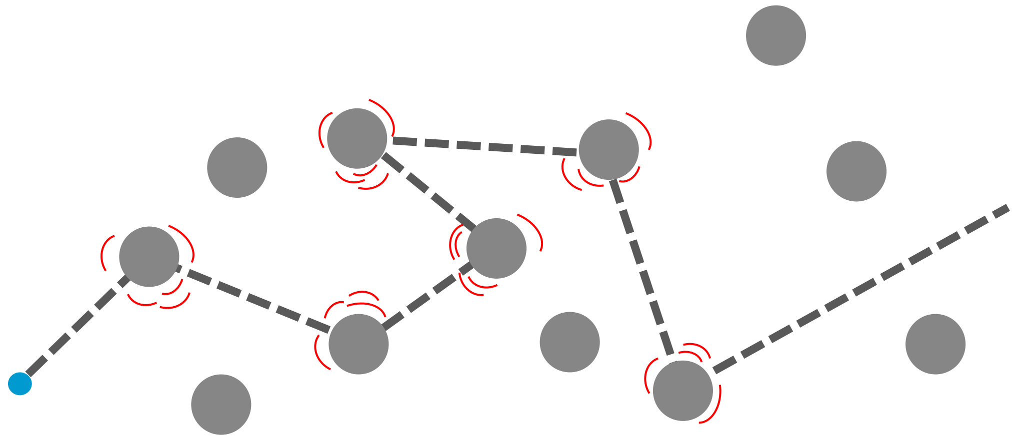
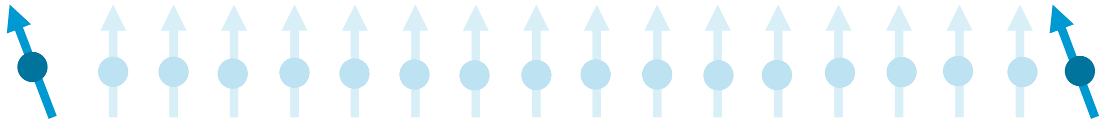


- Another property of electrons: spin
- Spintronics
- Make currents with spins, but how ?



Classification

Scope

	Bound e^-	Quasi-free e^-
Dia	Lamor Diamagnetism	Landau Diamagnetism
Para	Langevin Paramagnetism	Pauli Paramagnetism
IM	Cooperative Magnetism	Band Ferromagnetism

Spin-operators

Spin-operators

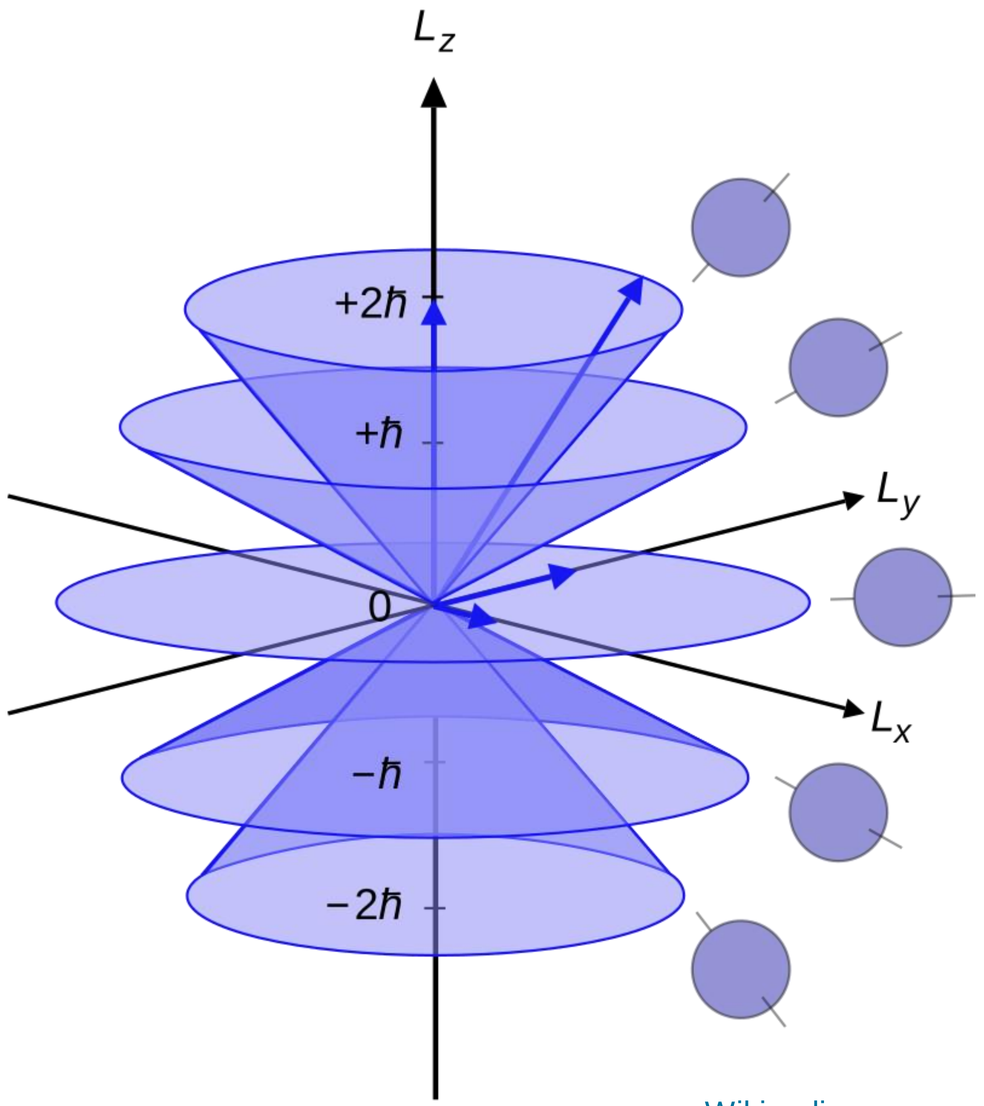
$$\left[\hat{S}_i^x, \hat{S}_j^y\right] = \mathrm{i}\hat{S}_i^z \delta_{i,j} \qquad + \text{cyclic permutation}$$

$$\hat{S}_i^+ = \hat{S}_i^x + \mathrm{i}\hat{S}_i^y$$

$$\hat{S}_i^- = \hat{S}_i^x - \mathrm{i}\hat{S}_i^y$$

$$\left[\hat{S}_i^z, \hat{S}_j^\pm\right] = \pm\hat{S}_i^\pm \delta_{i,j}$$

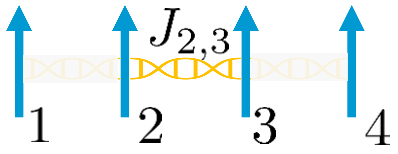
$$\left[\hat{S}_i^+, \hat{S}_j^-\right] = 2\hat{S}_i^z \delta_{i,j}$$



Wikipedia.com

For S = 2

Heisenberg Theory of Ferromagnetism

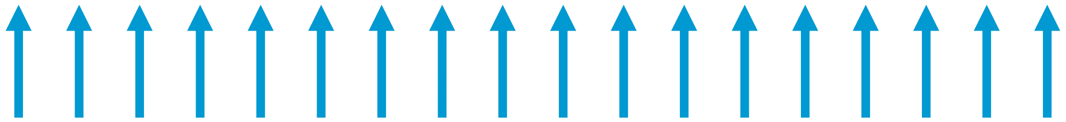


$$\begin{aligned}\mathcal{H} &= -\sum_{i,\Delta} J_{i,i+\Delta} \cdot \hat{S}_i \cdot \hat{S}_{i+\Delta} \\ &= -\sum_{i,\Delta} J_{i,i+\Delta} \cdot \left(\hat{S}_i^x \hat{S}_{i+\Delta}^x + \hat{S}_i^y \hat{S}_{i+\Delta}^y + \hat{S}_i^z \hat{S}_{i+\Delta}^z \right) \\ &= -\sum_{i,\Delta} J_{i,i+\Delta} \cdot \left(\frac{1}{2} \left\{ \hat{S}_i^+ \hat{S}_{i+\Delta}^- + \hat{S}_i^- \hat{S}_{i+\Delta}^+ \right\} + \hat{S}_i^z \hat{S}_{i+\Delta}^z \right)\end{aligned}$$

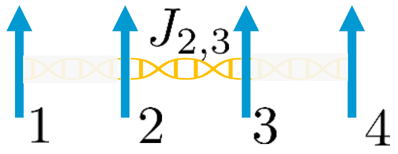
W.Heisenberg Z.Physik **49**, 619-636, (1928)

Groundstate

$$|0\rangle = |S, S, S, \ldots, S\rangle$$

$|0\rangle$ 

Heisenberg Theory of Ferromagnetism



$$\mathcal{H} = -\sum_{i,\Delta} J_{i,i+\Delta} \cdot \hat{S}_i \cdot \hat{S}_{i+\Delta}$$
$$= -\sum_{i,\Delta} J_{i,i+\Delta} \cdot \left(\hat{S}_i^x \hat{S}_{i+\Delta}^x + \hat{S}_i^y \hat{S}_{i+\Delta}^y + \hat{S}_i^z \hat{S}_{i+\Delta}^z \right)$$
$$= -\sum_{i,\Delta} J_{i,i+\Delta} \cdot \left(\frac{1}{2} \left\{ \hat{S}_i^+ \hat{S}_{i+\Delta}^- + \hat{S}_i^- \hat{S}_{i+\Delta}^+ \right\} + \hat{S}_i^z \hat{S}_{i+\Delta}^z \right)$$

Groundstate

$$|0\rangle = |S, S, S, \ldots, S\rangle$$

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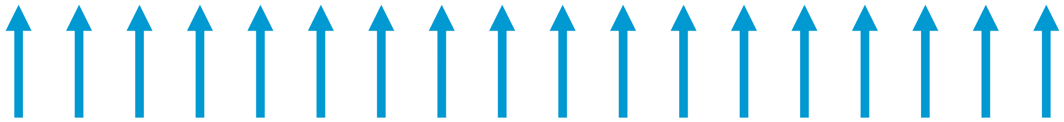
An eigenstate, with the eigenenergy E_0

$$\mathcal{H}|0\rangle = -\sum_{i,\Delta} J \cdot \left(\frac{1}{2} \left\{ \underline{\hat{S}_i^+ \hat{S}_{i+\Delta}^-} + \underline{\hat{S}_i^- \hat{S}_{i+\Delta}^+} \right\} + \hat{S}_i^z \hat{S}_{i+\Delta}^z \right) |0\rangle$$
$$= \underline{0} - zJN \cdot S^2 |0\rangle = E_0 |0\rangle$$

$|0\rangle$ 

Bloch Theory of Ferromagnetism

Excitations

Groundstate $|0\rangle = |S, S, S, \ldots, S\rangle$

How do excitations of this state look like?

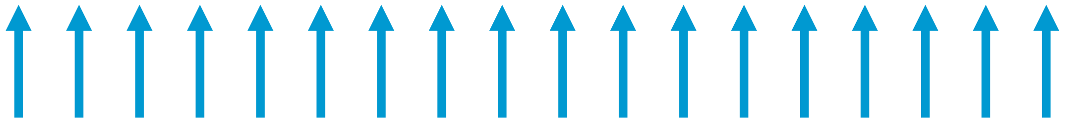
Zur Theorie des Ferromagnetismus.

Von **F. Bloch,** zurzeit in Utrecht.

(Eingegangen am 1. Februar 1930.)

F.Bloch. Z.Physik **61**, *206-219* (1930)

$|0\rangle$



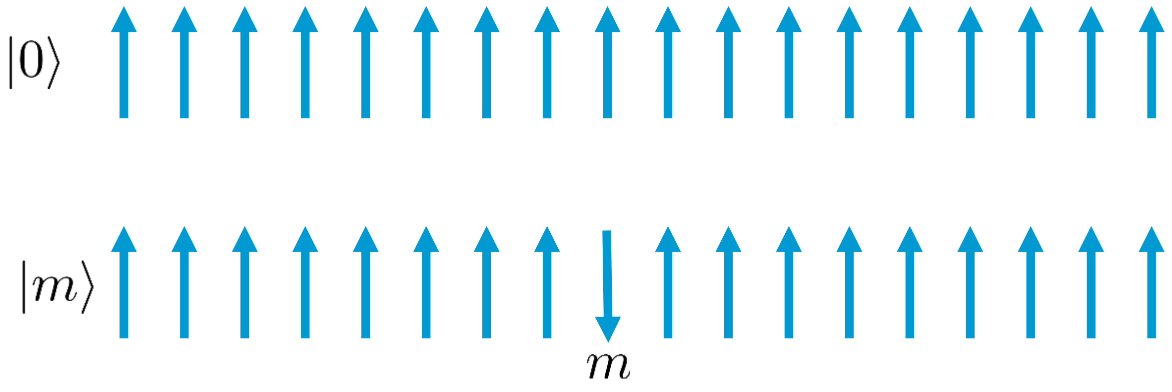
Bloch Theory of Ferromagnetism

Excitations

His approach was to consider one flipped spin

$$
\begin{aligned}
|m\rangle &= \frac{S_m^-}{\sqrt{2S}}|0\rangle \\
&= |S, S, \ldots, \underbrace{S-1}_{m}, \ldots, S\rangle
\end{aligned}
$$

Not an eigenstate anymore



Bloch Theory of Ferromagnetism

$$\mathcal{H}|m\rangle = -\sum_{i,\Delta} J \cdot \left(\frac{1}{2}\left\{\underline{\hat{S}_i^+ \hat{S}_{i+\Delta}^-}|m\rangle + \underline{\hat{S}_i^- \hat{S}_{i+\Delta}^+}|m\rangle\right\} + \hat{S}_i^z \hat{S}_{i+\Delta}^z|m\rangle\right)$$

Excitations

His approach was to consider one flipped spin

$$|m\rangle = \frac{S_m^-}{\sqrt{2S}}|0\rangle$$

$$= |S, S, \ldots, \underbrace{S-1}_{m}, \ldots, S\rangle$$

Not an eigenstate anymore

$$= -J\sum_{i,\Delta} \cdot \left(\frac{1}{2}\left\{\delta_{i,m}\hat{S}_i^+ \hat{S}_{i+\Delta}^-|m\rangle + \delta_{i+\Delta,m}\hat{S}_i^- \hat{S}_{i+\Delta}^+|m\rangle\right\} + \hat{S}_i^z \hat{S}_{i+\Delta}^z|m\rangle\right)$$

$$= -J \cdot \left(\frac{1}{2}\left\{\sum_{\Delta} \hat{S}_m^+ \hat{S}_{m+\Delta}^-|m\rangle + \sum_{\Delta} \hat{S}_{m-\Delta}^- \hat{S}_m^+|m\rangle\right\} + \sum_{i,\Delta} \hat{S}_i^z \hat{S}_{i+\Delta}^z|m\rangle\right)$$

$$= -J \cdot \left(\frac{1}{2}\left\{\sum_{\Delta} 2S|m+\Delta\rangle + \sum_{\Delta} 2S|m+\Delta\rangle\right\} + \sum_{i,\Delta} \hat{S}_i^z \hat{S}_{i+\Delta}^z|m\rangle\right)$$

$$= (E_0 + 2zJS)|m\rangle - \underline{2JS\sum_{\Delta}|m+\Delta\rangle}$$

E.Rastelli, *Statistical Mechanics of Magnetic Excitations* (2013)

Bloch Theory of Ferromagnetism

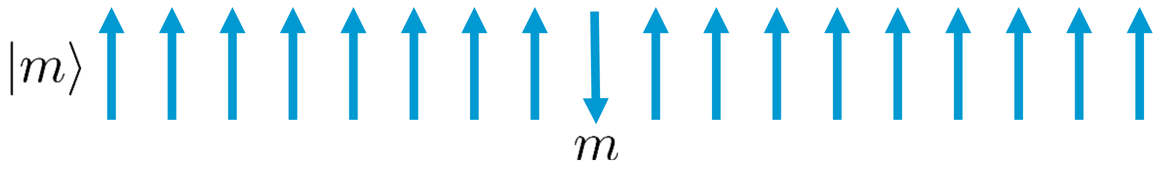
Search for Eigenstates

Not an eigenstate anymore.

The total magnetization was reduced by 1!

$$\mathcal{H}|m\rangle = \mathcal{H}S_m^-|0\rangle$$
$$= (E_0 + 2zJS)\,|m\rangle - 2JS\sum_{\Delta}|m+\Delta\rangle$$

$$\hat{S}^z_{\text{tot.}}|m\rangle = (NS-1)\,|m\rangle$$



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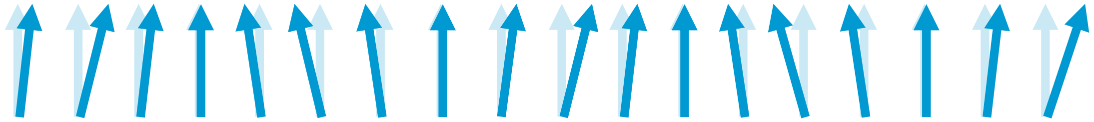
Hence we can guess an eigenstate

$$|\vec{k}\rangle \propto \sum_n \qquad |n\rangle$$

$|\vec{k}\rangle$ 

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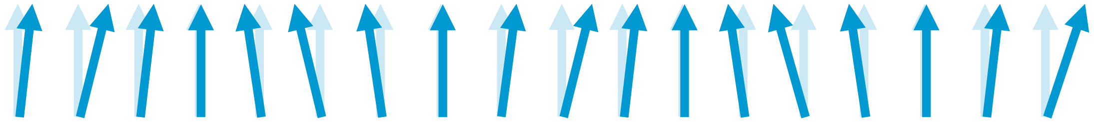
$$|\vec{k}\rangle = \frac{1}{\sqrt{N}}\sum_n \exp\left(\mathrm{i}\vec{k}\cdot\vec{r}_n\right)|n\rangle$$

$|\vec{k}\rangle$ 

Bloch Theory of Ferromagnetism

$$|\vec{k}\rangle = \frac{1}{\sqrt{N}} \sum_n \exp\left(\mathrm{i}\vec{k}\cdot\vec{r}_n\right) |n\rangle$$

Properties of the eigenstates

The total **magnetization** is reduced

As well as an increase in energy

$$\begin{aligned}\hat{S}^z_{\text{tot.}}|\vec{k}\rangle &= \frac{1}{\sqrt{N}} \sum_n \exp\left(\mathrm{i}\vec{k}\cdot\vec{r}_n\right) \hat{S}^z_{\text{tot.}}|n\rangle \\ &= \frac{1}{\sqrt{N}} \sum_n \exp\left(\mathrm{i}\vec{k}\cdot\vec{r}_n\right) (NS-1)|n\rangle \\ &= (NS-1)|\vec{k}\rangle\end{aligned}$$

But the average x and y component are still zero?

$$\langle\vec{k}|\hat{S}^x_i|\vec{k}\rangle = 0$$

$$\langle\vec{k}|\hat{S}^y_i|\vec{k}\rangle = 0$$

$$\langle\vec{k}|\hat{S}^z_i|\vec{k}\rangle = S - \frac{1}{N}$$

Bloch Theory of Ferromagnetism

Properties of the eigenstates

The total magnetization is reduced

As well as an increase in **energy**

$$\mathcal{H}|\vec{k}\rangle = \dots$$

Ferromagnetic Dispersion relation

$$\mathcal{H}|\vec{k}\rangle = \left((E_0 + 2zJS)\,|\vec{k}\rangle + 2JS \sum_{\Delta>0} - \exp\left(-\mathrm{i}\vec{k}\cdot\vec{r}_\Delta\right) - \exp\left(\mathrm{i}\vec{k}\cdot\vec{r}_\Delta\right)|\vec{k}\rangle \right)$$

Properties of the eigenstates

$$= \left((E_0 + 2zJS)\,|\vec{k}\rangle + 2JS \sum_{\Delta>0} -2\cos\left(\vec{k}\cdot\vec{r}_\Delta\right)|\vec{k}\rangle \right)$$

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$$= \left(E_0|\vec{k}\rangle + 2JS \sum_{\Delta} \left(1 - \cos\left(\vec{k}\cdot\vec{r}_\Delta\right)\right)|\vec{k}\rangle \right)$$

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Properties of the eigenstates

$$\varepsilon_k \overset{r\,=\,1.}{=} 2\,J_1\left[1 - \cos\frac{2\,\pi\,k}{N}\right],$$

F.Bloch. Z.Physik **61**, *206-219* (1930)

Same result with semiclassical calculation

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&= \left((E_0 + 2zJS)\,|\vec{k}\rangle + 2JS\sum_{\Delta>0} -2\cos\left(\vec{k}\cdot\vec{r}_\Delta\right)|\vec{k}\rangle\right)\\
&= \left((E_0 + 2zJS)\,|\vec{k}\rangle + 2JS\sum_{\Delta} - \cos\left(\vec{k}\cdot\vec{r}_\Delta\right)|\vec{k}\rangle\right)\\
&= \left(E_0|\vec{k}\rangle + 2JS\sum_{\Delta}\left(1 - \cos\left(\vec{k}\cdot\vec{r}_\Delta\right)\right)|\vec{k}\rangle\right)
\end{aligned}$$

Same result with semiclassical calculation

$$\mathcal{H}|\vec{k}\rangle = \left(E_0|\vec{k}\rangle + 4JS\left(1 - \cos\left(\vec{k}\cdot\vec{r}_\Delta\right)\right)|\vec{k}\rangle\right)$$

$$\Delta E = 4JS\left(1 - \cos\left(\vec{k}\cdot\vec{r}_\Delta\right)\right)$$

For a linear chain with NN

S.Blundell, *Magnetism in Condensed matter* (2000)

Ferromagnetic Dispersion relation

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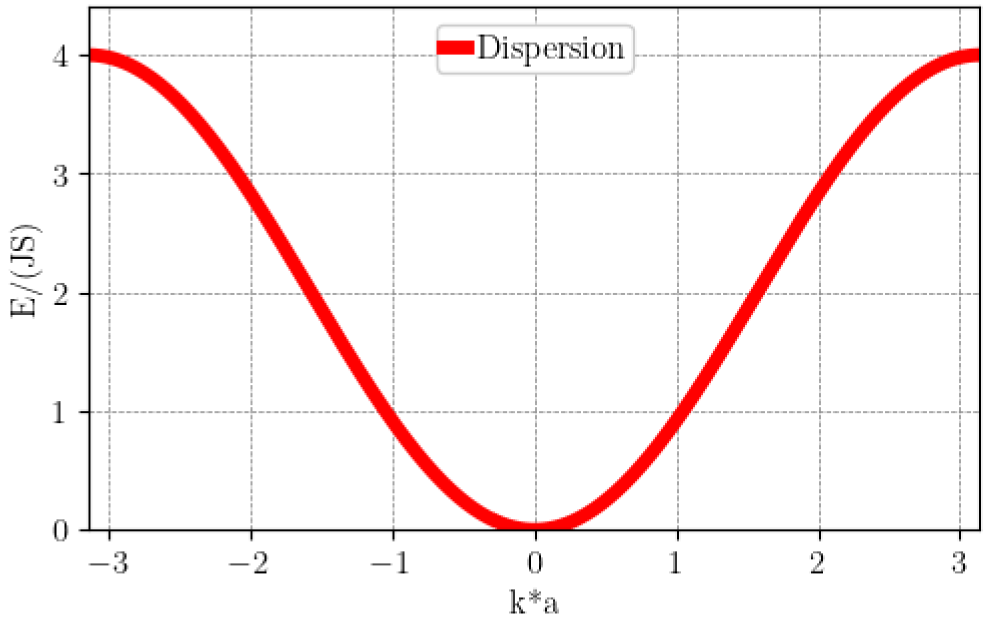

Ferromagnetic Dispersion relation with a magnetic Field

Addition of Zeeman term

$$
\begin{aligned}
\mathcal{H}_Z &= -\sum_{i,\Delta} J \cdot \hat{S}_i \cdot \hat{S}_{i+\Delta} - \sum_i \vec{B} \cdot \hat{S}_i \\
&= -\sum_{i,\Delta} J \cdot \hat{S}_i \cdot \hat{S}_{i+\Delta} - \sum_i B^z \cdot \hat{S}_i^z \\
&= -\sum_{i,\Delta} J \cdot \left(\frac{1}{2} \left\{ \hat{S}_i^+ \hat{S}_{i+\Delta}^- + \hat{S}_i^- \hat{S}_{i+\Delta}^+ \right\} + \hat{S}_i^z \hat{S}_{i+\Delta}^z \right) + B^z \hat{S}_i^z
\end{aligned}
$$

Ferromagnetic Dispersion relation with a magnetic Field

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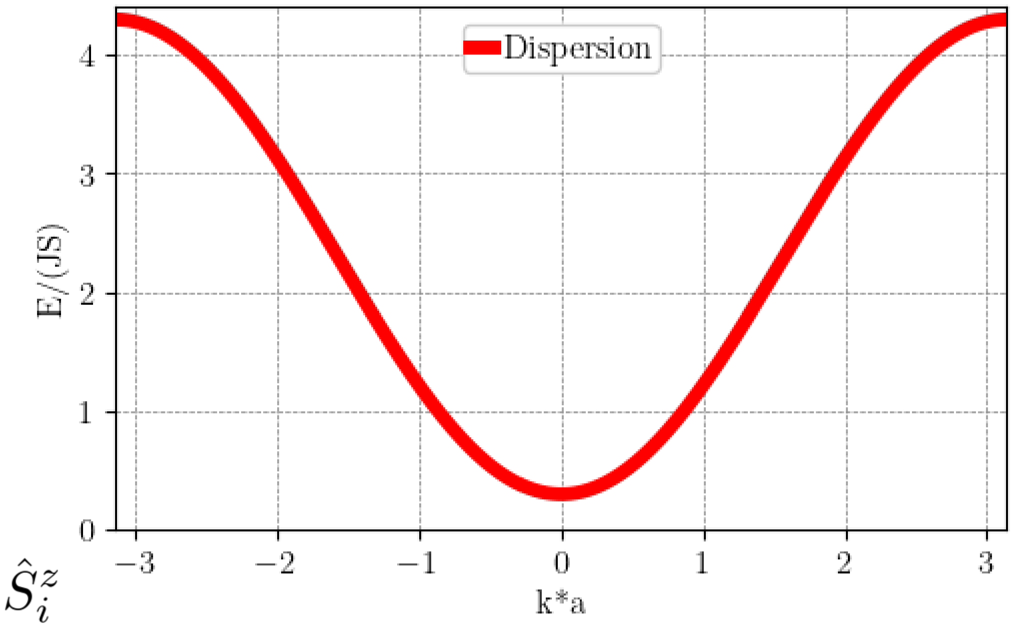

$$\mathcal{H}_Z |\vec{k}\rangle = \left(E_{0,Z} + B^z + 2JS \sum_{\Delta} \left(1 - \cos(\vec{k}\vec{r}_\Delta) \right) \right) |k\rangle$$

$$E_{0,Z} = -zJN \cdot S^2 - NSB^z$$

Recap

First excited state

Delocalization of a “flipped” spin over all sites

$$|\vec{k}\rangle = \frac{1}{\sqrt{N}} \sum_n \exp\left(\mathrm{i}\vec{k} \cdot \vec{r}_n\right) |n\rangle$$

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Collective excitation

$$|\langle m|\vec{k}\rangle|^2 = \left|\frac{1}{\sqrt{N}} \sum_n \exp\left(\mathrm{i}\vec{k}\cdot\vec{r}_n\right)\langle m|n\rangle\right|^2$$
$$= \left|\frac{1}{\sqrt{N}} \sum_n \exp\left(\mathrm{i}\vec{k}\cdot\vec{r}_n\right)\delta_{m,n}\right|^2$$
$$= \frac{1}{N}\left|\exp\left(\mathrm{i}\vec{k}\cdot\vec{r}_m\right)\right|^2 = \frac{1}{N}$$

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Collective excitation

By comparison to phonons:

-Well defined momentum $\hbar\vec{k}$

-Energy $\hbar E(\vec{k})$

$\left.\begin{array}{l} \\ \\ \end{array}\right\}$ Quasiparticle

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Reduces magnetization by 1 $\rightarrow$ Integer spin $\rightarrow$ Boson

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-Energy $\hbar E(\vec{k})$

Quasiparticle

Reduces magnetization by 1 → Integer spin → Boson

Magnon

Magnons as Bosons

statistischen
Gewicht 1 zu zählen. Der Sachverhalt ist derselbe, wie er von der Statistik eines Einstein-Bose-Gases her bekannt ist;

F.Bloch. Z.Physik **61**, *206-219* (1930)

Thermodynamical treatment

$$\hat{S}^z_{\mathrm{tot.}}|\vec{k}\rangle = (NS-1)|\vec{k}\rangle$$

Since it is a boson it must fulfill Bose-Einstein-statistic

$$n_{\mathrm{magnon}} \approx \int_0^\infty \frac{\mathrm{DOS}(\omega)\mathrm{d}\omega}{\exp(\hbar\omega/k_B T)-1}$$

$$= \ldots \propto T^{3/2}$$

Bose factor

Number of
magnons @ T

Magnons as Bosons

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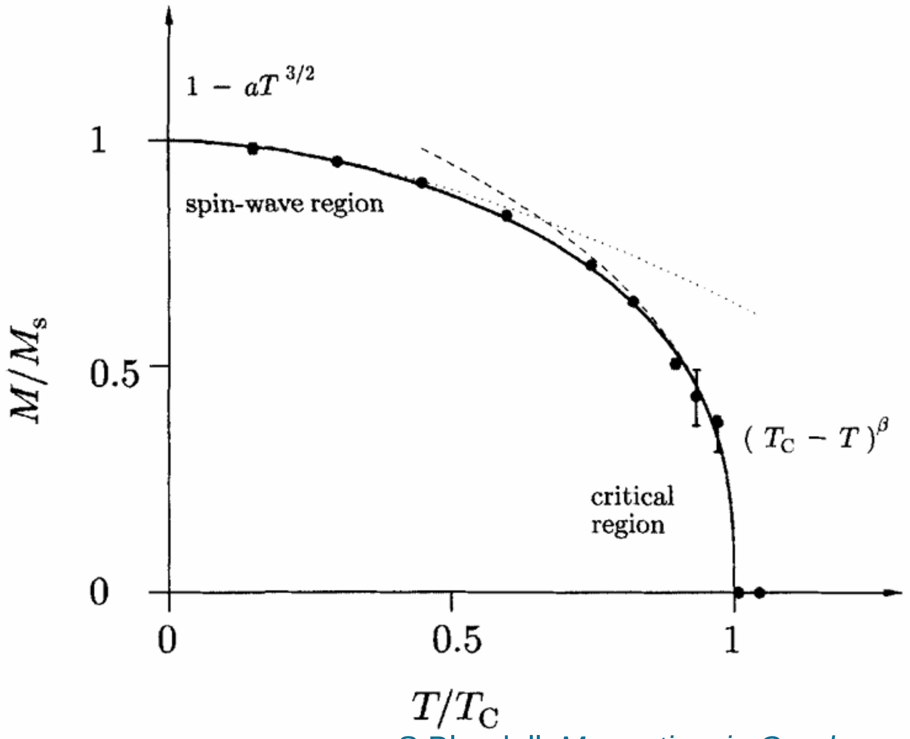

S.Blundell, *Magnetism in Condensed matter* (2000)

Spin-Boson Transformation

How to describe Magnons

The spin operators are not bosonic

Magnons are bosonic

Alternative approach:
LLG (analytically or numerically)
Or Schwinger representation
Or Dyson–Maleev representation

Conditions

i. The **transformation** needs to be **Hermitian**, raising and lowering operators written as creation and annihilation boson operators need to be Hermitian conjugate of each other

ii. The **transformation** must be **unitary** to preserve the commutation relations between the spin operators.

iii. Must satisfy the **equality between the matrix elements** of the spin operators on $|0\rangle$ and the bosons on $|n\rangle$

E.Rastelli, *Statistical Mechanics of Magnetic Excitations* (2013)

A Spin-Boson Transformation

Define higher magnon states

$$|n_\sigma\rangle = |S, S, \ldots, \underbrace{S - \sigma_n}_{n}, \ldots, S\rangle$$

$$\hat{S}_n^+ |n_\sigma\rangle = \sqrt{2S}\sqrt{1 - \frac{\sigma_n - 1}{2S}}\sqrt{\sigma_n}|n_{\sigma-1}\rangle$$

$$\hat{S}_n^- |n_\sigma\rangle = \sqrt{2S}\sqrt{\sigma_n + 1}\sqrt{1 - \frac{\sigma_n}{2S}}|n_{\sigma+1}\rangle$$

DECEMBER 15, 1940 PHYSICAL REVIEW VOLUME 58

Field Dependence of the Intrinsic Domain Magnetization of a Ferromagnet

T. HOLSTEIN
New York University, New York, New York
AND
H. PRIMAKOFF*
Polytechnic Institute of Brooklyn, Brooklyn, New York

(Received July 31, 1940)

A Spin-Boson Transformation

Define higher magnon states

$$|n_\sigma\rangle = |S, S, \ldots, \underbrace{S - \sigma_n}_{n}, \ldots, S\rangle$$

$$\hat{S}_n^+ |n_\sigma\rangle = \sqrt{2S}\sqrt{1 - \frac{\sigma_n - 1}{2S}}\sqrt{\sigma_n}|n_{\sigma-1}\rangle$$

$$\hat{S}_n^- |n_\sigma\rangle = \sqrt{2S}\sqrt{\sigma_n + 1}\sqrt{1 - \frac{\sigma_n}{2S}}|n_{\sigma+1}\rangle$$

Only for $\sigma \leq 2S$

$$\hat{S}_n^- |n_{2S}\rangle = \sqrt{2S}\sqrt{2S + 1}\sqrt{1 - \frac{2S}{2S}}|n_{2S+1}\rangle = 0$$

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A Spin-Boson Transformation

Compare the "eigenvalues"

$$|n_\sigma\rangle = |S, S, \dots, \underbrace{S - \sigma_n}_{n}, \dots, S\rangle$$

$$\hat{S}_n^+ |n_\sigma\rangle = \sqrt{2S}\sqrt{1 - \frac{\sigma_n - 1}{2S}}\sqrt{\sigma_n}|n_{\sigma-1}\rangle$$

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Only for $\sigma \leq 2S$

$$\hat{S}_n^- |n_{2S}\rangle = \sqrt{2S}\sqrt{2S + 1}\sqrt{1 - \frac{2S}{2S}}|n_{2S+1}\rangle = 0$$

Consider the states to be like 2nd quantized number states

$$\hat{a}_n |n_\sigma\rangle = \sqrt{\sigma_n}|n_{\sigma-1}\rangle$$

$$\hat{a}_n^\dagger |n_\sigma\rangle = \sqrt{\sigma_n + 1}|n_{\sigma+1}\rangle$$

A Spin-Boson Transformation

Compare the "eigenvalues"

$$|n_\sigma\rangle = |S, S, \ldots, \underbrace{S - \sigma_n}_{n}, \ldots, S\rangle$$

$$\hat{S}_n^+|n_\sigma\rangle = \sqrt{2S}\sqrt{1 - \frac{\boxed{\sigma_n - 1}}{2S}}\boxed{\sqrt{\sigma_n}}|n_{\sigma-1}\rangle$$

$$\hat{S}_n^-|n_\sigma\rangle = \sqrt{2S}\boxed{\sqrt{\sigma_n + 1}}\sqrt{1 - \frac{\boxed{\sigma_n}}{2S}}|n_{\sigma+1}\rangle$$

Consider the states to be like 2nd quantized number states

$$\boxed{\hat{a}_n^\dagger \hat{a}_n |n_{\sigma-1}\rangle = (\sigma_n - 1)|n_{\sigma-1}\rangle}$$

$$\boxed{\hat{a}_n^\dagger \hat{a}_n |n_\sigma\rangle = \sigma_n |n_\sigma\rangle}$$

$$\boxed{\hat{a}_n |n_\sigma\rangle = \sqrt{\sigma_n}|n_{\sigma-1}\rangle}$$

$$\boxed{\hat{a}_n^\dagger |n_\sigma\rangle = \sqrt{\sigma_n + 1}|n_{\sigma+1}\rangle}$$

$$\hat{S}_n^+ = \sqrt{2S}\sqrt{1 - \frac{\boxed{\hat{a}_n^\dagger \hat{a}_n}}{2S}}\boxed{\hat{a}_n}$$

$$\hat{S}_n^- = \sqrt{2S}\boxed{\hat{a}_n^\dagger}\sqrt{1 - \frac{\boxed{\hat{a}_n^\dagger \hat{a}_n}}{2S}}$$

The Holstein Primakoff Spin-Boson Transformation

Combined

$$\hat{S}_n^- = \sqrt{2S}\hat{a}_n^\dagger\sqrt{1-\frac{\hat{a}_n^\dagger\hat{a}_n}{2S}}$$

$$\hat{S}_n^+ = \sqrt{2S}\sqrt{1-\frac{\hat{a}_n^\dagger\hat{a}_n}{2S}}\hat{a}_n$$

$$\hat{S}_n^z = S - \hat{a}_n^\dagger\hat{a}_n$$

A magnon reduces the z-comp of the magnetization

$$\boxed{\begin{aligned}\hat{S}_j^z &= S - \hat{a}_j^\dagger\hat{a}_j\\ \hat{S}_j^+ &= \hat{S}_j^x + \mathrm{i}\hat{S}_j^y = \sqrt{2S-\hat{a}_j^\dagger\hat{a}_j}\cdot\hat{a}_j\\ \hat{S}_j^- &= \hat{S}_j^x - \mathrm{i}\hat{S}_j^y = \hat{a}_j^\dagger\cdot\sqrt{2S-\hat{a}_j^\dagger\hat{a}_j}\end{aligned}}$$

T-Holstein, H.Primakoff PR **58**, 1098, (1940)

The Holstein Primakoff Spin-Boson Transformation

Checking the conditions

(i) Fulfilled

(ii) Does fulfill the Boson commutation relations

(iii) Fulfilled by definition

$$\left.\begin{matrix}\text{(i)}\\\text{(ii)}\\\text{(iii)}\end{matrix}\right\}\ \text{for}\quad \sigma \leq 2S$$

$$\boxed{\begin{aligned}\hat{S}_j^z &= S - \hat{a}_j^\dagger \hat{a}_j\\ \hat{S}_j^+ &= \hat{S}_j^x + \mathrm{i}\hat{S}_j^y = \sqrt{2S - \hat{a}_j^\dagger \hat{a}_j} \cdot \hat{a}_j\\ \hat{S}_j^- &= \hat{S}_j^x - \mathrm{i}\hat{S}_j^y = \hat{a}_j^\dagger \cdot \sqrt{2S - \hat{a}_j^\dagger \hat{a}_j}\end{aligned}}$$

E.Rastelli, *Statistical Mechanics of Magnetic Excitations* (2013)

$$\left[\hat{S}_i^z, \hat{S}_j^\pm\right] = \pm \hat{S}_i^\pm \delta_{i,j} \quad ✓$$

$$\left[\hat{S}_i^+, \hat{S}_j^-\right] = \ldots = 2\hat{S}_i^z \delta_{i,j} \quad ✓$$

Holstein-Primakoff(HP)

$$\boxed{\begin{aligned}\hat{S}_j^z &= S - \hat{a}_j^\dagger \hat{a}_j \\ \hat{S}_j^+ &= \hat{S}_j^x + \mathrm{i}\hat{S}_j^y = \sqrt{2S - \hat{a}_j^\dagger \hat{a}_j} \cdot \hat{a}_j \\ \hat{S}_j^- &= \hat{S}_j^x - \mathrm{i}\hat{S}_j^y = \hat{a}_j^\dagger \cdot \sqrt{2S - \hat{a}_j^\dagger \hat{a}_j}\end{aligned}}$$

T-Holstein, H.Primakoff PR **58**, 1098, (1940)

$$\langle \hat{a}^\dagger \hat{a} \rangle < 2S$$

Applications

HP framework is a powerful method for calculating dispersions and higher order interactions

Linearized Holstein-Primakoff

$$\hat{S}_j^z = S - \hat{a}_j^\dagger \hat{a}_j$$
$$\hat{S}_j^+ = \hat{S}_j^x + \mathrm{i}\hat{S}_j^y = \sqrt{2S - \hat{a}_j^\dagger \hat{a}_j} \cdot \hat{a}_j$$
$$\hat{S}_j^- = \hat{S}_j^x - \mathrm{i}\hat{S}_j^y = \hat{a}_j^\dagger \cdot \sqrt{2S - \hat{a}_j^\dagger \hat{a}_j}$$

T-Holstein, H.Primakoff PR **58**, 1098, (1940)

Sacrifices

Linearized HP

$$\hat{S}_j^z = S - \hat{a}_j^\dagger \hat{a}_j$$
$$\hat{S}_j^y = \frac{\hat{S}_j^+ - \hat{S}_j^-}{2\mathrm{i}} \approx \frac{\sqrt{2S}}{2\mathrm{i}}\left(\hat{a}_j - \hat{a}_j^\dagger\right)$$
$$\hat{S}_j^x = \frac{\hat{S}_j^+ + \hat{S}_j^-}{2} \approx \frac{\sqrt{2S}}{2}\left(\hat{a}_j + \hat{a}_j^\dagger\right)$$
$$\hat{S}_j^+ \approx \sqrt{2S} \cdot \hat{a}_j$$
$$\hat{S}_j^- \approx \sqrt{2S} \cdot \hat{a}_j^\dagger$$

Does not fulfill the boson commutation relations

Hence (ii) is violated

$$[\hat{a}_i^\dagger, \hat{a}_j^\dagger] = 0$$
$$[\hat{a}_i, \hat{a}_j] = 0$$

$$\left[\hat{S}_i^+, \hat{S}_j^-\right] = 2\hat{S}_i^z \delta_{i,j}$$

$$[\hat{a}_i, \hat{a}_j^\dagger] = \frac{1}{2S}\left[\hat{S}_i^+, \hat{S}_j^-\right] = \frac{1}{S}\hat{S}_i^z \delta_{i,j} \neq \delta_{i,j}$$
$$\approx \frac{S}{S}\delta_{i,j} = \delta_{i,j}$$

Holstein-Primakoff

$$\hat{S}_j^z = S - \hat{a}_j^\dagger \hat{a}_j$$
$$\hat{S}_j^+ = \hat{S}_j^x + \mathrm{i}\hat{S}_j^y = \sqrt{2S - \hat{a}_j^\dagger \hat{a}_j} \cdot \hat{a}_j$$
$$\hat{S}_j^- = \hat{S}_j^x - \mathrm{i}\hat{S}_j^y = \hat{a}_j^\dagger \cdot \sqrt{2S - \hat{a}_j^\dagger \hat{a}_j}$$

T-Holstein, H.Primakoff PR **58**, 1098, (1940)

Simplified application

Only linear Spin-Wave theory using the linearized HP

$$\mathcal{H} = -\sum_{i,\Delta} J \cdot \hat{S}_i \cdot \hat{S}_{i+\Delta}$$
$$= -\sum_{i,\Delta} J \cdot \left(\frac{1}{2} \left\{ \hat{S}_i^+ \hat{S}_{i+\Delta}^- + \hat{S}_i^- \hat{S}_{i+\Delta}^+ \right\} + \hat{S}_i^z \hat{S}_{i+\Delta}^z \right)$$
$$= -\sum_{i,\Delta} J \cdot \left(S \left\{ \hat{a}_i \hat{a}_{i+\Delta}^\dagger + \hat{a}_i^\dagger \hat{a}_{i+\Delta} \right\} + (S - \hat{n}_i)(S - \hat{n}_{i+\Delta}) \right)$$
$$= -NJS^2 - \sum_{i,\Delta} JS \cdot \left\{ \hat{a}_{i+\Delta}^\dagger \hat{a}_i + \hat{a}_i^\dagger \hat{a}_{i+\Delta} - \hat{n}_i - \hat{n}_{i+\Delta} \right\} + \underbrace{\sum_{i,\Delta} J \hat{n}_i \hat{n}_{i+\Delta}}_{\approx 0}$$
$$\approx -NJS^2 - \sum_{i,\Delta} JS \cdot \left\{ \hat{a}_{i+\Delta}^\dagger \hat{a}_i + \hat{a}_i^\dagger \hat{a}_{i+\Delta} - \hat{n}_i - \hat{n}_{i+\Delta} \right\}$$

Linearized HP

$$\hat{S}_j^z = S - \hat{a}_j^\dagger \hat{a}_j$$
$$\hat{S}_j^y = \frac{\hat{S}_j^+ - \hat{S}_j^-}{2\mathrm{i}} \approx \frac{\sqrt{2S}}{2\mathrm{i}} \left(\hat{a}_j - \hat{a}_j^\dagger \right)$$
$$\hat{S}_j^x = \frac{\hat{S}_j^+ + \hat{S}_j^-}{2} \approx \frac{\sqrt{2S}}{2} \left(\hat{a}_j + \hat{a}_j^\dagger \right)$$
$$\hat{S}_j^+ \approx \sqrt{2S} \cdot \hat{a}_j$$
$$\hat{S}_j^- \approx \sqrt{2S} \cdot \hat{a}_j^\dagger$$

Holstein-Primakoff

Rewrite the Hamiltonian

$$\mathcal{H} \approx -NJS^2 - \sum_{i,\Delta} JS \cdot \left\{ \hat{a}^\dagger_{i+\Delta}\hat{a}_i + \hat{a}^\dagger_i \hat{a}_{i+\Delta} - \hat{n}_i - \hat{n}_{i+\Delta} \right\}$$

$$\approx -NJS^2 - \sum_{i,j} \mathscr{H}^{\mathrm{SW}}_{i,j} \cdot \hat{a}^\dagger_i \hat{a}_j$$

Time evolution of an operator :

Linearized HP

$$\boxed{\begin{aligned}
\hat{S}^z_j &= S - \hat{a}^\dagger_j \hat{a}_j \\
\hat{S}^y_j &= \frac{\hat{S}^+_j - \hat{S}^-_j}{2\mathrm{i}} \approx \frac{\sqrt{2S}}{2\mathrm{i}} \left(\hat{a}_j - \hat{a}^\dagger_j\right) \\
\hat{S}^x_j &= \frac{\hat{S}^+_j + \hat{S}^-_j}{2} \approx \frac{\sqrt{2S}}{2} \left(\hat{a}_j + \hat{a}^\dagger_j\right) \\
\hat{S}^+_j &\approx \sqrt{2S} \cdot \hat{a}_j \\
\hat{S}^-_j &\approx \sqrt{2S} \cdot \hat{a}^\dagger_j
\end{aligned}}$$

Holstein-Primakoff

Rewrite the Hamiltonian

$$\mathcal{H} \approx -NJS^2 - \sum_{i,\Delta} JS \cdot \left\{\hat{a}^\dagger_{i+\Delta}\hat{a}_i + \hat{a}^\dagger_i \hat{a}_{i+\Delta} - \hat{n}_i - \hat{n}_{i+\Delta}\right\}$$

$$\approx -NJS^2 - \sum_{i,j} \mathscr{H}^{\mathrm{SW}}_{i,j} \cdot \hat{a}^\dagger_i \hat{a}_j$$

Time evolution of an operator :

$$\begin{aligned} \mathrm{i}\hbar \frac{\partial}{\partial t}\hat{a}_n &= [\mathcal{H}, \hat{a}_n] \\ &= 0 - \sum_{i,j} \mathscr{H}^{\mathrm{SW}}_{i,j} \cdot \left[\hat{a}^\dagger_i \hat{a}_j, \hat{a}_n\right] \\ &= -\sum_{i,j} \mathscr{H}^{\mathrm{SW}}_{i,j} \cdot \delta_{i,n}\hat{a}_j \\ &= -\sum_{j} \mathscr{H}^{\mathrm{SW}}_{n,j} \cdot \hat{a}_j \end{aligned}$$

Problem: coupling between all sites!
Practically unsolvable for real systems

Linearized HP

$$\hat{S}^z_j = S - \hat{a}^\dagger_j \hat{a}_j$$

$$\hat{S}^y_j = \frac{\hat{S}^+_j - \hat{S}^-_j}{2\mathrm{i}} \approx \frac{\sqrt{2S}}{2\mathrm{i}}\left(\hat{a}_j - \hat{a}^\dagger_j\right)$$

$$\hat{S}^x_j = \frac{\hat{S}^+_j + \hat{S}^-_j}{2} \approx \frac{\sqrt{2S}}{2}\left(\hat{a}_j + \hat{a}^\dagger_j\right)$$

$$\hat{S}^+_j \approx \sqrt{2S} \cdot \hat{a}_j$$

$$\hat{S}^-_j \approx \sqrt{2S} \cdot \hat{a}^\dagger_j$$

Holstein-Primakoff

The new Hamiltonian

$$\mathcal{H} \approx -NJS^2 - \sum_{i,\Delta} JS \cdot \left\{ \hat{a}^\dagger_{i+\Delta} \hat{a}_i + \hat{a}^\dagger_i \hat{a}_{i+\Delta} - \hat{n}_i - \hat{n}_{i+\Delta} \right\}$$

$$\approx -NJS^2 - \sum_{i,j} \mathscr{H}^{\mathrm{SW}}_{i,j} \cdot \hat{a}^\dagger_i \hat{a}_j$$

Time evolution of an operator :

$$\mathrm{i}\hbar \frac{\partial}{\partial t} \hat{a}_n = - \sum_j \mathscr{H}^{\mathrm{SW}}_{n,j} \cdot \hat{a}_j$$

Problem: coupling between all sites!
Practically unsolvable for real systems

Going to Fourier-space

Lattice Fourier transformation

$$\hat{a}(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \hat{a}_n \exp(-\mathrm{i}\vec{k}\vec{r}_n),$$

$$\hat{a}^\dagger(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \hat{a}^\dagger_n \exp(\mathrm{i}\vec{k}\vec{r}_n).$$

Holstein-Primakoff

The new Hamiltonian

$$\mathcal{H} \approx -NJS^2 - \sum_{i,\Delta} JS \cdot \left\{ \hat{a}^\dagger_{i+\Delta}\hat{a}_i + \hat{a}^\dagger_i \hat{a}_{i+\Delta} - \hat{n}_i - \hat{n}_{i+\Delta} \right\}$$

$$\approx -NJS^2 - \sum_{i,j} \mathscr{H}^{\mathrm{SW}}_{i,j} \cdot \hat{a}^\dagger_i \hat{a}_j$$

Time evolution of an operator :

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\hat{a}_n = -\sum_j \mathscr{H}^{\mathrm{SW}}_{n,j} \cdot \hat{a}_j$$

Problem: coupling between all sites!
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$$\hat{a}(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \hat{a}_n \exp(-\mathrm{i}\vec{k}\vec{r}_n),$$

$$\hat{a}^\dagger(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \hat{a}^\dagger_n \exp(\mathrm{i}\vec{k}\vec{r}_n).$$

Time evolution

$$\frac{\partial}{\partial t}\hat{a}(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \left(\frac{\partial}{\partial t}\hat{a}_n \right) \exp(-\mathrm{i}\vec{k}\vec{r}_n),$$

Holstein-Primakoff

The new Hamiltonian

$$\mathcal{H} \approx -NJS^2 - \sum_{i,\Delta} JS \cdot \left\{ \hat{a}^\dagger_{i+\Delta}\hat{a}_i + \hat{a}^\dagger_i \hat{a}_{i+\Delta} - \hat{n}_i - \hat{n}_{i+\Delta} \right\}$$

$$\approx -NJS^2 - \sum_{i,j} \mathscr{H}^{\mathrm{SW}}_{i,j} \cdot \hat{a}^\dagger_i \hat{a}_j$$

Time evolution of an operator :

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\hat{a}_n = -\sum_j \mathscr{H}^{\mathrm{SW}}_{n,j} \cdot \hat{a}_j$$

Problem: coupling between all sites!
practically unsolvable for real systems

Going to Fourier-space

Lattice Fourier transformation

$$\hat{a}(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \hat{a}_n \exp(-\mathrm{i}\vec{k}\vec{r}_n),$$

$$\hat{a}^\dagger(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \hat{a}^\dagger_n \exp(\mathrm{i}\vec{k}\vec{r}_n).$$

Time evolution

$$\frac{\partial}{\partial t}\hat{a}(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \left(\frac{\partial}{\partial t}\hat{a}_n\right) \exp(-\mathrm{i}\vec{k}\vec{r}_n),$$

Plug in

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\hat{a}(\vec{k}) = \frac{1}{\sqrt{N}} \sum_n \left(-\sum_j \mathscr{H}^{\mathrm{SW}}_{n,j} \cdot \hat{a}_j\right) \exp(-\mathrm{i}\vec{k}\vec{r}_n)$$

$$= \frac{1}{\sqrt{N}} \sum_n \sum_j \mathscr{H}^{\mathrm{SW}}_{n,j} \cdot \hat{a}_j \exp(-\mathrm{i}\vec{k}\vec{r}_n)$$

$$= \dots$$

Holstein-Primakoff

$$\mathcal{H} \approx E_0 + \sum_{\vec{k}} \hbar\omega_{\vec{k}} \hat{a}^\dagger(\vec{k})\hat{a}(\vec{k})$$

Solving the differential equation

$$\frac{\partial}{\partial t}\hat{a}(\vec{k}) = \ldots = -\frac{\mathrm{i}}{\hbar}\mathscr{H}^{\mathrm{SW}}_{\vec{k}}\hat{a}(\vec{k}) \quad \Rightarrow \quad \hat{a}(\vec{k},t) = \hat{a}(\vec{k},0)\exp\left(-\mathrm{i}\omega_{\vec{k}}t\right)$$

With the spin-wave frequency

$$\begin{aligned}\hbar\omega_{\vec{k}} &= \mathscr{H}^{\mathrm{SW}}_{\vec{k}} \\ &= S\left(J_0 - J_{\vec{k}}\right)\end{aligned}$$

L. Rósza, Lecture Notes (2022)

$$J_{\vec{k}} \propto \sum_j J_{j,0}\exp(-\mathrm{i}\vec{k}(\vec{r}_j - \vec{r}_0)) \qquad J_0 = J_{\vec{k}=0}$$

Holstein-Primakoff

$$\mathcal{H} \approx E_0 + \sum_{\vec{k}} \hbar\omega_{\vec{k}} \hat{a}^\dagger(\vec{k})\hat{a}(\vec{k})$$

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$$\frac{\partial}{\partial t}\hat{a}(\vec{k}) = \ldots = -\frac{\mathrm{i}}{\hbar}\mathscr{H}_{\vec{k}}^{\mathrm{SW}}\hat{a}(\vec{k}) \quad \Rightarrow \quad \hat{a}(\vec{k},t) = \hat{a}(\vec{k},0)\exp\left(-\mathrm{i}\omega_{\vec{k}}t\right)$$

With the spin-wave frequency

$$\begin{aligned}\hbar\omega_{\vec{k}} &= \mathscr{H}_{\vec{k}}^{\mathrm{SW}} \\ &= S\left(J_0 - J_{\vec{k}}\right)\end{aligned}$$

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$$\hbar\omega_{\vec{k}} = \ldots = JS\sum_{\Delta}\left(1 - \cos\left(\vec{k}\cdot\vec{r}_\Delta\right)\right)$$

Holstein-Primakoff

$$\boxed{\mathcal{H} \approx E_0 + \sum_{\vec{k}} \hbar\omega_{\vec{k}} \hat{a}^\dagger(\vec{k})\hat{a}(\vec{k})}$$

Solving the differential equation

$$\frac{\partial}{\partial t}\hat{a}(\vec{k}) = \ldots = -\frac{\mathrm{i}}{\hbar}\mathscr{H}^{\mathrm{SW}}_{\vec{k}}\hat{a}(\vec{k}) \quad \Rightarrow \quad \hat{a}(\vec{k},t) = \hat{a}(\vec{k},0)\exp\left(-\mathrm{i}\omega_{\vec{k}}t\right)$$

With the spin-wave frequency
$$\hbar\omega_{\vec{k}} = \mathscr{H}^{\mathrm{SW}}_{\vec{k}} = S\left(J_0 - J_{\vec{k}}\right)$$

L. Rósza, Lecture Notes (2022)

$$\hat{a}_j(t) = \frac{1}{\sqrt{N}}\sum_{\vec{k}}\exp(\mathrm{i}\vec{k}\vec{r}_j)\hat{a}(\vec{k},t) = \frac{1}{\sqrt{N}}\sum_{\vec{k}}\exp(\mathrm{i}\vec{k}\vec{r}_j)\hat{a}(\vec{k},0)\cdot\exp(-\mathrm{i}\omega_{\vec{k}}t)$$

$$J_{\vec{k}} \propto \sum_j J_{j,0}\exp(-\mathrm{i}\vec{k}(\vec{r}_j - \vec{r}_0)) \qquad J_0 = J_{\vec{k}=0}$$

$$\hat{a}_j(\vec{k}_0,t) = \underbrace{\frac{1}{\sqrt{N}}\hat{a}(\vec{k}_0,0)}_{\hat{A}}\cdot\exp(\mathrm{i}\vec{k}_0\vec{r}_j - \mathrm{i}\omega_{\vec{k}_0}t)$$

$$\hbar\omega_{\vec{k}} = \ldots = JS\sum_{\Delta}\left(1-\cos\left(\vec{k}\cdot\vec{r}_\Delta\right)\right)$$

$$\boxed{\hat{a}_j(\vec{k}_0,t) = \hat{A}\cdot\exp(\mathrm{i}\vec{k}_0\vec{r}_j - \mathrm{i}\omega_{\vec{k}_0}t)}$$

Holstein-Primakoff

Back to Real-space

$$\hat{a}_j(\vec{k}_0, t) = \frac{1}{\sqrt{N}} \exp(\mathrm{i}\vec{k}_0 \vec{r}_j - \mathrm{i}\omega_{\vec{k}_0} t)\hat{a}(\vec{k}_0, 0)$$

$$\boxed{\hat{a}_j(\vec{k}_0, t) = \hat{A} \exp(\mathrm{i}\vec{k}_0 \vec{r}_j - \mathrm{i}\omega_{\vec{k}_0} t)}$$

Linearized HP

$$\boxed{\begin{aligned} \hat{S}_j^z &= S - \hat{a}_j^\dagger \hat{a}_j \\ \hat{S}_j^y &\approx \frac{\sqrt{2S}}{2\mathrm{i}} \left(\hat{a}_j - \hat{a}_j^\dagger\right) \\ \hat{S}_j^x &\approx \frac{\sqrt{2S}}{2} \left(\hat{a}_j + \hat{a}_j^\dagger\right) \end{aligned}}$$

Holstein-Primakoff

Back to Real-space

$$\hat{a}_j(\vec{k}_0, t) = \frac{1}{\sqrt{N}} \exp(\mathrm{i}\vec{k}_0\vec{r}_j - \mathrm{i}\omega_{\vec{k}_0} t)\hat{a}(\vec{k}_0, 0)$$

$$\boxed{\hat{a}_j(\vec{k}_0, t) = \hat{A}\exp(\mathrm{i}\vec{k}_0\vec{r}_j - \mathrm{i}\omega_{\vec{k}_0} t)}$$

S.Blundell, *Magnetism in Condensed matter* (2000)

Linearized HP

$$\boxed{\begin{aligned} \hat{S}_j^z &= S - \hat{a}_j^\dagger \hat{a}_j \\ \hat{S}_j^y &\approx \frac{\sqrt{2S}}{2\mathrm{i}}\left(\hat{a}_j - \hat{a}_j^\dagger\right) \\ \hat{S}_j^x &\approx \frac{\sqrt{2S}}{2}\left(\hat{a}_j + \hat{a}_j^\dagger\right) \end{aligned}}$$

$$\boxed{\begin{aligned} \hat{S}_j^x(t) &= \sqrt{2S}\hat{A}\cdot\cos(\vec{k}_0\vec{r}_j + \omega_{\vec{k}_0} t + \phi_A) \\ \hat{S}_j^y(t) &= \sqrt{2S}\hat{A}\cdot\sin(\vec{k}_0\vec{r}_j + \omega_{\vec{k}_0} t + \phi_A) \\ \hat{S}_j^z(t) &= S - |\hat{A}|^2 \end{aligned}}$$

This explains why the average we calculated earlier was 0

Holstein-Primakoff & Bogoliubov transformation

In-plane anisotropy

Considering more interactions

$$\mathcal{H} = -\sum_{i\Delta} J \cdot \hat{S}_i \cdot \hat{S}_{i+\Delta} - \sum_i \hat{S}_i \cdot \mathscr{K}_i \cdot \hat{S}_i$$

$$\mathscr{K}_i = \begin{pmatrix} K & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Holstein-Primakoff & Bogoliubov transformation

In-plane anisotropy

$$\mathscr{K}_i = \begin{pmatrix} K & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Considering more interactions

$$\mathcal{H} = -\sum_{i\Delta} J \cdot \hat{S}_i \cdot \hat{S}_{i+\Delta} - \sum_i \hat{S}_i \cdot \mathscr{K}_i \cdot \hat{S}_i$$

Linearized HP

$$\boxed{\begin{aligned} \hat{S}^z_j &= S - \hat{a}^\dagger_j \hat{a}_j \\ \hat{S}^y_j &\approx \frac{\sqrt{2S}}{2\mathrm{i}} \left(\hat{a}_j - \hat{a}^\dagger_j\right) \\ \hat{S}^x_j &\approx \frac{\sqrt{2S}}{2} \left(\hat{a}_j + \hat{a}^\dagger_j\right) \end{aligned}}$$

$$\begin{aligned} \hat{S}_i \cdot \mathscr{K}_i \cdot \hat{S}_i &= K \hat{S}^x_i \cdot \hat{S}^x_i \\ &\approx K \frac{S}{2} \left(\hat{a}_i \hat{a}_i + \hat{a}_i \hat{a}^\dagger_i + \hat{a}^\dagger_i \hat{a}_i + \hat{a}^\dagger_i \hat{a}^\dagger_i\right) \\ &= K \frac{S}{2} \left(\hat{a}_i \hat{a}_i + 2\hat{a}^\dagger_i \hat{a}_i + \hat{a}^\dagger_i \hat{a}^\dagger_i\right) + K \frac{S}{2} \end{aligned}$$

$$\mathcal{H}_{\mathrm{iso}} \approx -E_0 - \sum_{i,j} \mathscr{H}^{\mathrm{SW}}_{i,j} \cdot \hat{a}^\dagger_i \hat{a}_j$$

Holstein-Primakoff & Bogoliubov transformation

In-plane anisotropy

Considering more interactions

$$\mathcal{H} = -\sum_{i\Delta} J \cdot \hat{S}_i \cdot \hat{S}_{i+\Delta} - \sum_i \hat{S}_i \cdot \mathscr{K}_i \cdot \hat{S}_i$$

$$\mathscr{K}_i = \begin{pmatrix} K & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Linearized HP

$$\boxed{\begin{aligned} \hat{S}_j^z &= S - \hat{a}_j^\dagger \hat{a}_j \\ \hat{S}_j^y &\approx \frac{\sqrt{2S}}{2\mathrm{i}} \left(\hat{a}_j - \hat{a}_j^\dagger\right) \\ \hat{S}_j^x &\approx \frac{\sqrt{2S}}{2} \left(\hat{a}_j + \hat{a}_j^\dagger\right) \end{aligned}}$$

$$\begin{aligned} \hat{S}_i \cdot \mathscr{K}_i \cdot \hat{S}_i &= K \hat{S}_i^x \cdot \hat{S}_i^x \\ &\approx K \frac{S}{2} \left(\hat{a}_i \hat{a}_i + \hat{a}_i \hat{a}_i^\dagger + \hat{a}_i^\dagger \hat{a}_i + \hat{a}_i^\dagger \hat{a}_i^\dagger\right) \\ &= K \frac{S}{2} \left(\hat{a}_i \hat{a}_i + 2\hat{a}_i^\dagger \hat{a}_i + \hat{a}_i^\dagger \hat{a}_i^\dagger\right) + K \frac{S}{2} \end{aligned}$$

$$\mathcal{H} \approx -\tilde{E}_0 - \sum_{i,j} \tilde{\mathscr{H}}_{i,j}^{\mathrm{SW}} \cdot \hat{a}_i^\dagger \hat{a}_j + \frac{KS}{2} \left(\underline{\hat{a}_i \hat{a}_i} + \underline{\hat{a}_i^\dagger \hat{a}_i^\dagger}\right)$$

How to diagonalize this ?

Holstein-Primakoff & Bogoliubov transformation

How to diagonalize this

$$\mathcal{H} = -\tilde{E}_0 - \sum_{i,j} \tilde{\mathscr{H}}^{\mathrm{SW}}_{i,j} \cdot \hat{a}_i^\dagger \hat{a}_j + \frac{KS}{2}\left(\underline{\hat{a}_i\hat{a}_i} + \underline{\hat{a}_i^\dagger \hat{a}_i^\dagger}\right)$$

$$= -\tilde{E}_0 - \left(\hat{a}^\dagger, \hat{a}\right)\begin{pmatrix} H & \mathcal{K} \\ \mathcal{K}^\star & H^\star \end{pmatrix}\begin{pmatrix} \hat{a} \\ \hat{a}^\dagger \end{pmatrix}$$

$$\hat{a} = \left(\hat{a}_1, \ldots, \hat{a}_N\right)^T$$

$$\hat{a}^\dagger = \left(\hat{a}_1^\dagger, \ldots, \hat{a}_N^\dagger\right)^T$$

Holstein-Primakoff & Bogoliubov transformation

How to diagonalize this

$$\mathcal{H} = -\tilde{E}_0 - \sum_{i,j} \tilde{\mathscr{H}}^{\mathrm{SW}}_{i,j} \cdot \hat{a}_i^\dagger \hat{a}_j + \frac{KS}{2} \left(\underline{\hat{a}_i \hat{a}_i} + \underline{\hat{a}_i^\dagger \hat{a}_i^\dagger} \right)$$

$$= -\tilde{E}_0 - \left(\hat{a}^\dagger, \hat{a}\right) \underbrace{\begin{pmatrix} H & \mathcal{K} \\ \mathcal{K}^\star & H^\star \end{pmatrix}}_{=\mathcal{M}} \begin{pmatrix} \hat{a} \\ \hat{a}^\dagger \end{pmatrix}$$

$$\hat{a} = \left(\hat{a}_1, \ldots, \hat{a}_N\right)^T$$

$$\hat{a}^\dagger = \left(\hat{a}_1^\dagger, \ldots, \hat{a}_N^\dagger\right)^T$$

Solve this:

$$\underbrace{\begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix}}_{=:\mathscr{M}} = \begin{pmatrix} U & V \\ V & U \end{pmatrix} \begin{pmatrix} H & \mathcal{K} \\ \mathcal{K} & H^\star \end{pmatrix} \begin{pmatrix} U & V \\ V & U \end{pmatrix}$$

With this we need a new set of operators

$$\hat{\alpha} = U\hat{a} + V\hat{a}^\dagger$$
$$\hat{\alpha}^\dagger = V\hat{a} + U\hat{a}^\dagger$$

$$\rightarrow$$

$$\hat{\alpha}_i = u_i \hat{a}_i + v_i \hat{a}_i^\dagger$$
$$\hat{\alpha}_i^\dagger = v_i \hat{a}_i + u_i \hat{a}_i^\dagger$$

B.Latz Thesis, Heidelberg, (2009)

Holstein-Primakoff & Bogoliubov transformation

How to diagonalize this

$$\mathcal{H} = -\tilde{E}_0 - \sum_{i,j} \tilde{\mathscr{H}}^{\mathrm{SW}}_{i,j} \cdot \hat{a}_i^\dagger \hat{a}_j + \frac{KS}{2}\left(\underline{\hat{a}_i\hat{a}_i} + \underline{\hat{a}_i^\dagger\hat{a}_i^\dagger}\right)$$

$$= -\tilde{E}_0 - (\hat{a}^\dagger, \hat{a}) \begin{pmatrix} H & \mathcal{K} \\ \mathcal{K} & H^\star \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{a}^\dagger \end{pmatrix}$$

$$\hat{\alpha} = U\hat{a} + V\hat{a}^\dagger$$

$$\hat{\alpha}^\dagger = V\hat{a} + U\hat{a}^\dagger$$

$$\Downarrow$$

$$\hat{\alpha}_i = u_i\hat{a}_i + v_i\hat{a}_i^\dagger$$

$$\hat{\alpha}_i^\dagger = v_i\hat{a}_i + u_i\hat{a}_i^\dagger$$

They still fulfill bosonic commutation relations

$$\left[\hat{\alpha}_i, \hat{\alpha}_j^\dagger\right] = \left[u_i\hat{a}_i + v_i\hat{a}_i^\dagger, u_j\hat{a}_j^\dagger + v_j\hat{a}_j\right]$$

$$= u_iu_j \underbrace{\left[\hat{a}_i, \hat{a}_j^\dagger\right]}_{=\delta_{i,j}} + v_iv_j \underbrace{\left[\hat{a}_i^\dagger, \hat{a}_j\right]}_{=-\delta_{i,j}}$$

$$= \left(u_i^2 - v_i^2\right)\delta_{i,j}$$

$$[\hat{\alpha}_i^\dagger, \hat{\alpha}_j^\dagger] = 0$$

$$[\hat{\alpha}_i, \hat{\alpha}_j] = 0$$

Holstein-Primakoff & Bogoliubov transformation

How to diagonalize this

$$\mathcal{H} = -\tilde{E}_0 - \sum_{i,j} \tilde{\mathscr{H}}^{\mathrm{SW}}_{i,j} \cdot \hat{a}_i^\dagger \hat{a}_j + \frac{KS}{2}\left(\underline{\hat{a}_i \hat{a}_i} + \underline{\hat{a}_i^\dagger \hat{a}_i^\dagger}\right)$$

$$= -\tilde{E}_0 - \left(\hat{a}^\dagger, \hat{a}\right) \begin{pmatrix} H & \mathcal{K} \\ \mathcal{K} & H^\star \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{a}^\dagger \end{pmatrix}$$

$$= -\tilde{E}_0 - \sum_{i,j} \mathscr{M}^{\mathrm{diag}}_{i,j} \cdot \hat{\alpha}_i^\dagger \hat{\alpha}_j$$

$$\hat{\alpha}_i = u_i \hat{a}_i + v_i \hat{a}_i^\dagger$$

$$\hat{\alpha}_i^\dagger = v_i \hat{a}_i + u_i \hat{a}_i^\dagger$$

$$|u_i|^2 - |v_j|^2 = 1$$

Holstein-Primakoff & Bogoliubov transformation

$$\hat{\alpha}_i = u_i \hat{a}_i + v_i \hat{a}_i^\dagger$$
$$\hat{\alpha}_i^\dagger = v_i \hat{a}_i + u_i \hat{a}_i^\dagger$$

$$|u_i|^2 - |v_j|^2 = 1$$

How to diagonalize this

$$\mathcal{H} = -\tilde{E}_0 - \sum_{i,j} \tilde{\mathscr{H}}_{i,j}^{\mathrm{SW}} \cdot \hat{a}_i^\dagger \hat{a}_j + \frac{KS}{2}\left(\underline{\hat{a}_i \hat{a}_i} + \underline{\hat{a}_i^\dagger \hat{a}_i^\dagger}\right)$$

$$= -\tilde{E}_0 - (\hat{a}^\dagger, \hat{a}) \begin{pmatrix} H & \mathcal{K} \\ \mathcal{K} & H^\star \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{a}^\dagger \end{pmatrix}$$

$$= -\tilde{E}_0 - \sum_{i,j} \mathscr{M}_{i,j}^{\mathrm{diag}} \cdot \hat{\alpha}_i^\dagger \hat{\alpha}_j$$

Lattice Fourier transformation

$$\mathcal{H} = \sum_{\vec{k}} \mathscr{M}_{\vec{k}} \cdot \hat{\alpha}^\dagger(\vec{k}) \hat{\alpha}(\vec{k})$$

Visualising a Magnon

$$\frac{\mathrm{d}\vec{S}_i}{\mathrm{d}t} = -\frac{\gamma}{(1+\alpha^2)\mu_S}\vec{S}_i \times \left(\vec{H}_i(t) + \alpha\vec{S}_i \times \vec{H}_i(t)\right)$$

$$\vec{H}_i(t) = -\frac{\partial \mathcal{H}}{\partial \vec{S}_i}$$

Numerical methods

Integrate using the Landau Lifshitz Gilbert equation.

Time integration with Heun’s methods

Additional property is the damping α

Engine_LLG\Test.ipynb

Visualising a Magnon

$$\frac{\mathrm{d}\vec{S}_i}{\mathrm{d}t} = -\frac{\gamma}{(1+\alpha^2)\mu_S}\vec{S}_i \times \left(\vec{H}_i(t) + \alpha\vec{S}_i \times \vec{H}_i(t)\right)$$

$$\vec{H}_i(t) = \vec{\xi}_i(t) - \frac{\partial\mathcal{H}}{\partial\vec{S}_i}$$

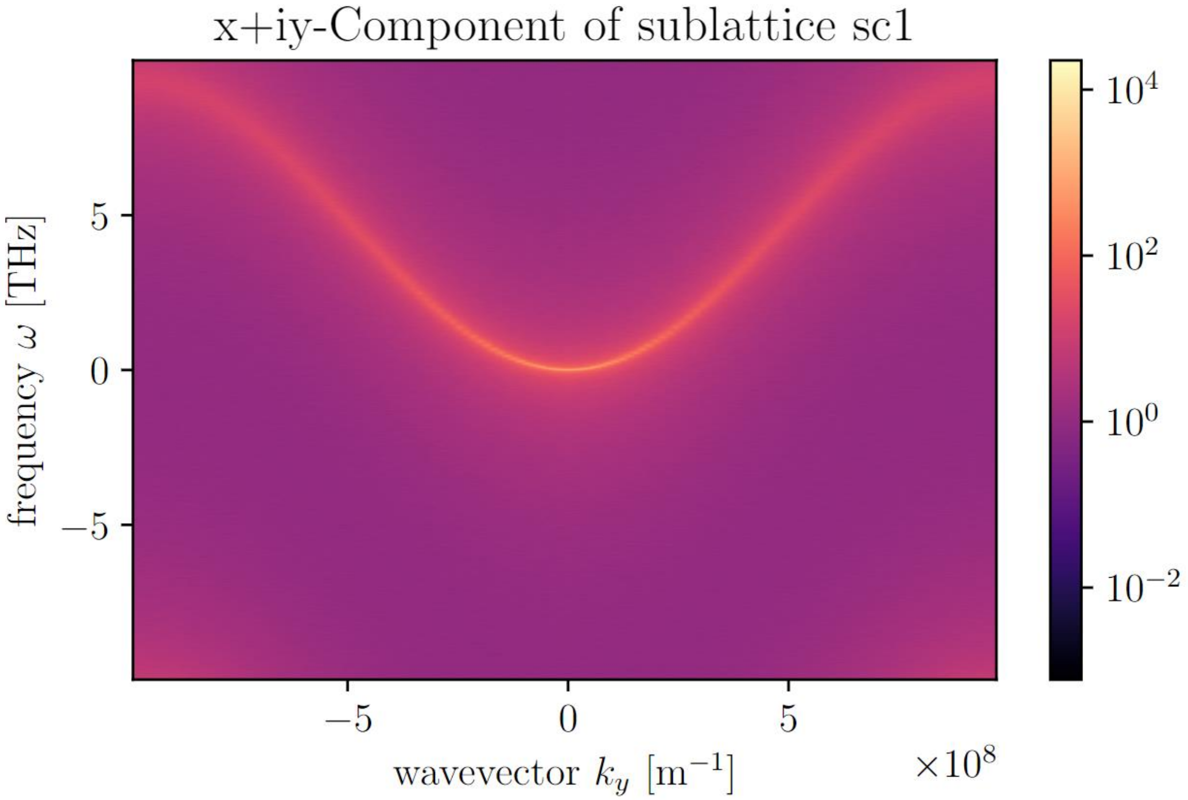
Numerical methods

Integrate using the Landau Lifshitz Gilbert equation.

Time integration with Heun’s methods

Additional property is the damping α

Engine_LLG\Test.ipynb



Outlook

Current research

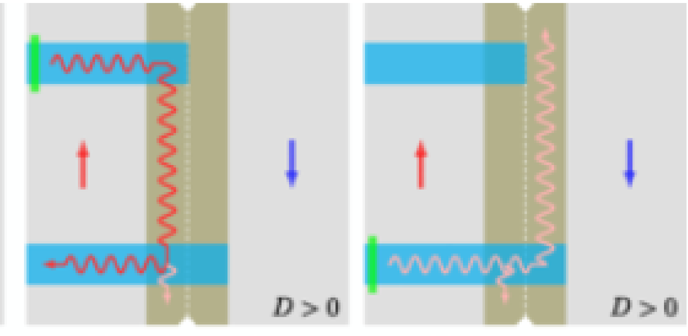
Easily obtain highly interesting dispersion relations.

Can be used for computing, without energy loss through Joule heating

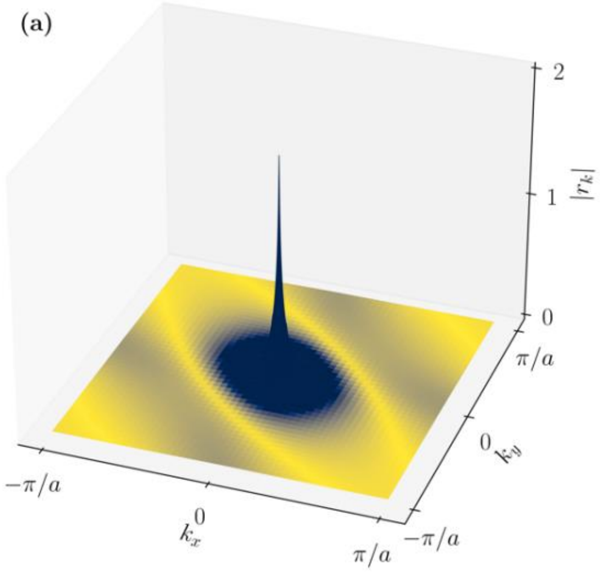
This is ongoing research in
- Spin wave diodes *J.Lan et.al.* PRX **5**, 041049, (2015)
- Spin wave transitors *A.Chumak et.al.* Nature C. **5**, 4700, (2014)

The most modern research even includes Altermagnets, which have distinct symmetry enforced properties regarding the dispersion in different directions

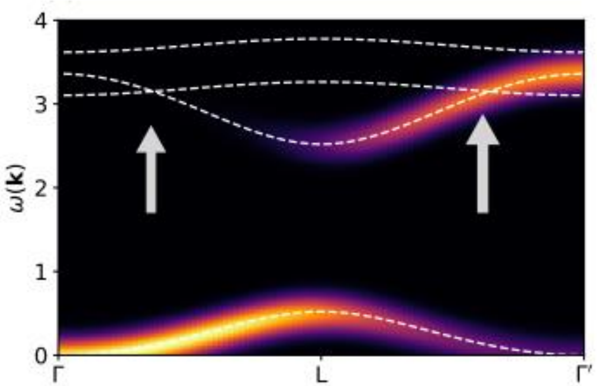
Topological magnons *P. McClarty* Annual Reviews **13**, 171-190, (2022)
Squeezed magnons *D. Wuhrer et.al. PR* **5**, 043124, (2023)



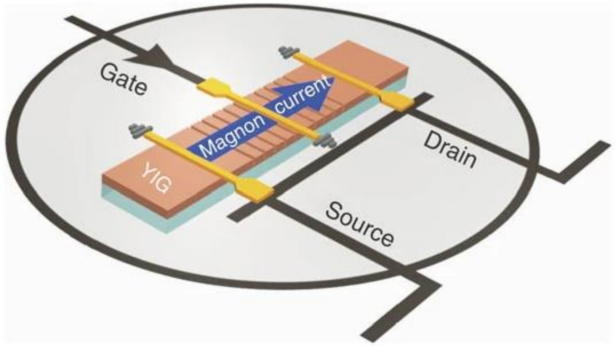
J.Lan et.al. PRX **5**, 041049, (2015)



D. Wuhrer et.al. PR **5**, 043124, (2023)



Magnon transistor scheme



A.Chumak et.al. Nature C. **5**, 4700, (2014)

Heisenbergs Theorie des Ferromagnetismus

Heisenbergs Erweiterung

- $$\boxed{\mathcal{H}_{\text{Heisenberg}} = -\sum_{\langle i,j \rangle} J_{ij} \cdot S_i \cdot S_j}$$

W.Heisenberg Z.Physik **49**, 619-636, (1928)

- „Beweis durch Simulation“

Erfolge

- Luis Neél verwendete das Heisenberg Modell zur Entdeckung/Beschreibung von Antiferro-und Ferrimagnetismus.

$$\vec{\mathfrak{J}} = \lambda \vec{\mathfrak{J}}_a + \mu \vec{\mathfrak{J}}_b, \qquad (9)$$

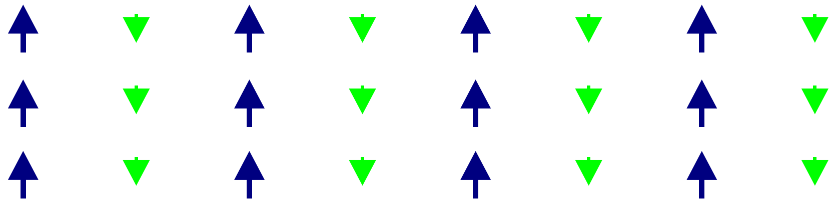
en désignant par λ et μ les proportions des ions magnétiques réparties sur chacun des deux groupes de sites.

L. Néel An. de phys. **12**, 137-198 (1948)

- Simulationen/Berechnungen von Magnonen/Dispersionen
  - Aktuelle Forschung zu (gequetschten) Magnonen
  - Forschungsgebiet der Spintronik

- Nicht nur direkte Wechselwirkung

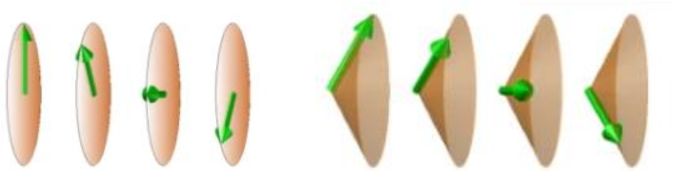
Weitere Arten von magnetischer Ordnung



Wikipedia.com

- Ferrimagnete (1930er)
  - Antiferromagnete mit unterschiedlicher Magnetisierung für verschiedene Untergitter

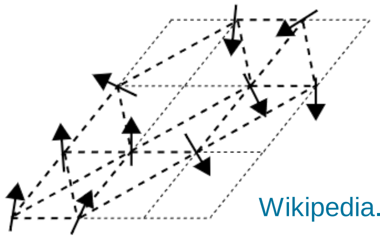
  L. Néel An. de phys. **12**, 137-198 (1948)



N.Jiang et al. Nature **11**,1601 (2020)

- Helimagnetismus (1959)

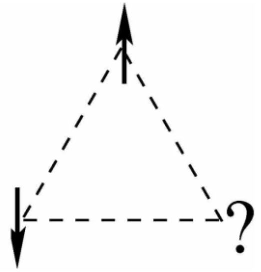
  A. Yoshimori J. Phys. Soc. Jpn. **14**, 807-821 (1959)



Wikipedia.com

- Spin Glässer(1973)
  - Zufälliges aber kooperatives Einfrieren von Spins

  D.Sherrington et al. PRL **35**,26 (1975)



Fkf.mpg.de

- Frustrierter Magnetismus (1950-77)
  - Kollinear und Nicht-kollinear

  G. Toulouse, Commun. Phys. **2**, 115 (1977).

Obacht

Pfeildarstellung der Spins ist semi-klassisch. Stets beachten dass es sich um quantenmechansiche Magnetisierungsdichten handelt.

NEWSFLASH

TECHNOLOGY — INTERNATIONAL BUSINESS TIMES UK

Revolutionary 'Magic Magnet' Altermagnetism Paves the Way for Advanced Electronic Devices

20.02.24

Experimental Evidence for a New Type of Magnetism

January 18, 2024 • *Physics* 17, s10

APS

Physik — Süddeutsche Zeitung

Eine neue Art von Magnetismus

28.02.24

ALTERMAGNETISMUS

Neue Art von Magnetismus entdeckt

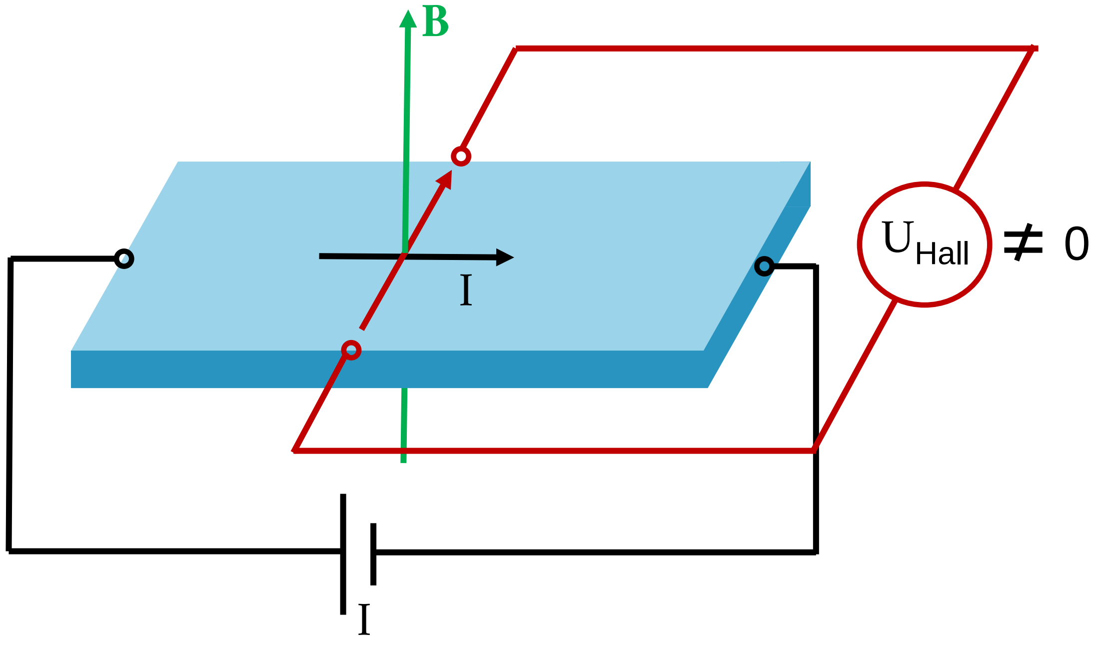
Spektrum der Wissenschaft

22.02.24

Woher kommt die Aufruhr?

Hall Effekt mit B-Feld

- Mit B-Feld bereits bekannt



Woher kommt die Aufruhr?

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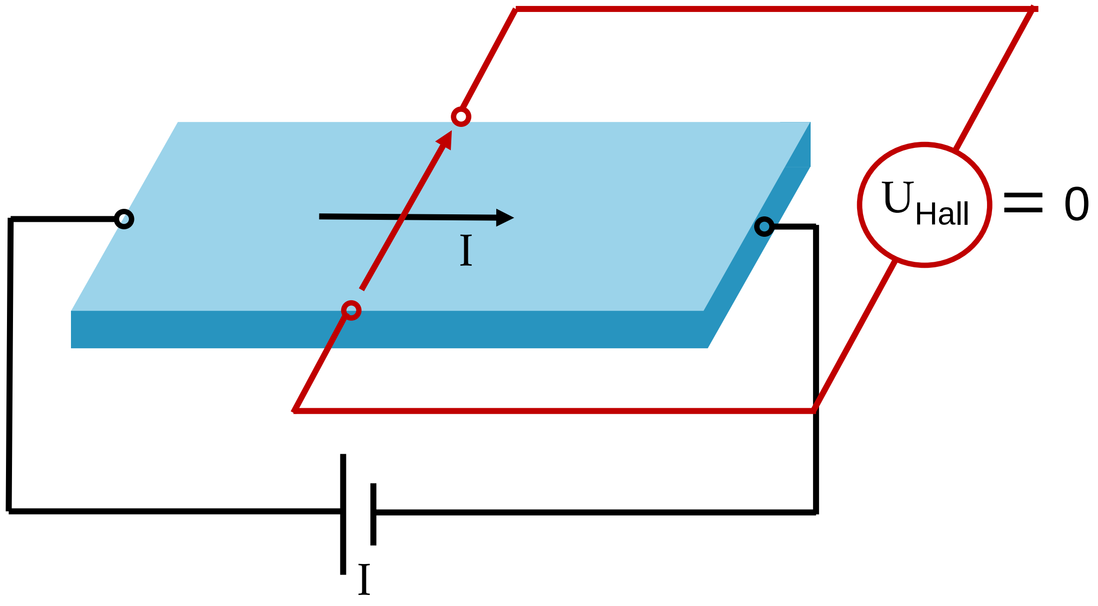

Woher kommt die Aufruhr?

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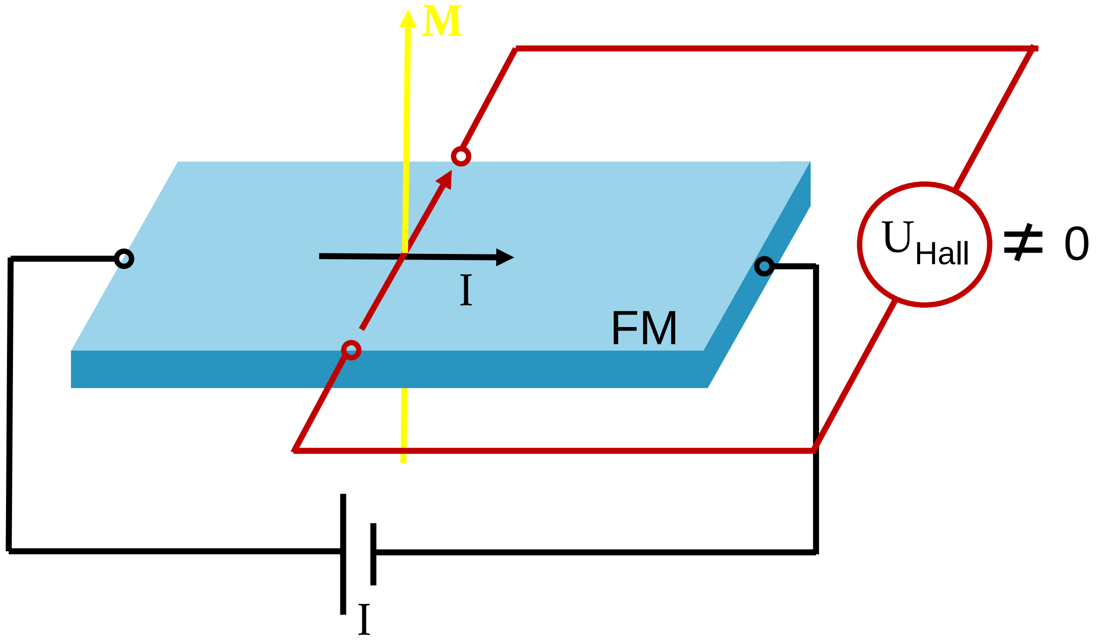


Anormaler Hall Effekt

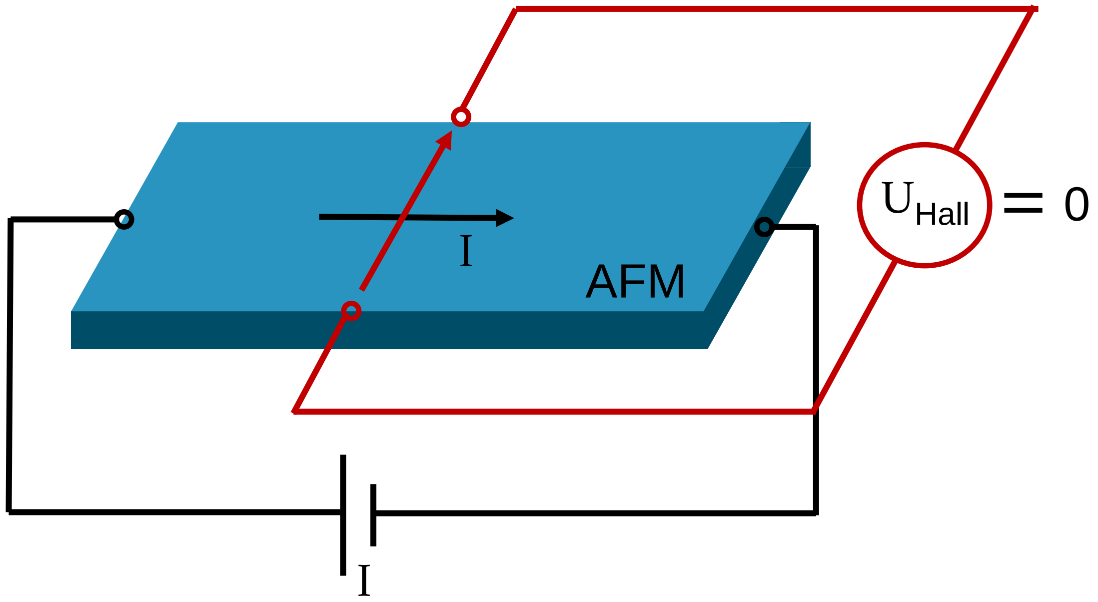
Kein externes Feld B = 0

- FM: $M \neq 0$ $U_{\text{Hall}} \neq 0$

Woher kommt die Aufruhr?

Hall Effekt mit B-Feld

- Mit B-Feld bereits bekannt



Anormaler Hall Effekt

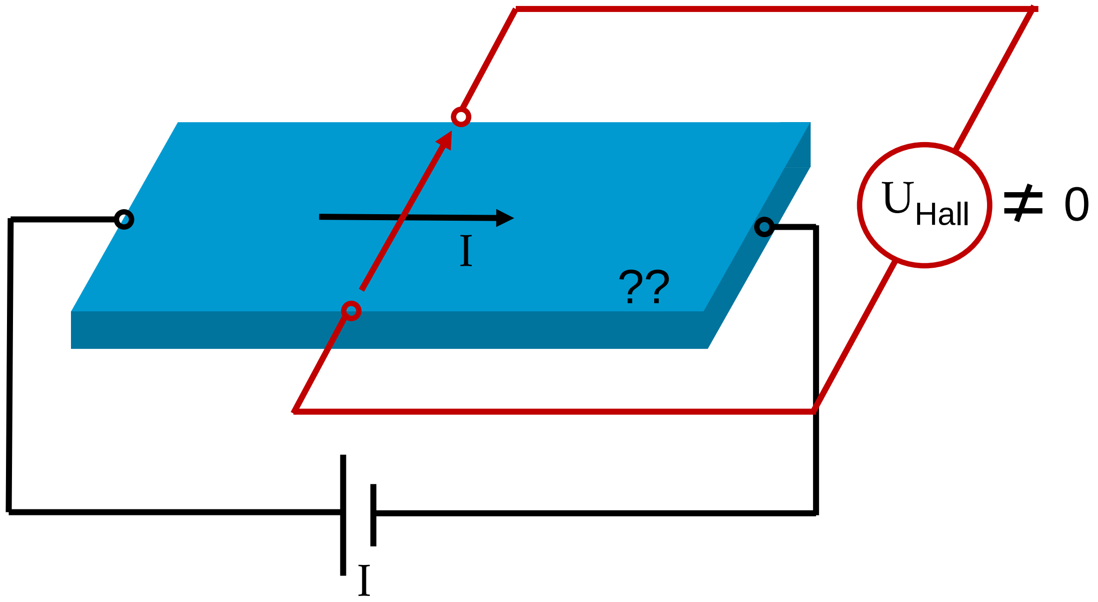
Kein externes Feld B = 0

- FM: $M \neq 0$ $U_{\text{Hall}} \neq 0$
- AFM: $M = 0$ $U_{\text{Hall}} = 0$

Woher kommt die Aufruhr?

Hall Effekt mit B-Feld

- Mit B-Feld bereits bekannt



Anormaler Hall Effekt

Kein externes Feld B = 0

- FM: $M \neq 0$ $\quad U_{\mathrm{Hall}} \neq 0$
- AFM: $M = 0$ $\quad U_{\mathrm{Hall}} = 0$
- ??: $M = 0$ $\quad U_{\mathrm{Hall}} \neq 0$
  - (in manchen Richtungen)

H. Reichlova et al. arXiv:2012.15651

Symmetrien

Zeitumkehroperator

- Gegeben durch den Operator $\mathcal{T}: t \rightarrow -t$

- Angewandt auf ein paar bekannte Größen

$$\mathcal{T}: v \rightarrow \frac{\mathrm{d}x}{-\mathrm{d}t} = -v$$

$$\mathcal{T}: j \rightarrow \frac{\mathrm{d}q}{-\mathrm{d}t} = -j$$

$$\mathcal{T}: B \rightarrow -B$$

$$\mathcal{T}: S \rightarrow -S$$

Symmetrien

Zeitumkehroperator

- Gegeben durch den Operator $\mathcal{T} : t \to -t$
- Angewandt auf ein paar bekannte Größen

$$\mathcal{T} : v \to \frac{\mathrm{d}x}{-\,\mathrm{d}t} = -v$$

$$\mathcal{T} : j \to \frac{\mathrm{d}q}{-\,\mathrm{d}t} = -j$$

$$\mathcal{T} : B \to -B$$

$$\mathcal{T} : S \to -S$$

Spin-Gruppen

- Eine Symmetrie stellen wir da durch
  - Spin-Raum $[T_s||T_g]$ Gitter-Raum

L.Šmejkal et al. PRX **12**, *031042* (2022)

- Frage zur Klassifizierung: Kann eine Gittertransformation die Zeitumkehr aufheben?

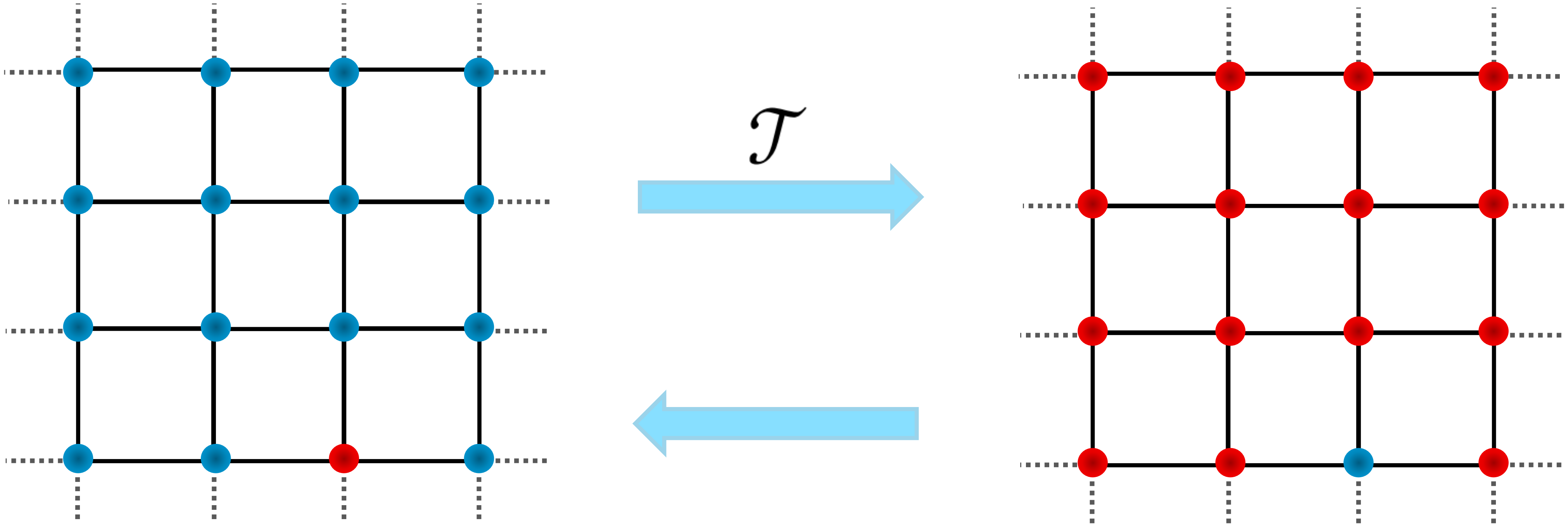
L.Šmejkal et al. PRX **12**, *031042* (2022)

Phasen des Magnetismus

Frage zur Klassifizierung: Kann eine Gittertransformation die Zeitumkehr aufheben?

Ferromagnetismus

- Starke Magnetisierung

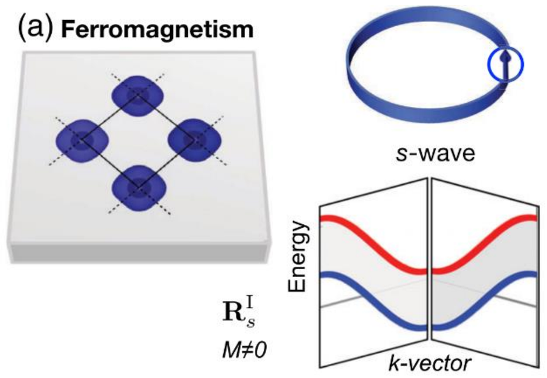


Phasen des Magnetismus

Frage zur Klassifizierung: Kann eine Gittertransformation die Zeitumkehr aufheben?

Folge: Bandaufspaltung

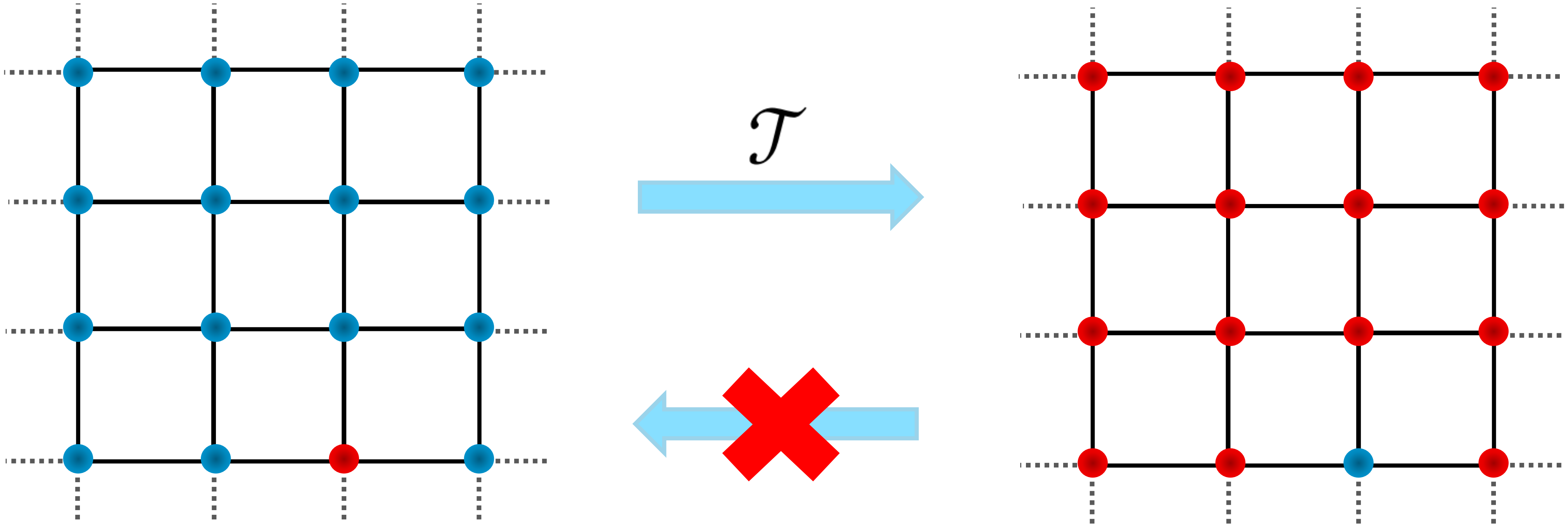
Keine Zeitumkehrsymmetrie



L.Šmejkal *et al.* PRX **12**, *040501* (2022)

Ferromagnetismus: Nein

– Starke Magnetisierung

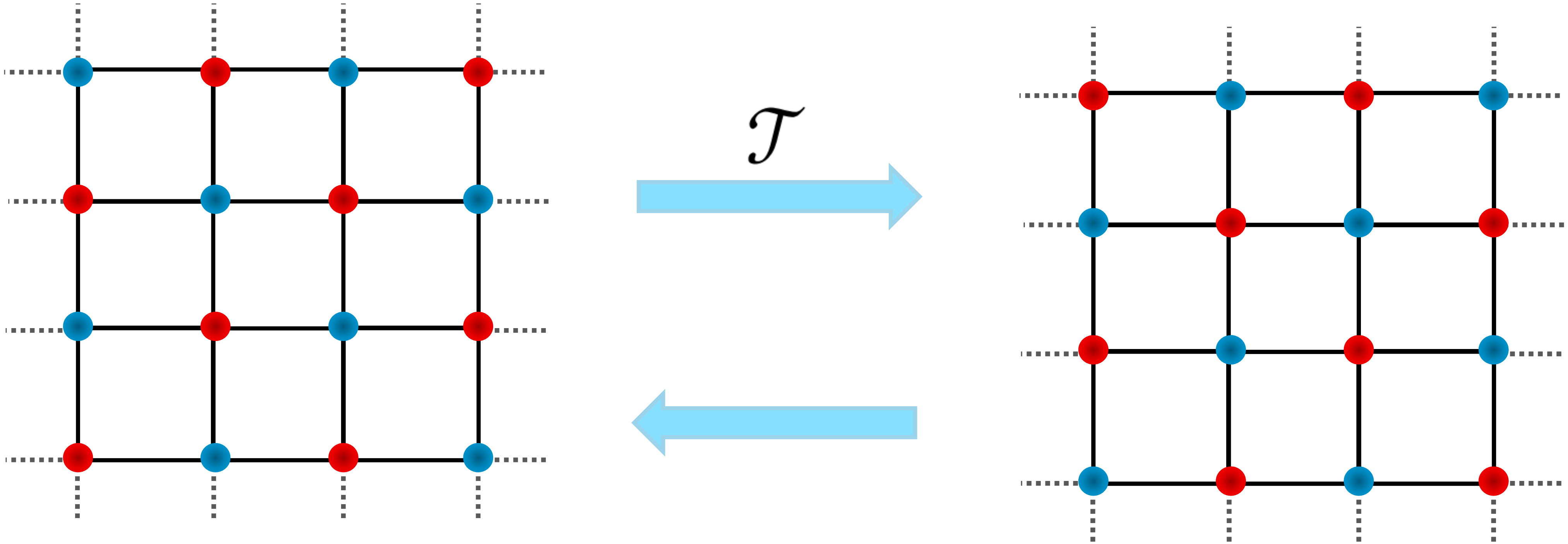


Phasen des Magnetismus

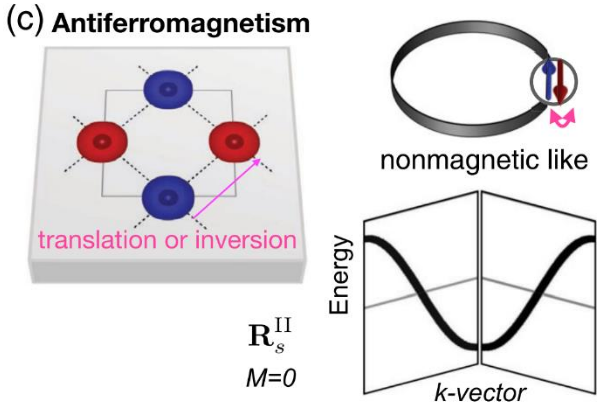
Frage zur Klassifizierung: Kann eine Gittertransformation die Zeitumkehr aufheben?

Antiferromagnetismus

- Keine Magnetisierung



Phasen des Magnetismus



L.Šmejkal et al. PRX **12**, 040501 (2022)

Frage zur Klassifizierung: Kann eine Gittertransformation die Zeitumkehr aufheben?

Folge: Bandentartung

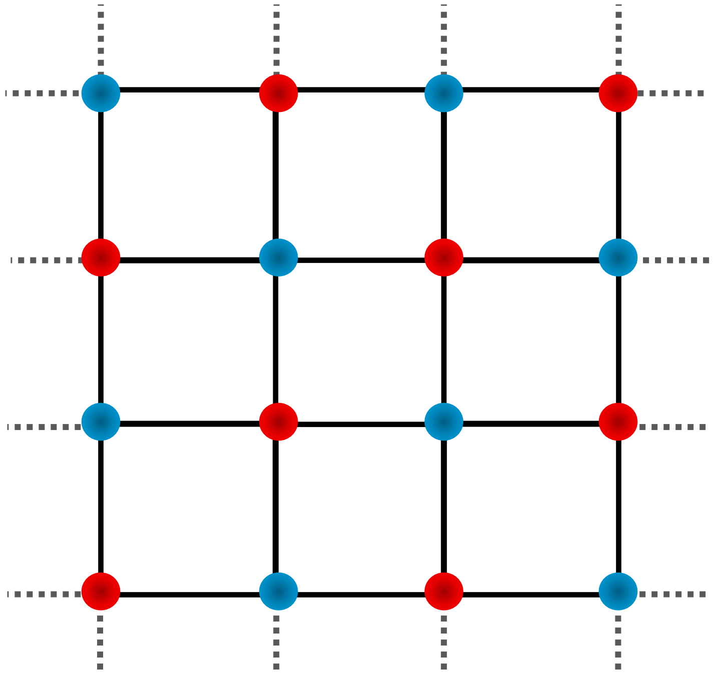
Symmetrien: Zeitumkehr + Translation $[C_2||\mathbf{t}]$

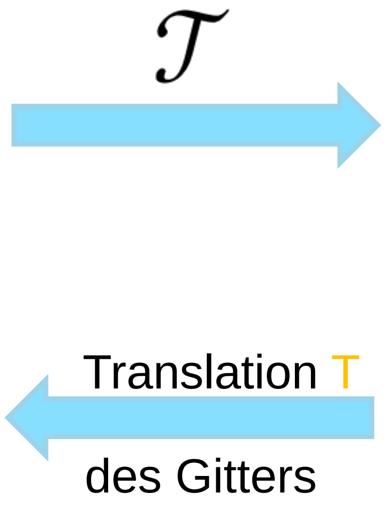
Zeitumkehr + Inversion $[C_2||\bar{E}]$

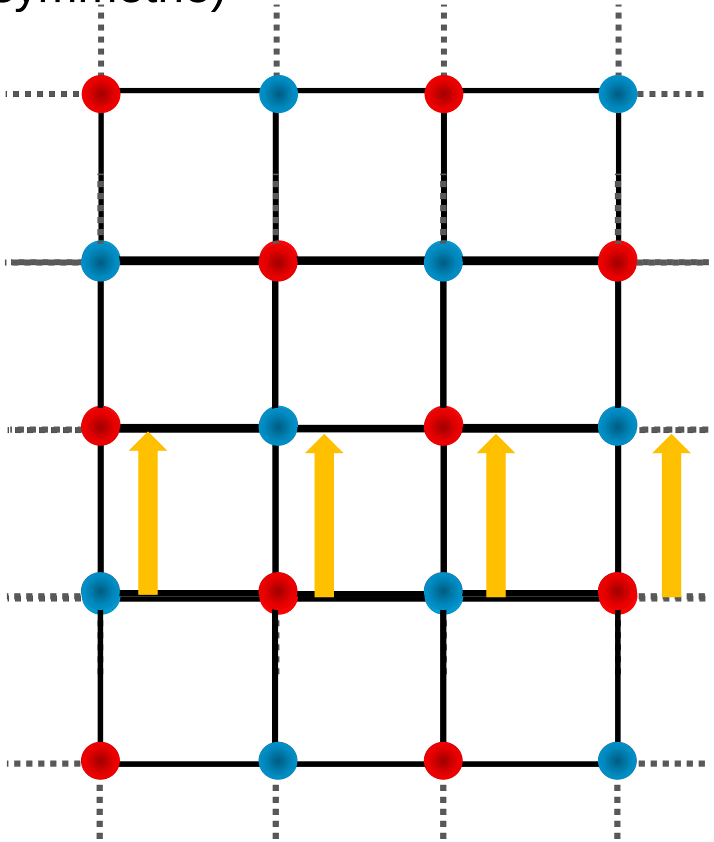
(Zeitumkehrsymmetrie)

Antiferromagnetismus:Ja

- Keine Magnetisierung





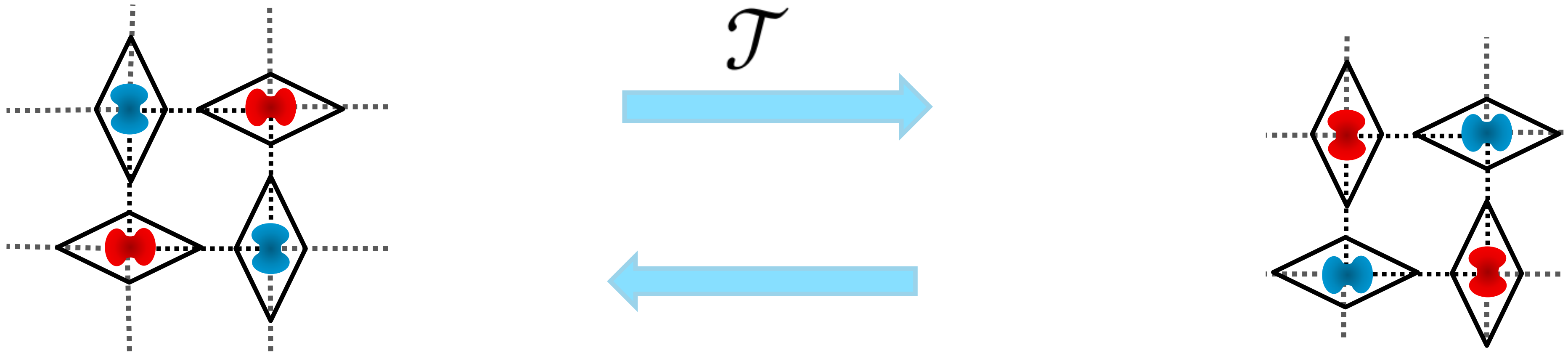


Phasen des Magnetismus

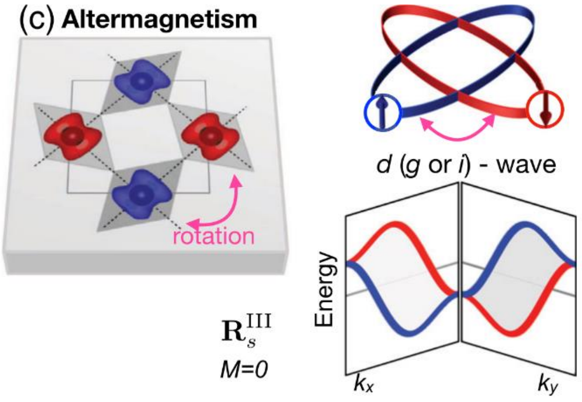
Frage zur Klassifizierung: Kann eine Gittertransformation die Zeitumkehr aufheben?

Altermagnetismus

- Keine Magnetisierung



Phasen des Magnetismus



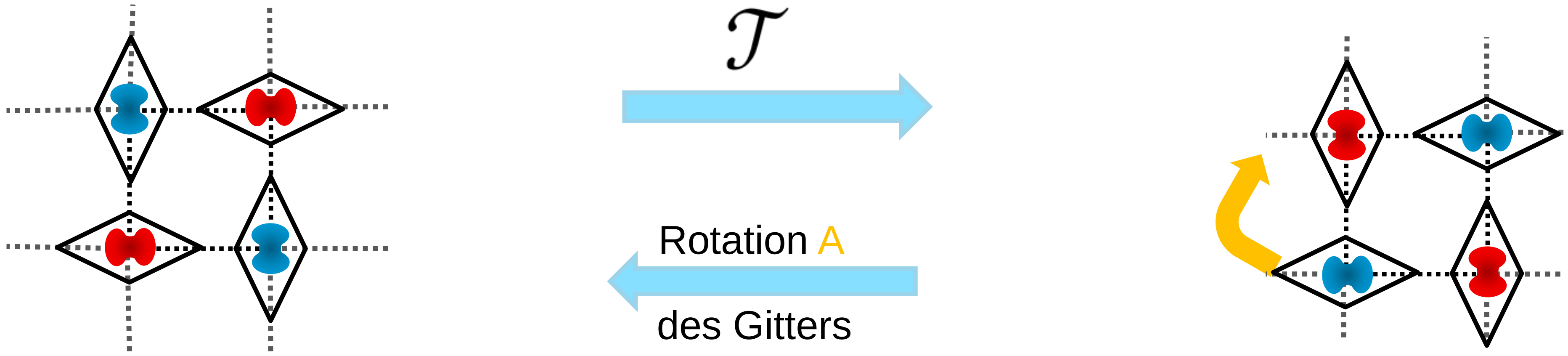
L.Šmejkal *et al.* PRX **12**, *040501* (2022)

Frage zur Klassifizierung: Kann eine Gittertransformation die Zeitumkehr aufheben?

Folge: Bandaufspaltung?
Symmetrien: Zeitumkehr + Rotation $[C_2||A]$
Keine Zeitumkehrsymmetrie

Altermagnetismus: Ja

- Keine Magnetisierung



Symmetrie Beschreibung der Phasen

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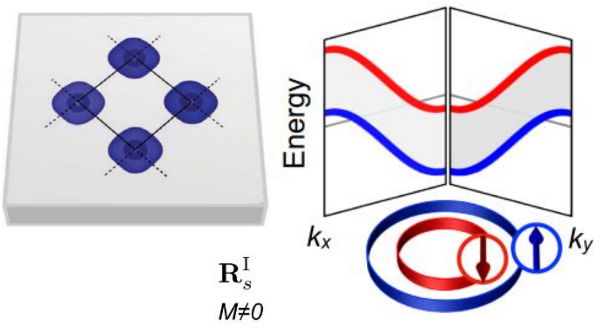
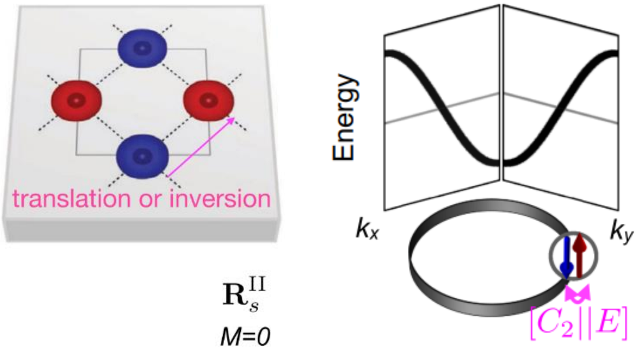
Ferromagnetismus



- Spin-Gruppe: $[E||\mathbf{G}]$

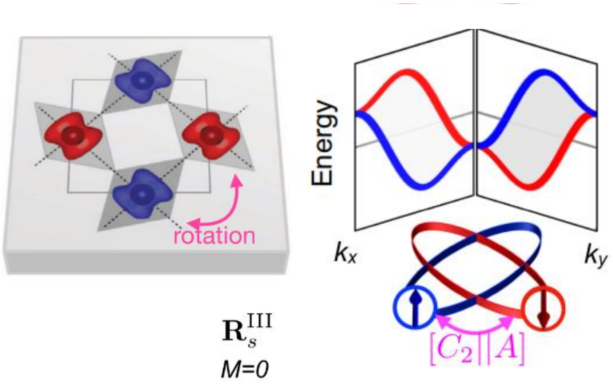
Antiferromagnetismus



- Spin-Gruppe:

$$[E||\mathbf{G}] + [C_2||\mathbf{G}]$$

Altermagnetismus



- Spin-Gruppe:

$$[E||\mathbf{H}] + [C_2||\mathbf{AH}]$$

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Symmetrie Beschreibung der Phasen

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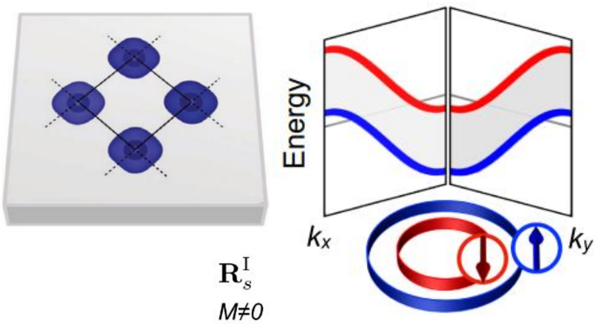
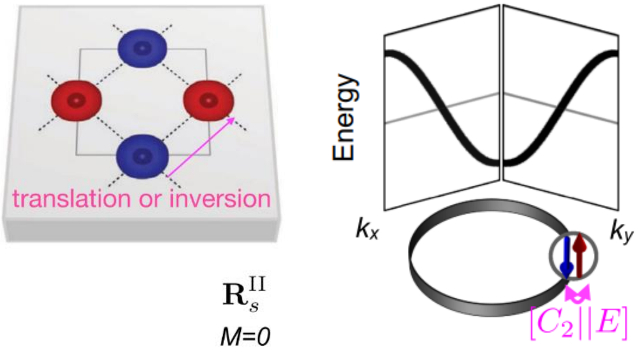
Ferromagnetismus



- Spin-Gruppe: $[E||\mathbf{G}]$

- Magnetisierung
- Isotrope aufgeteilte Energiebänder

Antiferromagnetismus

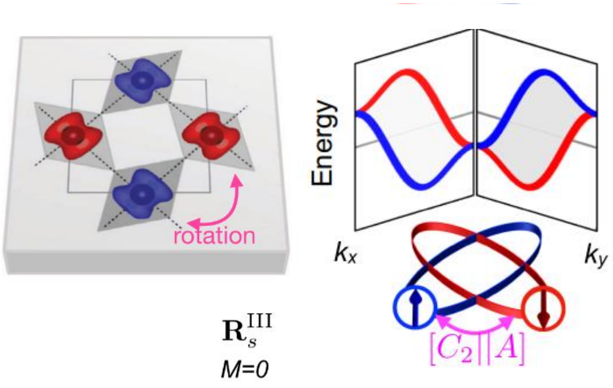


- Spin-Gruppe:

$$[E||\mathbf{G}] + [C_2||\mathbf{G}]$$

- Keine Magnetisierung
- Isotrope entartete Energiebänder
  - (Im nichtrelativistischen Limit)

Altermagnetismus



- Spin-Gruppe:

$$[E||\mathbf{H}] + [C_2||\mathbf{AH}]$$

- Keine Magnetisierung
- Alternierende Spin-Polarisation
  - Im k- und r-Raum
- Gleichbesetze und aufgeteilte Up u. Down Bänder
  - (Im nichtrelativistischen Limit)

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Symmetrie Beschreibung der Phasen

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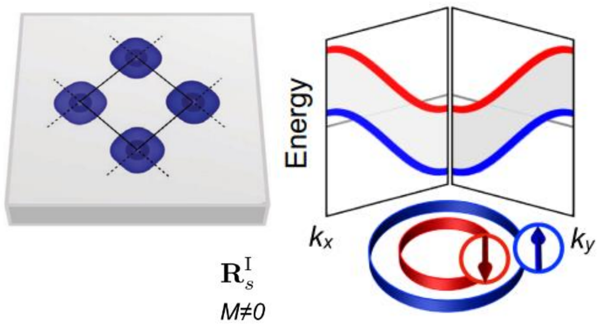
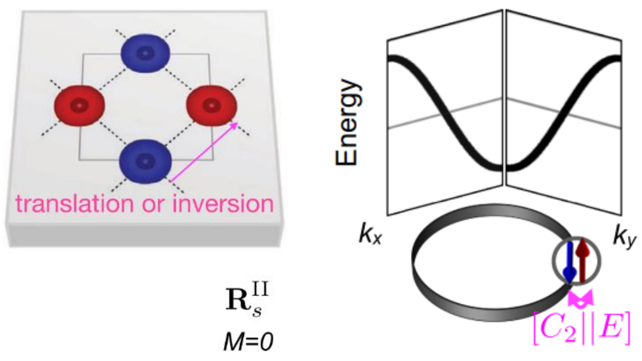
Ferromagnetismus



- Spin-Gruppe: $[E||\mathbf{G}]$

- Magnetisierung
- Isotrope aufgeteilte Energiebänder
  - Magnet-Transport-Effekte

Antiferromagnetismus

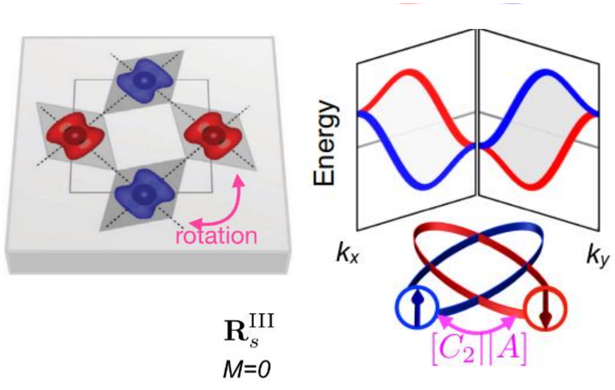


- Spin-Gruppe:

$$[E||\mathbf{G}] + [C_2||\mathbf{G}]$$

- Keine Magnetisierung
  - Robust zu externen Felder
  - Keine Streufelder
- Isotrope entartete Energiebänder
  - (Im nichtrelativistischen Limit)

Altermagnetismus



- Spin-Gruppe:

$$[E||\mathbf{H}] + [C_2||\mathbf{AH}]$$

- Keine Magnetisierung
- Alternierende Spin-Polarisation
  - Im k- und r-Raum
- Gleichbesetze und aufgeteilte Up u. Down Bänder
  - (Im nichtrelativistischen Limit)

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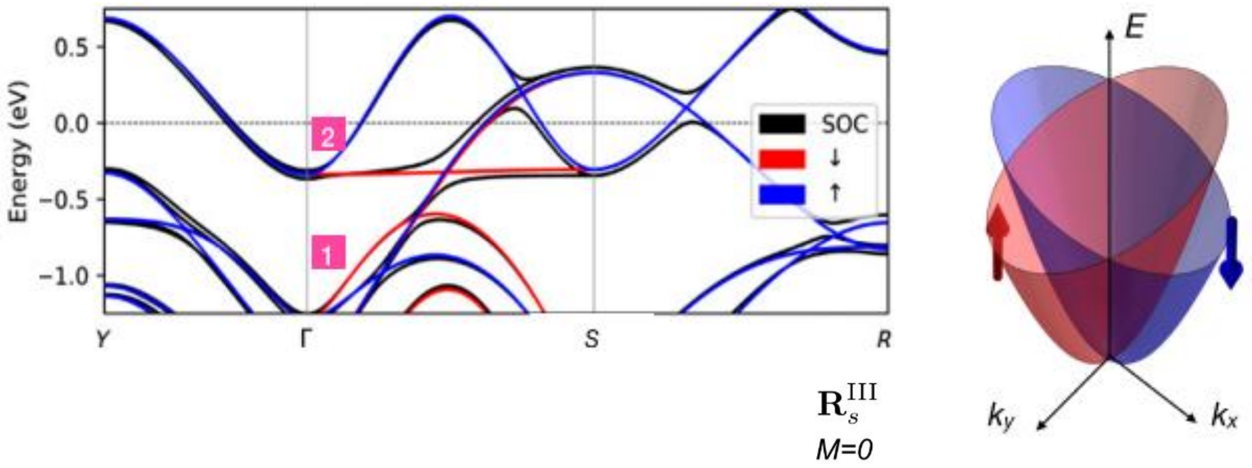
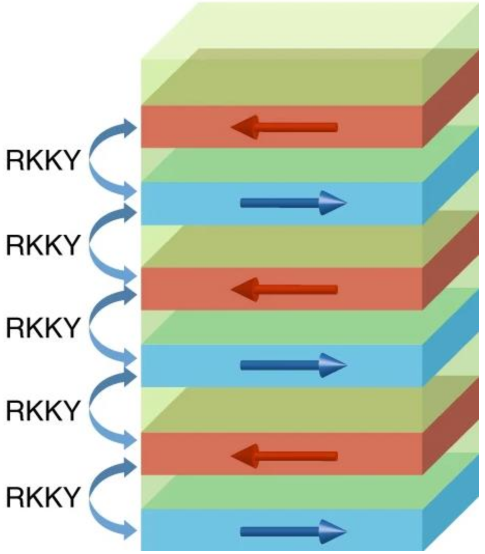
Altermagnetismus



L.Šmejkal et al. PRX **12**, *040501* (2022)

Vorteile/Einfluss

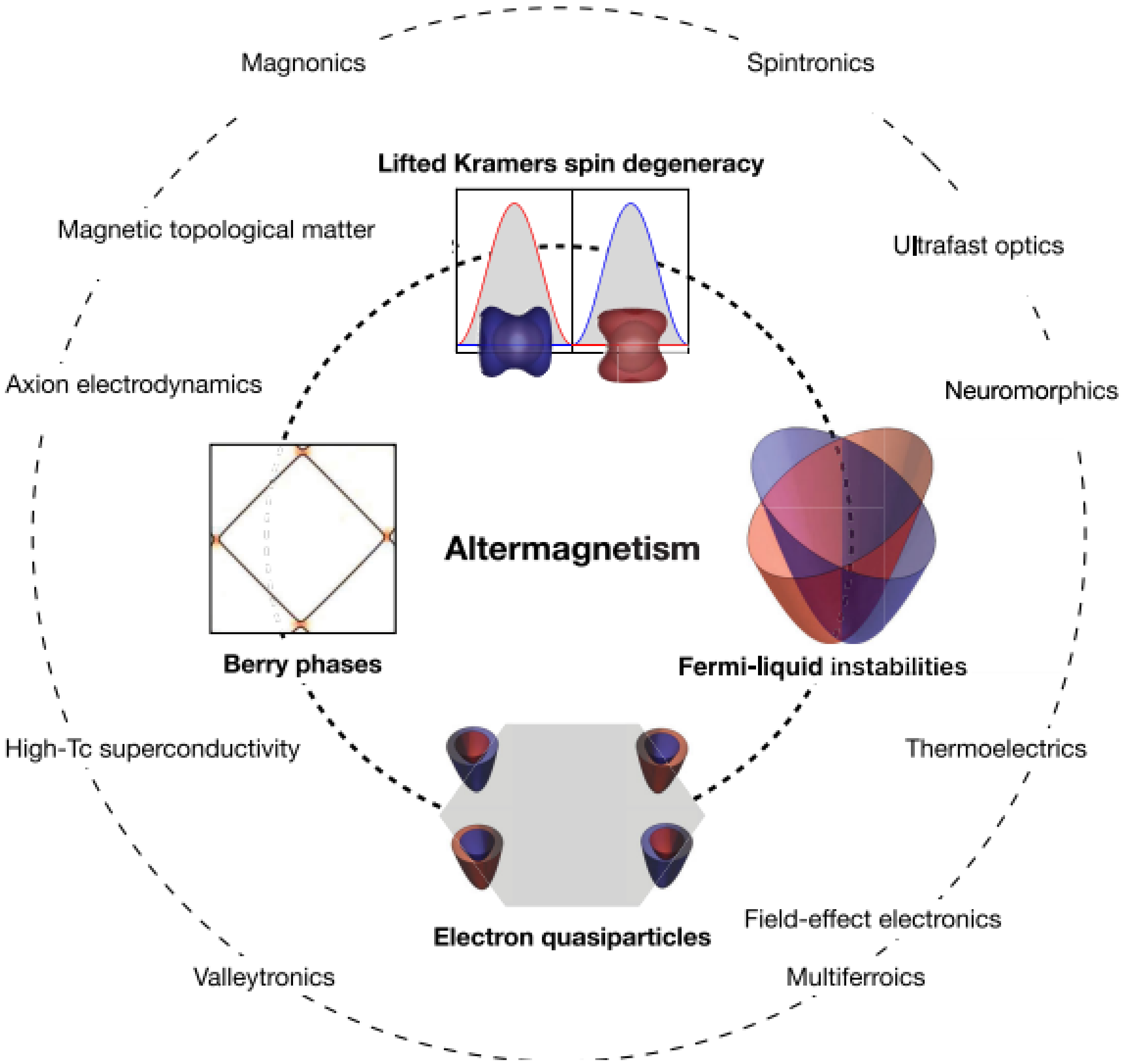
- Anomaler Hall Effekt/GMR/TMR
- Robust, weil keine Magnetisierung
  - keine Streumagentisierung (aufwendiges SAFS(GMR-Stacks) im Moment)
- Spinwellen im THz-Bereich
- Spin Dynamik im ps-Bereich (FM µs-Bereich)
- „einfache“ Symmetrie Klassifizierung erlaubt Folgerung der beobachteten Eigenschaften
  - Relativistische Effekt nicht nötig aber addierbar



R.A.Duine et al. Nature **14**, *217-219* (2018)

Altermagnetismus

Ausblick



L.Šmejkal et al. PRX **12**, *040501* (2022)