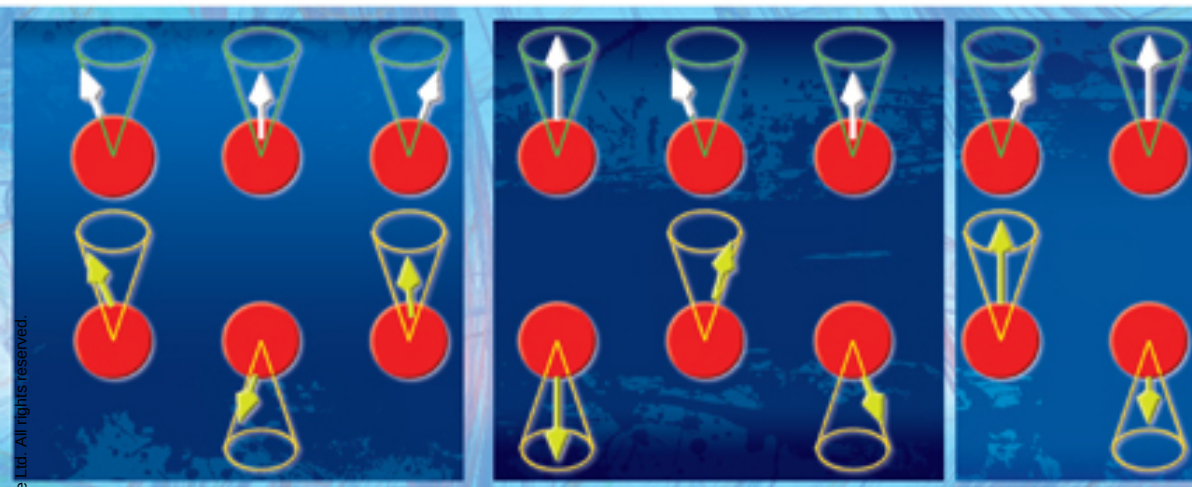


# Statistical Mechanics of Magnetic Excitations



From Spin Waves to Stripes and Checkerboards

Enrico Rastelli

Series on Advances in Statistical Mechanics – Vol. 18

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Enrico Rastelli

Institute of Materials for Electronics and Magnetism, CNR  
and Department of Physics, University of Parma, Italy



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# PREFACE

Elementary excitations in solid state physics are the key to understand the low temperature properties of the matter. Indeed, their energy and lifetime are deduced from the real and imaginary part of the complex pole of the related Green function that can be experimentally investigated through its relationship with the correlation function on the basis of the linear response theory.

This book is devoted to the elementary excitations in magnetic periodic structures, the spin waves or magnons, which are shown to behave like an ideal Bose gas at low temperature and an interacting Bose system at higher temperature.

The theoretical research in this field has been keeping up during the time since the pioneer investigation on spin oscillations in ferromagnets by Bloch and twenty years later in antiferromagnets by Anderson. The well-established model Hamiltonian used to describe a variety of insulating magnetic materials is the Heisenberg Hamiltonian. A fundamental contribution to the statistical treatment of the Heisenberg Hamiltonian was given by Dyson who established a connection between spin and Bose operators obtaining exact results for the isotropic ferromagnet in the low temperature limit.

The discovery of non-collinear magnetic order like helices and spirals stimulated theoreticians to understand the origin of such intriguing magnetic configurations. The basic underlying mechanism is the frustration entered by competing exchange interactions or lattice structure.

In the 1980's, the interest shifted towards the low-dimensional magnets in connection with the discovery of the high- $T_c$  superconductors (quasi-2D magnetic systems), the availability of excellent magnetic surfaces and ultrathin magnetic films. Theoretical interest in 2D magnetic models dates back to the exact evaluation of the partition function of the  $S = \frac{1}{2}$ , nearest neighbour (NN) 2D Ising model (Onsager, 1944) and the discovery of a new type of transition in 2D planar rotator (sometimes referred improperly as XY) model (Berezinskii, Kosterlitz and Thouless, 1970). Subsequently, the experimental investigation of quasi-1D magnetic systems like  $ABX_3$  (where A is an alkali element, B is a transition metal and X is a halogen) and the theoretical discovery that antiferromagnetic chains with integer-spin show statistical properties different from the half-integer-spin chains (Haldane, 1980) stimulated the study of 1D magnets.

From the experimental point of view, the use of neutrons as a probe to investigate both the kind of magnetic order and the elementary excitations spectrum looking at the elastic and inelastic part of the neutron scattering cross-section was a milestone in the comprehension of the magnetism.

In Chap. 1, we deduce the Heisenberg Hamiltonian starting from the hydrogen molecule and we present some exact results for small ring of spin  $S = \frac{1}{2}$ . The low lying energy states of a ferromagnet are obtained.

In Chap. 2, we discuss the spin-boson transformations and we give the low temperature thermodynamic functions for non-interacting spin waves.

In Chap. 3, the interaction between the spin waves is accounted for using the method of the equation of motion of the Green function.

In Chap. 4, the systematic perturbation theory for the Green function is introduced making use of the Feynman diagrams.

In Chap. 5, the two-magnon bound states in ferromagnets are investigated and their dependence on the dimensionality and anisotropy is discussed.

In Chap. 6, we present explicit calculations of the spin waves in ferromagnets with planar anisotropy and show the way to overcome the problem of the kinematical consistency entered by the spin-boson transformation in systems with planar anisotropy.

In Chap. 7, we obtain the ground states and the spin waves of magnetic systems with non-collinear order. The exchange competition or the lattice structure leads to helix-configurations that can be checked in neutron scattering experiments.

In Chap. 8, we obtain the ground-state configurations and the spin waves in multilayers. The explicit calculation for ultrathin films and for a semi-infinite medium is explicitly performed.

In Chap. 9, we study the ground-state configuration and spin waves excitations in presence of long range dipole-dipole interaction. The explicit calculation for the high- $T_c$  superconductor  $\text{ErBa}_2\text{Cu}_3\text{O}_{6+x}$  is performed including a crystalline electric field calculation to fit the model with the experimental data.

In Chap. 10, we study the effect of long range dipole-dipole interaction in 2D systems and we show how unusual stripe and checkerboard configurations originate in anisotropic systems as the Ising model. Then, we present results at finite temperature obtained by Monte Carlo simulations.

Chapters 1, 2, 5, 7 and 8 are appropriate for an upper level of undergraduate course of magnetism. Chapters 3, 4 and 6 are suitable for a postgraduate course. Finally, Chapters 9 and 10 have to be considered as a stimulus for candidates to the theoretical research in magnetism.

Let me conclude this introduction by thanking the people who contributed to my scientific formation. First of all, I would thank Prof. Luciano Reatto who introduced me to the study of the statistical mechanics and of the non-collinear magnetic systems. Then, I would thank Prof. Per-Anker Lindgård who addressed me towards a thorough study of the properties of the different spin-boson transformations and Prof. Steven Lovesey who pointed out the importance of the neutron scattering as

a tool of investigation of the spin waves. I am particularly grateful to Prof. Brooks Harris for having introduced me to the many-body theory of magnetic insulators.

A very special thank goes to Prof. Armando Tassi who assisted me in writing this book and was my fundamental collaborator in my research activities in the past 40 years. Let me thank other collaborators with whom I was in touch in different periods of my research activity: Prof. Alberto Pimpinelli, Dr. Silvia Sedazzari, Dr. Albino Carbognani and, particularly, Dr. Sofia Regina who gave me a relevant contribution in Monte Carlo simulations.

Finally, I would like to thank the people who convinced me to write this book: Prof. Davide Cassi and my family. Their insistence and stimulus played a key role in pushing me to undertake this venture.



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