# Classification and Regression Trees

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# Classification and Regression Trees

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## Cross-References

Recursive partitioning

## Keywords

Decision tree, discriminant analysis, missing value, recursive partitioning, selection bias

## Abstract

Classification and regression trees are prediction models constructed by recursively partitioning a data set and fitting a simple model to each partition. Their name derives from the usual practice of describing the partitioning process by a decision tree. This article reviews some widely available algorithms and compares their capabilities, strengths and weaknesses in two examples.

Classification and regression trees are machine learning methods for constructing prediction models from data. The models are obtained by recursively partitioning the data space and fitting a simple prediction model within each partition. As a result, the partitioning can be represented graphically as a decision tree. Classification trees are designed for dependent variables that take a finite number of unordered values, with prediction error measured in terms of misclassification cost. Regression trees are for dependent variables that take continuous or ordered discrete values, with prediction error typically measured by the squared difference between the observed and predicted values. This article gives an introduction to the subject by reviewing some widely available algorithms and comparing their capabilities, strengths and weakness in two examples.

## Classification trees

In a classification problem, we have a training sample of $n$ observations on a class variable $Y$ that takes values 1, 2, ..., $k$, and $p$ predictor variables, $X_1, \ldots, X_p$. Our goal is to find a model for predicting the values of $Y$ from new $X$ values. In theory, the solution is simply a partition of the $X$ space into $k$ disjoint sets, $A_1, A_2, \ldots, A_k$, such

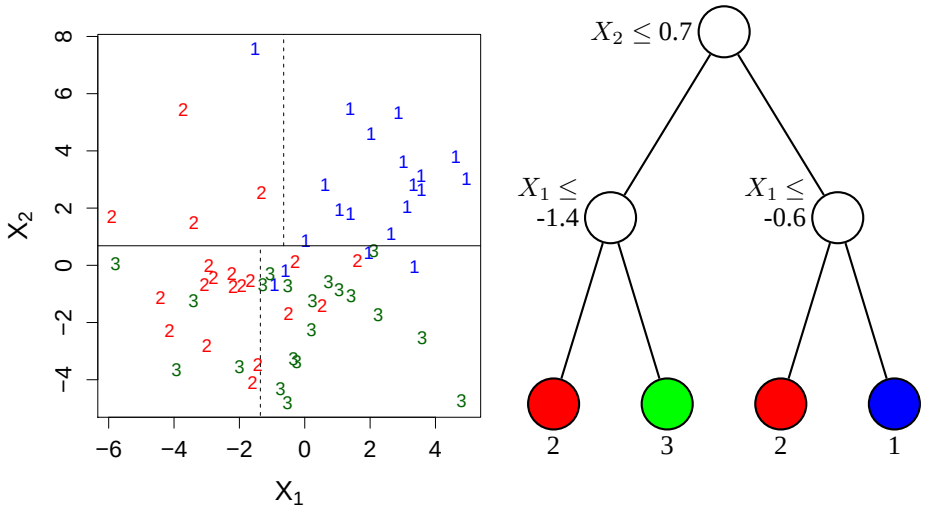


Figure 1: Partitions (left) and decision tree structure (right) for a CART classification model with three classes labeled 1, 2, 3. At each intermediate node, a case goes to the left child node if and only if the condition is satisfied. The predicted class is given beneath each leaf node.

that the predicted value of $Y$ is $j$ if $X$ belongs to $A_j$, for $j = 1, 2, \ldots, k$. If the $X$ variables take ordered values, two classical solutions are linear discriminant analysis [17] and nearest-neighbor classification [13]. These methods yield sets $A_j$ with piecewise linear and nonlinear, respectively, boundaries that are not easy to interpret if $p$ is large.

Classification tree methods yield rectangular sets $A_j$ by recursively partitioning the data set one $X$ variable at a time. This makes the sets easier to interpret. For example, Figure 1 gives an example where there are three classes and two $X$ variables. The left panel plots the data points and partitions and the right panel shows the corresponding decision tree structure. A key advantage of the tree structure is its applicability to any number of variables, whereas the plot on its left is limited to at most two.

The first published classification tree algorithm is THAID [16, 35]. Employing a measure of node impurity based on the distribution of the observed $Y$ values in the node, THAID splits a node by exhaustively searching over all $X$ and $S$ for the split $\{X \in S\}$ that minimizes the total impurity of its two child nodes. If $X$ takes ordered values, the set $S$ is an interval of the form $(-\infty, c]$. Otherwise, $S$ is a subset of the values taken by $X$. The process is applied recursively on the data in each child node. Splitting stops if the relative decrease in impurity is below a pre-specified threshold. Algorithm 1 gives the pseudocode for the basic steps.

**Algorithm 1** *Pseudocode for tree construction by exhaustive search*

1. *Start at the root node.*

2. *For each $X$, find the set $S$ that minimizes the sum of the node impurities in the two child nodes and choose the split $\{X^* \in S^*\}$ that gives the minimum over all $X$ and $S$.*

3. *If a stopping criterion is reached, exit. Otherwise, apply step 2 to each child node in turn.*

C4.5 [38] and CART [5] are two later classification tree algorithms that follow this approach. C4.5 uses entropy for its impurity function while CART uses a generalization of the binomial variance called the Gini index. Unlike THAID, however, they first grow an overly large tree and then prune it to a smaller size to minimize an estimate of the misclassification error. CART employs ten-fold (default) cross-validation while C4.5 uses a heuristic formula to estimate error rates. CART is implemented in the R system [39] as RPART [43], which we use in the examples below.

Despite its simplicity and elegance, the exhaustive search approach has an undesirable property. Note that an ordered variable with $m$ distinct values has $(m-1)$ splits of the form $X \leq c$, and an unordered variable with $m$ distinct unordered values has $(2^{m-1}-1)$ splits of the form $X \in S$. Therefore if everything else is equal, variables that have more distinct values have a greater chance to be selected. This selection bias affects the integrity of inferences drawn from the tree structure.

Building on an idea that originated in the FACT [34] algorithm, CRUISE [21, 22], GUIDE [31] and QUEST [33] use a two-step approach based on significance tests to split each node. First, each $X$ is tested for association with $Y$ and the most significant variable is selected. Then an exhaustive search is performed for the set $S$. Because every $X$ has the same chance to be selected if each is independent of $Y$, this approach is effectively free of selection bias. Besides, much computation is saved as the search for $S$ is carried out only on the selected $X$ variable. GUIDE and CRUISE use chi-squared tests, and QUEST uses chi-squared tests for unordered and analysis of variance tests for ordered variables. CTree [18], another unbiased method, uses permutation tests. Pseudocode for the GUIDE algorithm is given in Algorithm 2. The CRUISE, GUIDE and QUEST trees are pruned the same way as CART.

**Algorithm 2** *Pseudocode for GUIDE classification tree construction*

1. *Start at the root node.*

2. *For each ordered variable $X$, convert it to an unordered variable $X'$ by grouping its values in the node into a small number of intervals. If $X$ is unordered, set $X' = X$.*

3. *Perform a chi-squared test of independence of each $X'$ variable versus $Y$ on the data in the node and compute its significance probability.*

4. *Choose the variable $X^*$ associated with the $X'$ that has the smallest significance probability.*

5. *Find the split set $\{X^* \in S^*\}$ that minimizes the sum of Gini indexes and use it to split the node into two child nodes.*

6. *If a stopping criterion is reached, exit. Otherwise, apply steps 2–5 to each child node.*

7. *Prune the tree with the CART method.*

CHAID [20] employs yet another strategy. If $X$ is an ordered variable, its data values in the node are split into ten intervals and one child node is assigned to each interval. If $X$ is unordered, one child node is assigned to each value of $X$. Then CHAID uses significance tests and Bonferroni corrections to try to iteratively merge pairs of child nodes. This approach has two consequences. First, some nodes may be split into more than two child nodes. Second, owing to the sequential nature of the tests and the inexactness of the corrections, the method is biased toward selecting variables with few distinct values.

CART, CRUISE and QUEST can allow splits on linear combinations of all the ordered variables while GUIDE can split on combinations of two variables at a time. If there are missing values, CART and CRUISE use alternate splits on other variables when needed, C4.5 sends each observation with a missing value in a split through every branch using a probability weighting scheme, QUEST imputes the missing values locally, and GUIDE treats missing values as belonging to a separate category. All except C4.5 accept user-specified misclassification costs and all except C4.5 and CHAID accept user-specified class prior probabilities. By default, all algorithms fit a constant model to each node, predicting $Y$ to be the class with the smallest misclassification cost. CRUISE can optionally fit bivariate linear discriminant models and GUIDE can fit bivariate kernel density and nearest neighbor models in the nodes. GUIDE also can produce ensemble models using bagging [3] and random forest [4] techniques. Table 1 summarizes the features of the algorithms.

To see how the algorithms perform in a real application, we apply them to a data set on new cars for the 1993 model year [26]. There are 93 cars and 25 variables. We let the $Y$ variable be the type of drive train, which takes three values (rear, front, or four-wheel drive). The $X$ variables are listed in Table 2. Three are unordered (`manuf`, `type`, and `airbag`, taking 31, 6, and 3 values, respectively), two binary-valued (`manual` and `domestic`), and the rest ordered. The class frequencies are rather unequal: 16 (17.2%) are rear, 67 (72.0%) are front, and 10 (10.8%) are four-wheel drive vehicles. To avoid randomness due to ten-fold cross-validation, we use leave-one-out (i.e., $n$-fold) cross-validation to prune the CRUISE, GUIDE, QUEST, and RPART trees in this article.

Figure 2 shows the results if the 31-valued variable `manuf` is excluded. The CHAID tree is not shown because it has no splits. The wide variety of variables selected in the splits is due partly to differences between the algorithms and partly to the absence of a dominant $X$ variable. Variable `passngr` is chosen by three algorithms (C4.5, GUIDE QUEST); `enginsz`, `fuel`, `length`, and `minprice` by two; and `hp`, `hwympg`,

Table 1 Comparison of classification tree methods. A check mark indicates presence of a feature. The codes are: *b* = missing value branch, *c* = constant model, *d* = discriminant model, *i* = missing value imputation, *k* = kernel density model, *l* = linear splits, *m* = missing value category, *n* = nearest neighbor model, *u* = univariate splits, *s* = surrogate splits, *w* = probability weights.

| Feature | C4.5 | CART | CHAID | CRUISE | GUIDE | QUEST |
|---|---|---|---|---|---|---|
| Unbiased splits | | | | √ | √ | √ |
| Split type | *u* | *u,l* | *u* | *u,l* | *u,l* | *u,l* |
| Branches/split | ≥2 | 2 | ≥2 | ≥2 | 2 | 2 |
| Interaction tests | | | | √ | √ | |
| Pruning | √ | √ | | √ | √ | √ |
| User-specified costs | | √ | √ | √ | √ | √ |
| User-specified priors | | √ | | √ | √ | √ |
| Variable ranking | | √ | | | √ | |
| Node models | *c* | *c* | *c* | *c,d* | *c,k,n* | *c* |
| Bagging & ensembles | | | | | √ | |
| Missing values | *w* | *s* | *b* | *i,s* | *m* | *i* |

`luggage`, `maxprice`, `rev`, `type`, and `width` by one each. Variables `airbag`, `citympg`, `cylin`, `midprice`, and `rpm` are not selected by any.

When GUIDE does not find a suitable variable to split a node, it looks for a linear split on a pair of variables. One such split, on `enginsz` and `rseat`, occurs at the node marked with an asterisk (*) in the GUIDE tree. Restricting the linear split to two variables allows the data and the split to be displayed in a plot, as shown in Figure 3. Clearly, no single split on either variable alone can do as well in separating the two classes there.

Figure 4 shows the C4.5, CRUISE and GUIDE trees when variable `manuf` is included. Now CHAID, QUEST and RPART give no splits. Comparing them with their counterparts in Figure 2, we see that the C4.5 tree is unchanged, the CRUISE tree has an additional split (on `manuf`), and the GUIDE tree is much shorter. This behavior is not uncommon when there are many variables with little or no predictive power: their introduction can substantially reduce the size of a tree structure and its prediction accuracy; see, e.g., [15] for more empirical evidence.

Table 3 reports the computational times used to fit the tree models on a computer with a 2.66Ghz Intel Core 2 Quad Extreme processor. The fastest algorithm is C4.5, which takes milliseconds. If `manuf` is excluded, the next fastest is RPART, at a tenth of a second. But if `manuf` is included, RPART takes more than three hours—a $10^5$-fold increase. This is a practical problem with the CART algorithm: because `manuf` takes 31 values, the algorithm must search through $2^{30} - 1$ (more than 1 billion) splits at the root node alone. (If $Y$ takes only two values, a computational shortcut [5, p. 101] reduces the number of searches to just 30 splits.) C4.5 is not similarly affected because it does not search for binary splits on unordered $X$ variables. Instead, C4.5 splits the node into one branch for each $X$ value and then merges some branches after the tree

Table 2 Predictor variables for the car data.

| Variable | Description | Variable | Description |
|---|---|---|---|
| manuf | Manufacturer (31 values) | rev | Engine revolutions per mile |
| type | Type (small, sporty, compact, midsize, large, van) | manual | Manual transmission available (yes, no) |
| minprice | Minimum price (in $1,000) | fuel | Fuel tank capacity (gallons) |
| midprice | Midrange price (in $1,000) | passngr | Passenger capacity (persons) |
| maxprice | Maximum price (in $1,000) | length | Length (inches) |
| citympg | City miles per gallon | whlbase | Wheelbase (inches) |
| hwympg | Highway miles per gallon | width | Width (inches) |
| airbag | Air Bags standard (0, 1, 2) | uturn | U-turn space (feet) |
| cylin | Number of cylinders | rseat | Rear seat room (inches) |
| enginzs | Engine size (liters) | luggage | Luggage capacity (cu. ft.) |
| hp | Maximum horsepower | weight | Weight (pounds) |
| rpm | Revolutions per minute at maximum horsepower | domestic | Domestic (U.S./non U.S. manufacturer) |

Table 3 Tree construction times on a 2.66Ghz Intel Core 2 Quad Extreme processor for the car data.

| | C4.5 | CRUISE | GUIDE | QUEST | RPART |
|---|---|---|---|---|---|
| Without `manuf` | 0.004s | 3.57s | 2.49s | 2.26s | 0.09s |
| With `manuf` | 0.003s | 4.00s | 1.86s | 2.54s | 3h 2m |

is grown. CRUISE, GUIDE and QUEST also are unaffected because they search exhaustively for splits on unordered variables only if the number of values is small. If the latter is large, these algorithms employ a technique in [34] that uses linear discriminant analysis on dummy variables to find the splits.

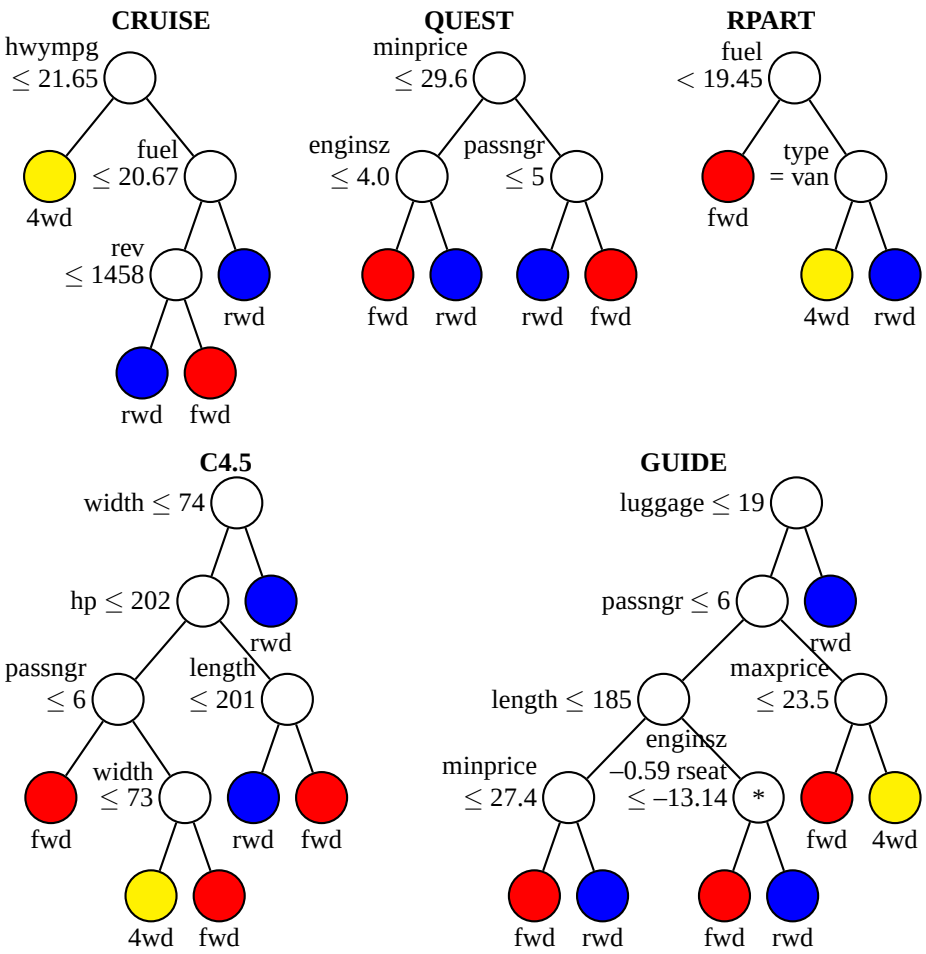


Figure 2: CRUISE, QUEST, RPART, C4.5 and GUIDE trees for car data without `manuf`. The CHAID tree is trivial with no splits. At each intermediate node, a case goes to the left child node if and only if the condition is satisfied. The predicted class is given beneath each leaf node.

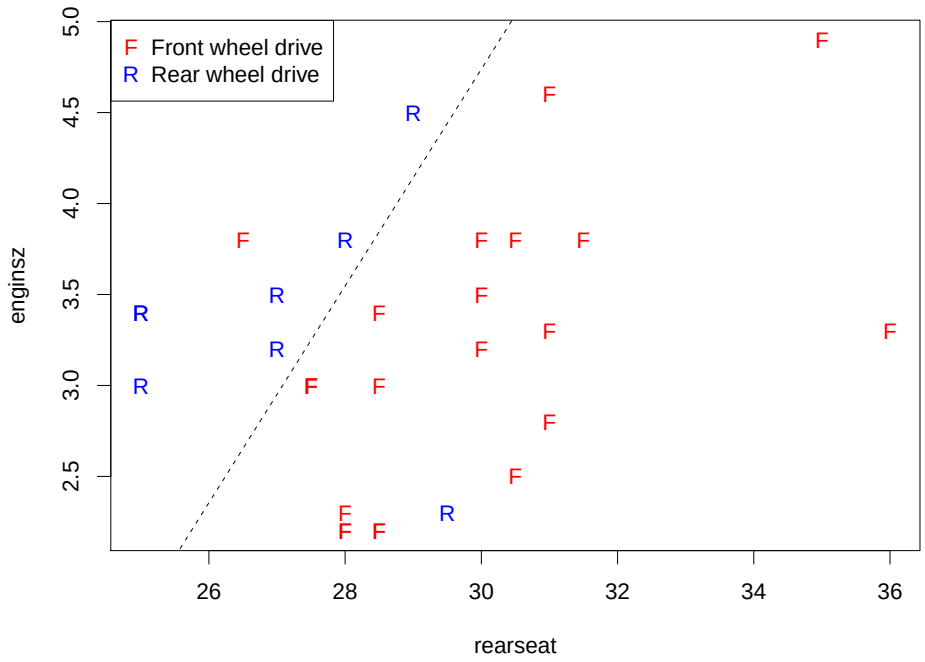


Figure 3: Data and split at the node marked with an asterisk (*) in the GUIDE tree in Figure 2.

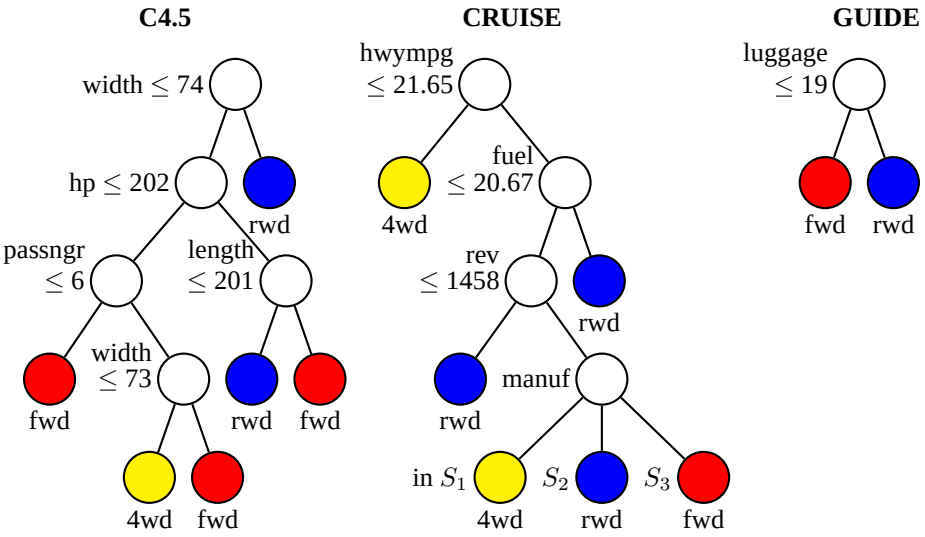


Figure 4: CRUISE, C4.5 and GUIDE trees for car data with `manuf` included. The CHAID, RPART and QUEST trees are trivial with no splits. Sets $S_1$ and $S_2$ are {Plymouth, Subaru} and {Lexus, Lincoln, Mercedes, Mercury, Volvo}, respectively, and $S_3$ is the complement of $S_1 \cup S_2$.

## Regression trees

A regression tree is similar to a classification tree, except that the $Y$ variable takes ordered values and a regression model is fitted to each node to give the predicted values of $Y$. Historically, the first regression tree algorithm is AID [36], which appeared several years before THAID. The AID and CART regression tree methods follow Algorithm 1, with the node impurity being the sum of squared deviations about the mean and the node prediction the sample mean of $Y$. This yields piecewise constant models. Although they are simple to interpret, the prediction accuracy of these models often lags behind that of models with more smoothness. It can be computationally impracticable, however, to extend this approach to piecewise linear models, because two linear models (one for each child node) must be fitted for every candidate split.

M5' [44], an adaptation of a regression tree algorithm due to Quinlan [37], uses a more computationally efficient strategy to construct piecewise linear models. It first constructs a piecewise constant tree and then fits a linear regression model to the data in each leaf node. Since the tree structure is the same as that of a piecewise constant model, the resulting trees tend to be larger than those from other piecewise linear tree methods. GUIDE [27] uses classification tree techniques to solve the regression problem. At each node, it fits a regression model to the data and computes the residuals. Then it defines a class variable $Y'$ taking values 1 or 2, depending on whether the sign of the residual is positive or not. Finally it applies Algorithm 2 to the $Y'$ variable to split the node into two. This approach has three advantages: (i) the splits are unbiased, (ii) only one regression model is fitted at each node, and (iii) because it is based on residuals, the method is neither limited to piecewise constant models nor to the least squares criterion. Table 4 lists the main features of CART, GUIDE and M5'.

Table 4 Comparison of regression tree methods. A check mark indicates presence of a feature. The codes are: *a* = missing value category, *c* = constant model, *g* = global mean/mode imputation, *l* = linear splits, *m* = multiple linear model, *p* = polynomial model, *r* = stepwise linear model, *s* = surrogate splits, *u* = univariate splits, *v* = least squares, *w* = least median of squares, quantile, Poisson, and proportional hazards.

| Feature | CART | GUIDE | M5' |
|---|---|---|---|
| Unbiased splits | | √ | |
| Split type | *u,l* | *u* | *u* |
| Branches/split | 2 | 2 | ≥2 |
| Interaction tests | | √ | |
| Pruning | √ | √ | √ |
| Variable importance ranking | √ | √ | |
| Node models | *c* | *c,m,p,r* | *c,r* |
| Missing value methods | *s* | *a* | *g* |
| Loss criteria | *v* | *v,w* | *v* |
| Bagging & ensembles | | √ | |

To compare CART, GUIDE and M5' with ordinary least squares (OLS) linear regression, we apply them to some data on smoking and pulmonary function in children [40]. The data, collected from 654 children aged 3–19, give the forced expiratory volume (FEV, in liters), gender (sex, M/F), smoking status (smoke, Y/N), age (years), and height (ht, inches) of each child. Using an OLS model for predicting FEV that includes all the variables, Kahn [19] finds that smoke is the only one not statistically significant. He also finds a significant age-smoke interaction if ht is excluded, but not if ht and its square are both included. This problem with interpreting OLS models often occurs when collinearity is present (the correlation between age and height is 0.8).

Figure 5 shows five regression tree models: (a) GUIDE piecewise constant (with sample mean of $Y$ as the predicted value in each node), (b) GUIDE best simple linear (with a linear regression model involving only one predictor in each node), (c) GUIDE best simple quadratic regression (with a quadratic model involving only one predictor in each node), (d) GUIDE stepwise linear (with a stepwise linear regression model in each node), and (e) M5' piecewise constant. The CART tree (from RPART) is a subtree of (a), with six leaf nodes marked by asterisks (*). In the piecewise polynomial models (b) and (c), the predictor variable is found independently in each node, and non-significant terms of the highest orders are dropped. For example for model (b) in Figure 5, a constant is fitted in the node containing females taller than 66.2, because the linear term for ht is not significant at the 0.05 level. Similarly, two of the three leaf nodes in model (c) are fitted with first degree polynomials in ht because the quadratic terms are not significant. Since the nodes, and hence the domains of the polynomials, are defined by the splits in the tree, the estimated regression coefficients typically vary between nodes.

Because the total model complexity is shared between the tree structure and the set of node models, the complexity of a tree structure often decreases as the complexity of the node models increases. Therefore the user can choose a model by trading off tree structure complexity against node model complexity. Piecewise constant models are mainly used for the insights their tree structures provide. But they tend to have low prediction accuracy, unless the data are are sufficiently informative and plentiful to yield a tree with many nodes. The trouble is that the larger the tree, the harder it is to derive insight from it. Trees (a) and (e) are quite large, but because they split almost exclusively on ht, we can infer from the predicted values in the leaf nodes that FEV increases monotonically with ht.

The piecewise simple linear (b) and quadratic (c) models reduce tree complexity without much loss (if any) of interpretability. Instead of splitting the nodes, ht now serves exclusively as the predictor variable in each node. This suggests that ht has strong linear and possibly quadratic effects. On the other hand, the splits on age and sex point to interactions between them and ht. These interactions can be interpreted with the help of Figures 6 and 7, which plot the data values of FEV and ht and the fitted regression functions, with a different symbol and color for each node. In Figure 6, the slope of ht is zero for the group of females taller than 66.2 inches, but it is constant and non-zero across the other groups. This indicates a three-way interaction involving age, ht and sex. A similar conclusion can be drawn from Figure 7, where there are

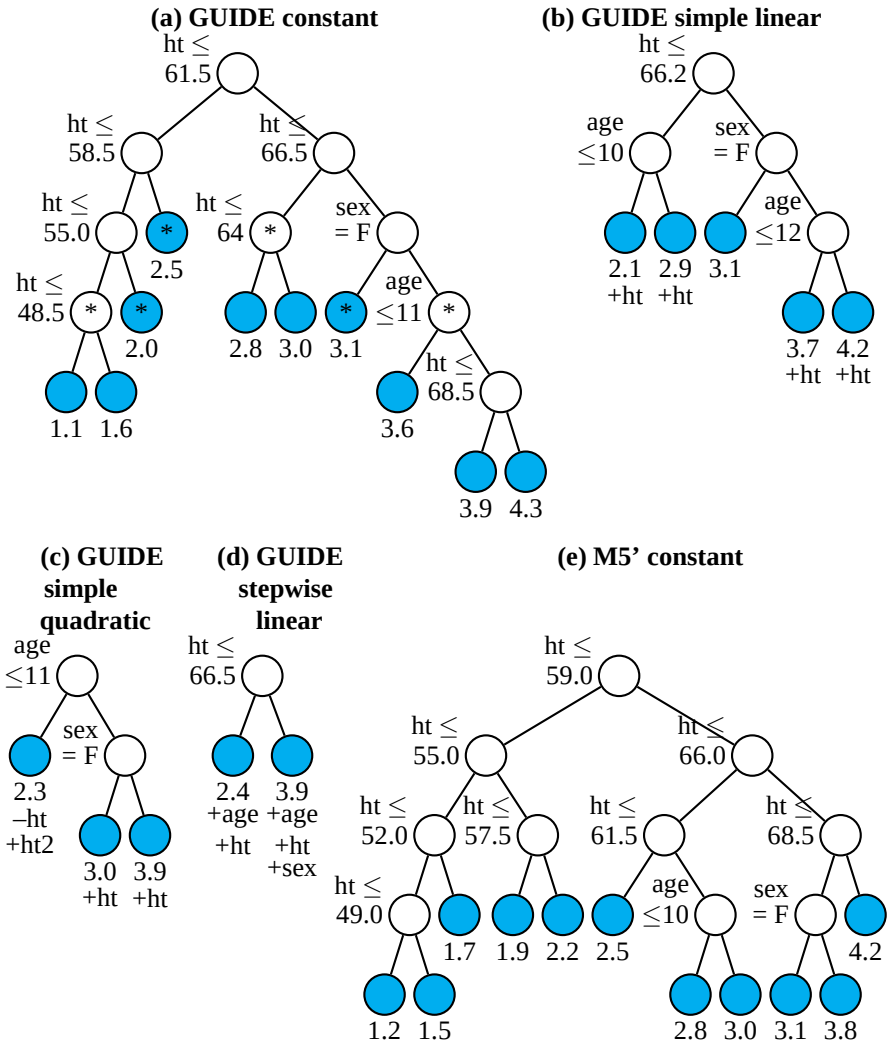


Figure 5: GUIDE piecewise constant, simple linear, simple quadratic, and stepwise linear, and M5' piecewise constant regression trees for predicting FEV. The RPART tree is a subtree of (a), with leaf nodes marked by asterisks (*). The mean FEV and linear predictors (with signs of the coefficients) are printed beneath each leaf node. Variable ht2 is the square of ht.

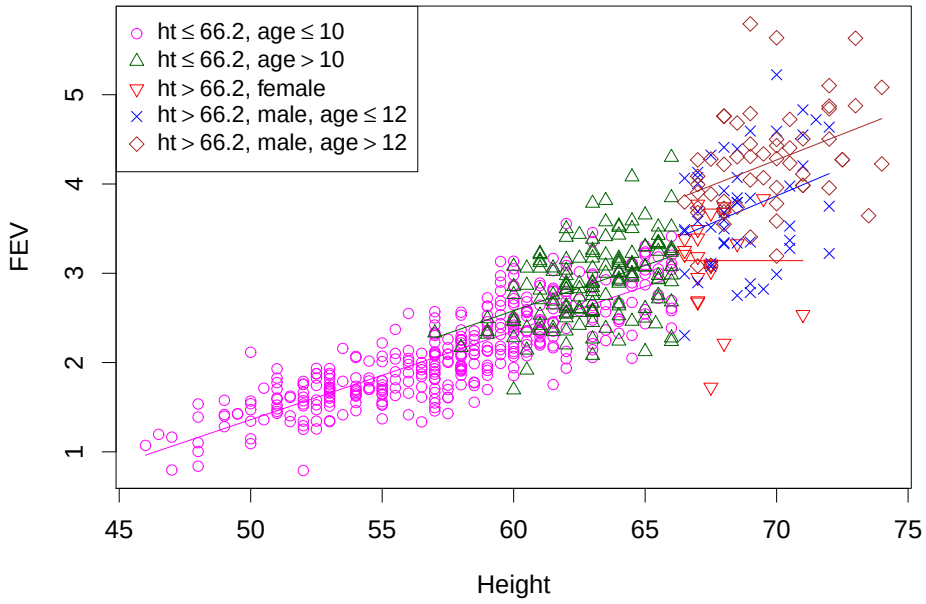


Figure 6: Data and fitted regression lines in the five leaf nodes of the GUIDE piecewise simple linear model in Figure 5(b).

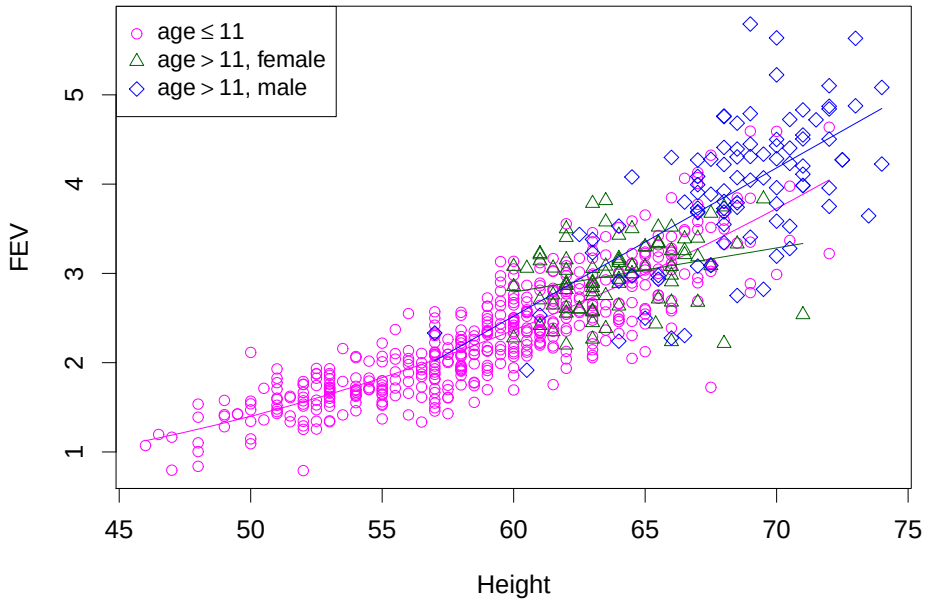


Figure 7: Data and fitted regression functions in the three leaf nodes of the GUIDE piecewise simple quadratic model in Figure 5(c).

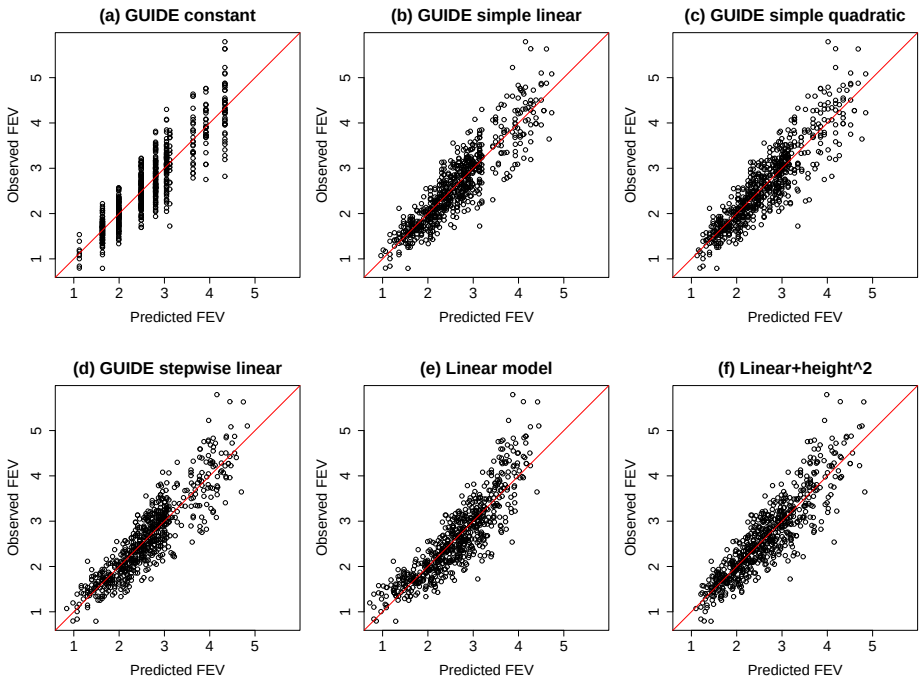


Figure 8: Observed versus predicted values for the tree models in Figure 5 and two ordinary least squares models.

only three groups, with the group of children aged 11 or below exhibiting a quadratic effect of `ht` on `FEV`. For children above age 11, the effect of `ht` is linear, but males have, on average, about a half liter more `FEV` than females. Thus the effect of `sex` seems to be due mainly to children older than 11 years. This conclusion is reinforced by the piecewise stepwise linear tree in Figure 5(d), where a stepwise linear model is fitted to each of the two leaf nodes. `Age` and `ht` are selected as linear predictors in both leaf nodes, but `sex` is selected only in the node corresponding to children taller than 66.5 inches, seventy percent of whom are above 11 years old.

Figure 8 plots the observed versus predicted values of the four GUIDE models and two OLS models containing all the variables, without and with the square of height. The discreteness of the predicted values from the piecewise constant model is obvious, as is the curvature in plot (e). The piecewise simple quadratic model in plot (c) is strikingly similar to plot (f) where the OLS model includes `ht`-squared. This suggests that the two models have similar prediction accuracy. Model (c) has an advantage over model (f), however, because the former can be interpreted through its tree structure and the graph of its fitted function in Figure 7.

This example shows that piecewise linear regression tree models can be valuable in providing visual information about the roles and relative importance of the predictor variables. For more examples, see [23, 30].

## Conclusion

Based on published empirical comparisons of classification tree algorithms, GUIDE appears to have, on average, the highest prediction accuracy and RPART the lowest, although the differences are not substantial for univariate splits [31]. RPART trees often have fewer leaf nodes than those of CRUISE, GUIDE and QUEST, while C4.5 trees often have the most by far. If linear combination splits are used, CRUISE and QUEST can yield accuracy as high as the best non-tree methods [21, 22, 24]. The computational speed of C4.5 is almost always the fastest while RPART can be fast or extremely slow, with the latter occurring when $Y$ takes more than two values and there are unordered variables taking many values. (Owing to its coding, the RPART software cannot accept unordered variables with more than thirty-two values.) GUIDE piecewise linear regression tree models typically have higher prediction accuracy than piecewise constant models. Empirical results [32] show that the accuracy of the piecewise linear trees can be comparable to that of spline-based methods and ensembles of piecewise constant trees.

Owing to space limitations, other approaches and extensions are not discussed here. For likelihood and Bayesian approaches, see [12, 42] and [10, 14], respectively. For Poisson and logistic regression trees, see [8, 29] and [6, 28], respectively. For quantile regression trees, see [9]. For regression trees applicable to censored data, see [1, 2, 11, 25, 41]. Asymptotic theory for the consistency of the regression tree function and derivative estimates may be found in [7, 8, 23]. The C source code for C4.5 may be obtained from `http://www.rulequest.com/Personal/`. RPART may be obtained from `http://www.R-project.org`. M5' is part of the WEKA [44] package at `http://www.cs.waikato.ac.nz/ml/weka/`. Software for CRUISE, GUIDE and QUEST may be obtained from `http://www/stat.wisc.edu/~loh/`.

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## Further Reading

E. Alpaydin. *Introduction to Machine Learning*, 2nd edition, MIT Press, 2010.

## Notes

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