1628 $\boxed{(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3}$

$$\int (3-x^2)^3\,dx = \int (27 - 27x^2 + 9x^4 - x^6)\,dx =$$ 2244 2245

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ ;$$ 2248

$$= 27x - 27\cdot\frac{x^3}{3} + 9\cdot\frac{x^5}{5} - \frac{x^7}{7} + C =$$ 2268 2272 2274

$$= 27x - 9x^3 + \frac{9}{5}x^5 - \frac{1}{7}x^7 + C$$ 2278

1629

$$\int x^2(5-x)^4\,dx = \left\{\begin{array}{l} x = 5-t \\ t = 5-x \\ dx = \frac{1}{t'}dt = -dt \\ \underline{dt = t'dx} \end{array}\right\} =$$

$$= \int -x^2(5-x)^4\,dt = \int -(5-t)^2(5-x)^4\,dt =$$

$$= \int -(5-t)^2 t^4\,dt = \int -(t-5)^2 t^4\,dt =$$

$$= \int -(t^2 - 10t + 25)\,t^4\,dt = \int (-t^6 + 10t^5 - 25t^4)\,dt =$$

$$= -\frac{1}{7}t^7 + \frac{10}{6}\cdot t^6 - \frac{25}{5}\cdot t^5 + C = -\frac{t^7}{7} + \frac{5}{3}t^6 - 5t^5 + C =$$

$$= -\frac{(5-x)^7}{7} + \frac{5(5-x)^6}{3} - 5(5-x)^5 + C$$

1630

$$\int (1-x)(1-2x)(1-3x)\,dx = \int (1 - 3x + 2x^2)\cdot$$

$$\cdot(1-3x)\,dx = \int ((1-3x)^2 + (1-3x)2x^2)\,dx =$$

$$= \int (1 - 6x + 9x^2 + 2x^2 - 6x^3)\,dx = \int (1 - 6x + 11x^2 - 6x^3)\,dx =$$

$$= x - \frac{6}{2}x^2 + \frac{11}{3}x^3 - \frac{6x^4}{4} + C = x - 3x^2 + \frac{11}{3}x^3 - \frac{3}{2}x^4 + C$$

1631 $\int \left(\frac{1-x}{x}\right)^2 dx = \int \frac{(1-x)^2}{x^2} dx =$

$= \int \frac{x^2-2x+1}{x^2} dx = \int 1 - \frac{2}{x} + \frac{1}{x^2} dx =$

$= x - 2\ln|x| + \int x^{-2} dx = x - 2\ln|x| +$

$+ \frac{x^{-1}}{-1} + C = x - 2\ln|x| - \frac{1}{x} + C$

$\int \frac{1}{x} dx = \ln|x| + C$; $\int a^x dx = \frac{a^x}{\ln a} + C$

1632 $\int \left(\frac{a}{x} + \frac{a^2}{x^2} + \frac{a^3}{x^3}\right) dx =$

$x^{-3} = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$

$= a \cdot \ln|x| + c + a^2 \int \frac{1}{x^2} dx + a^3 \int \frac{1}{x^3} dx =$

$= a\ln|x| - a^2 \frac{1}{x^2} - a^3 \frac{1}{2x^2} + C$

1633

$\int \frac{x+1}{\sqrt{x}} dx = \int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx =$

$= \int (x^{\frac{1}{2}} + x^{-\frac{1}{2}}) dx = 2\frac{x^{\frac{3}{2}}}{3} + \frac{2x^{\frac{1}{2}}}{1} + C =$

$= \frac{2}{3} x\sqrt{x} + 2\sqrt{x} + C$

1634 $\int \frac{\sqrt{x} - 2\sqrt[3]{x^2} + 1}{\sqrt[4]{x}} dx = \int \left(\frac{\sqrt{x}}{\sqrt[4]{x}} - 2\frac{\sqrt[3]{x^2}}{\sqrt[4]{x}} + \frac{1}{\sqrt[4]{x}}\right) dx =$

$= \int (x^{\frac{1}{4}} - 2x^{\frac{5}{12}} + x^{-\frac{1}{4}}) dx =$

$= \frac{4x^{\frac{5}{4}}}{5} - 2\frac{12x^{\frac{17}{12}}}{17} + \frac{4x^{\frac{3}{4}}}{3} + C =$

$= \frac{4}{5} x \sqrt[4]{x} - \frac{24}{17} x \sqrt[12]{x^5} + \frac{4}{3}\sqrt[4]{x^3} + C$

$\underline{1635}$ $\int \frac{(1-x)^3}{x\sqrt[3]{x}}dx = \int \frac{(1-x)^2(1-x)}{x^{\frac{4}{3}}}dx =$

$= \int \frac{(1-2x+x^2)(1-x)}{x^{\frac{4}{3}}}dx = \int \frac{1-x-2x+2x^2+x^2-x^3}{x^{\frac{4}{3}}}dx =$

$= \int \frac{-x^3+3x^2-3x+1}{x^{\frac{4}{3}}}dx = \int (-x^{3-\frac{4}{3}} + 3x^{2-\frac{4}{3}} -$

$- 3x^{1-\frac{4}{3}} + 1 \cdot x^{-\frac{4}{3}})dx = \int (-x^{\frac{5}{3}} + 3x^{\frac{2}{3}} -$

$- 3x^{-\frac{1}{3}} + x^{-\frac{4}{3}})dx = -\frac{3x^{\frac{8}{3}}}{8} + 3\frac{3x^{\frac{5}{3}}}{5} -$

$- 3\frac{3x^{\frac{2}{3}}}{2} + -\frac{3x^{-\frac{1}{3}}}{1} + C =$

$\underline{1636}$ $\int \left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}\,dx = \int (x^{\frac{3}{4}} - x^{\frac{3}{4}-2})dx =$

$\sqrt[2]{x^{\frac{3}{2}}} = x^{\frac{3}{4}}$

$= \int (x^{\frac{3}{4}} - x^{-\frac{5}{4}})dx = \frac{4x^{\frac{7}{4}}}{7} + \frac{4x^{-\frac{1}{4}}}{1} + C =$

$= \frac{4}{7}x\sqrt[4]{x^3} + \frac{4}{\sqrt[4]{x}} + C$

$\underline{1637}$ $\int \frac{(\sqrt[2]{2x} - \sqrt[3]{3x})^2}{x}dx = \int \frac{(2x - 2\sqrt[6]{72x^5} + \sqrt[3]{3x}^2)}{x}dx =$

$= \int (2 - 2\sqrt[6]{72}x^{-\frac{1}{6}} + \sqrt[3]{9}x^{-\frac{1}{3}})dx = 2x -$

$- 2\sqrt[6]{72} \cdot \frac{6}{5} \cdot \frac{x^{\frac{5}{6}}}{1} + \sqrt[3]{9} \cdot \frac{3}{2}x^{\frac{2}{3}} + C =$

$= 2x - \frac{12}{5}\sqrt[6]{72}\,x^{\frac{5}{6}} + \sqrt[3]{9} \cdot \frac{3}{2} \cdot x^{\frac{2}{3}} + C$

1638

$$\int \frac{\sqrt{x^4+x^{-4}+2}}{x^3}\,dx = \int \frac{\sqrt{\left(x^2+\frac{1}{x^2}\right)^2}}{x^3}\,dx =$$

$$= \int \frac{x^4+1}{x^5}\,dx = \int\left(\frac{1}{x}+\frac{1}{x^5}\right)dx = \ln|x| - \frac{1}{4x^4}+C$$

1639

$$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + C$$

$$\int \frac{x^2}{1+x^2}\,dx = \int \frac{x^2+1-1}{1+x^2}\,dx =$$

$$= \int\left(1-\frac{1}{1+x^2}\right)dx = x - \operatorname{arctg} x + C$$

1640

$$\int \frac{x^2}{1-x^2}\,dx = \int \frac{-(1-x^2-1)}{1-x^2}\,dx =$$

$$= \int\left(-1+\frac{1}{1-x^2}\right)dx = -x+\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|+C$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C$$

1641

$$\int \frac{x^2+3}{x^2-1}\,dx = \int \frac{x^2-1+4}{x^2-1}\,dx = \int\left(1+\frac{4}{x^2-1}\right)dx =$$

$$= x + 4\int\frac{1}{x^2-1}\,dx = x - 4\int\frac{1}{1-x^2}\,dx = x - 4\cdot\frac{1}{2\cdot 1}\cdot\ln\left|\frac{1+x}{1-x}\right|+C$$

$$= x + 2\ln\left|\frac{1-x}{1+x}\right|+C$$

$$\longrightarrow 2\ln\left|\frac{x-1}{x+1}\right| \ ?$$

1642

$$\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int\left(\frac{1}{\sqrt{1-x^2}}+\right.$$

$$\left.+\frac{1}{\sqrt{1+x^2}}\right)dx = \arcsin x + \ln\left|x+\sqrt{x^2+1}\right|+C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C;$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln\left|x+\sqrt{x^2\pm a^2}\right|+C$$

1643 $$\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx = \int\left(\frac{1}{\sqrt{x^2-1}} - \frac{1}{\sqrt{x^2+1}}\right)dx = \ln\left|x+\sqrt{x^2-1}\right| - \ln\left|x+\sqrt{x^2+1}\right|+C =$$
$$= \ln\left|\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2+1}}\right|+C$$

$$\int\frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C; \quad \int \frac{1}{1+x^2} = \operatorname{arctg} x + C$$

$$\int\frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln\left|x+\sqrt{x^2\pm a^2}\right|+C$$

1644 $$\int(2^x+3^x)^2\,dx = \int(2^x+3^x)(2^x+3^x)\,dx =$$
$$= \int(4^x+2\cdot 6^x+9^x)\,dx = \frac{4^x}{\ln 4}+2\frac{6^x}{\ln 6}+\frac{9^x}{\ln 9}+C$$

$$\int a^x\,dx = \frac{a^x}{\ln a}+C$$

1645 $$\int\frac{2^{x+1}-5^{x-1}}{10^x}\,dx = \int\left(2\left(\frac{2}{10}\right)^x - \frac{1}{5}\left(\frac{5}{10}\right)^x\right)dx =$$
$$= \int\left(2\left(\frac{1}{5}\right)^x-\frac{1}{5}\left(\frac{1}{2}\right)^x\right)dx = 2\frac{\left(\frac{1}{5}\right)^x}{\ln\frac{1}{5}} - \frac{1}{5}\frac{\left(\frac{1}{2}\right)^x}{\ln\frac{1}{2}}+C =$$
$$= -2\frac{\left(\frac{1}{5}\right)^x}{\ln 5}+\frac{1}{5}\frac{\left(\frac{1}{2}\right)^x}{\ln 2}+C$$

Почему не $\ln\frac{1}{5}$ и $\ln\frac{1}{2}$, а $\ln 5$ и $\ln 2$?

Ответ:

$$-\frac{2}{\ln 5}\left(\frac{1}{5}\right)^x+\frac{1}{5\ln 2}\left(\frac{1}{2}\right)^x+C$$

1646 $\int \frac{e^{3x}+1}{e^x+1}\,dx = \int \frac{(e^x+1)(e^{2x}-e^x+1)}{e^x+1}\,dx =$

$a^3+b^3=(a+b)(a^2-ab+b^2)$; $\int e^{ax}\,dx = \frac{1}{a}e^{ax}+C$

$= \int (e^{2x}-e^x+1)\,dx = \frac{1}{2}e^{2x} - e^x + x + C$

1647 $\int (1+\sin x+\cos x)\,dx =$

$\int \sin x\,dx = -\cos x + C$; $\int \cos x\,dx = \sin x + C$

$= x - \cos x + \sin x + C$

1648 $\int \sqrt{1-\sin 2x}\,dx$, $0 \le x \le \pi$ $= \int \sqrt{\sin^2 x+\cos^2 x-\sin 2x}\,dx =$

$= \int \sqrt{\sin^2 x - 2\sin x\cos x + \cos^2 x}\,dx = \int \sqrt{(\sin x-\cos x)^2}\,dx =$

$= \int (\sin x - \cos x)\,dx = -\cos x - \sin x + C$

1649 $\int ctg^2 x\,dx = \int \left(\frac{1}{\sin^2 t} - 1\right)dx =$

$1+tg^2 t = \frac{1}{\cos^2 t}$; $= -ctg\,x - x + C$

$1+ctg^2 t = \frac{1}{\sin^2 t}$;

$tg\,t \cdot ctg\,t = 1$;

$\int \frac{1}{\sin^2 x}\,dx = -ctg\,x + C$;

$\int \frac{1}{\cos^2 x}\,dx = tg\,x + C$

1650 $\int tg^2 x\,dx =$

$= \int \left(\frac{1}{\cos^2 t} - 1\right)dx =$

$= tg\,x - x + C$

1651 $\int (a\,\mathrm{sh}\,x + b\,\mathrm{ch}\,x)\,dx = a\,\mathrm{ch}\,x + b\,\mathrm{sh}\,x + C$

$\int \mathrm{sh}\,x\,dx = \mathrm{ch}\,x + C;$

$\int \mathrm{ch}\,x\,dx = \mathrm{sh}\,x + C$

1652 $\int \mathrm{th}^2 x\,dx = \int \left(1 - \frac{1}{\mathrm{ch}^2 x}\right) dx = x - \mathrm{th}\,x + C$

$\mathrm{th}^2 x = 1 - \frac{1}{\mathrm{ch}^2 x}$

$\int \frac{1}{\mathrm{ch}^2 x} = \mathrm{th}\,x + C$

1653 $\int \mathrm{cth}^2 x\,dx = \int \left(\frac{1}{\mathrm{sh}^2 x} + 1\right) dx = -\mathrm{cth}\,x + x + C$

$\mathrm{cth}^2 x = \frac{1}{\mathrm{sh}^2 x} + 1$

$\int \frac{1}{\mathrm{sh}^2 x} = -\mathrm{cth}\,x + C$

1655 $\int \frac{1}{x+a}\,dx = \left\{\begin{array}{l} t = x+a \\ dx = \frac{1}{t'}\,dt = 1dt \end{array}\right\} =$

$= \int \frac{1}{t}\,dt = \ln|t| + C = \ln|x+a| + C$

1656 $\int (2x-3)^{10}\,dx = \left\{\begin{array}{l} t = 2x-3 \\ dx = \frac{1}{t'}\,dt = \frac{1}{2}\,dt \end{array}\right\} =$

$= \frac{1}{2}\int t^{10}\,dt = \frac{1}{2}\cdot\frac{t^{11}}{11} + C = \frac{1}{22}(2x-3)^{11} + C$

1657 $\int \sqrt[3]{1-3x}\,dx = \left\{\begin{array}{l} t = 1-3x \\ dx = \frac{1}{t'}\,dt = -\frac{1}{3}\,dt \end{array}\right\} =$

$= -\frac{1}{3}\int t^{\frac{1}{3}}\,dt = -\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}\,t^{\frac{4}{3}}}{4} + C = -\frac{(3x-1)^{\frac{4}{3}}}{4} + C$

1658

$$\int \frac{1}{\sqrt{2-5x}}\,dx = \left\{ \begin{array}{l} t = 2-5x \\ dx = \frac{1}{t'}\,dt = -\frac{1}{5}\,dt \end{array} \right\} =$$

$$= -\frac{1}{5}\int \frac{1}{\sqrt{t}}\,dt = -\frac{1}{5}\int t^{-\frac{1}{2}}\,dt = -\frac{1}{5}\cdot 2\,\frac{t^{\frac{1}{2}}}{1} + C$$

$$= -\frac{2}{5}\sqrt{t} + C = \frac{2}{5}\sqrt{5x-2} + C$$

1659

$$\int \frac{1}{(5x-2)^{\frac{5}{2}}}\,dx = \int (5x-2)^{-\frac{5}{2}}\,dx =$$

$$= \left\{ \begin{array}{l} t = 5x-2 \\ dx = \frac{1}{t'}\,dt = \frac{1}{5}\,dt \end{array} \right\} = \frac{1}{5}\int t^{-\frac{5}{2}}\,dt =$$

$$= \frac{1}{5}\cdot\left(-\frac{2\,t^{-\frac{3}{2}}}{3}\right) + C = -\frac{2}{15}\cdot\frac{1}{t\sqrt{t}} + C =$$

$$= -\frac{2}{15}\cdot\frac{1}{(5x-2)\sqrt{5x-2}} + C$$

1660

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = \;?\; \quad -3 \text{ или } -\frac{3}{5}$$

$$= \int \frac{\sqrt[5]{(1-x)^2}}{1-x}\,dx = \int \frac{(1-x)^2}{(1-x)^5}\,dx =$$

$$= \int (1-x)^{-3}\,dx = -\frac{(1-x)^{-2}}{-2} + C =$$

$$= \frac{1}{2(1-x)^2} + C$$

$$\int u\,dv = uv - \int v\,du$$

2244

$$\int_0^{\sqrt{3}} x \operatorname{arctg} x\,dx = \left| \begin{array}{l} u = \operatorname{arctg} x, \\ du = u'dx = \frac{1}{1+x^2}\,dx \\ dv = x\,dx \\ v = \frac{x^2}{2} \end{array} \right| =$$

2244
2245
2248
2268
2272
2274
2278

$$= (\operatorname{arctg} x)\cdot\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\cdot\frac{1}{1+x^2}\,dx =$$

$$= \frac{x^2 \operatorname{arctg} x}{2} - \frac{1}{2}\int x^2\cdot\frac{1}{1+x^2}\,dx =$$

$$= \frac{x^2 \operatorname{arctg} x}{2} - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx =$$

$$= \frac{x^2 \operatorname{arctg} x}{2} - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right)dx =$$

$$= \frac{x^2 \operatorname{arctg} x}{2} - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg} x + C =$$

$$= \frac{1}{2}\left(x^2 \operatorname{arctg} x - x + \operatorname{arctg} x\right) + C =$$

$$= \frac{1}{2}\left((\sqrt{3})^2 \operatorname{arctg}\sqrt{3} - \sqrt{3} + \operatorname{arctg}\sqrt{3}\right) -$$

$$- \frac{1}{2}\left(0 - 0 + \underbrace{\operatorname{arctg} 0}_{0}\right) = \frac{1}{2}\left(3\cdot\frac{\pi}{3} -\right.$$

$$\left.- \sqrt{3} + \frac{\pi}{3}\right) - \frac{1}{2}\cdot 0 = \frac{1}{2}\left(\pi - \sqrt{3} + \frac{\pi}{3}\right) =$$

$$= \frac{1}{2}\left(\frac{4}{3}\pi - \sqrt{3}\right) = \frac{2}{3}\pi - \frac{\sqrt{3}}{2}$$

2245

$$\int_{-1}^{1} \frac{x}{\sqrt{5-4x}}\,dx = \left| \begin{aligned} u &= x \\ du &= u'dx = 1 \cdot dx \\ dv &= \frac{1}{\sqrt{5-4x}}\,dx \\ v &= -\frac{\sqrt{5-4x}}{2} \end{aligned} \right| =$$

$$\int u\,dv = uv - \int v\,du$$

$$= -x\frac{\sqrt{5-4x}}{2} - \int -\frac{\sqrt{5-4x}}{2}\,dx = -x\frac{\sqrt{5-4x}}{2} -$$

$$\left(-\frac{1}{4}\right)\cdot$$

$$+ \frac{1}{2}\int \sqrt{5-4x}\,dx = -\frac{1}{2}x\sqrt{5-4x} + \frac{1}{2}\cdot\frac{2}{3}(5-4x)^{\frac{3}{2}} + C$$

$$= -\frac{1}{2}x\sqrt{5-4x} - \frac{1}{12}\sqrt[2]{(5-4x)^3} + C =$$

$$\int \frac{1}{\sqrt{5-4x}}\,dx = \left| \begin{aligned} t &= 5-4x \\ dx &= \frac{1}{t'}dt = -\frac{1}{4}dt \end{aligned} \right| =$$

$$= -\frac{1}{4}\int \frac{1}{\sqrt{t}}\,dt = -\frac{1}{4}\left(\frac{2t^{\frac{1}{2}}}{1}\right) + C =$$

$t^{-\frac{1}{2}}$

$$= -\frac{1}{2}\sqrt{t} + C \quad \left| \int \sqrt{t} = \int t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} = \frac{2}{3}(5-4x)^{\frac{3}{2}} \right.$$

$$= \left(-\frac{1}{2}\cdot 1\sqrt{5-4} + \frac{1}{12}\sqrt[2]{(5-4)^3}\right) -$$

$$- \left(-\frac{1}{2}(-1)\sqrt{5+4} - \frac{1}{12}\sqrt[2]{(5+4)^3}\right) =$$

$$= -\frac{1}{2} - \frac{1}{12} - \frac{3}{2} + \frac{27}{12} = \frac{-6-1-18+27}{12} =$$

$$= \frac{2}{12} = \frac{1}{6}$$

22 48 $\int_0^{\ln 2} \sqrt{e^x - 1}\, dx = \left\{ \begin{array}{l} u = \sqrt{e^x - 1} \\ du = u' dx = \frac{e^x}{2\sqrt{e^x - 1}} dx \\ dv = dx \\ v = x \end{array} \right\} =$

$\boxed{\int u\,dv = uv - \int v\,du}$

$= \sqrt{e^x - 1}\, x - \int x \frac{e^x}{2\sqrt{e^x - 1}}\, dx =$

$= \sqrt{e^x - 1}\, x -$