Семинар №4. Мощности

$|A| = |B| \Leftrightarrow f: A \to B$, f – биективная функция

РАВНОМОЩНЫЕ МНОЖЕСТВА

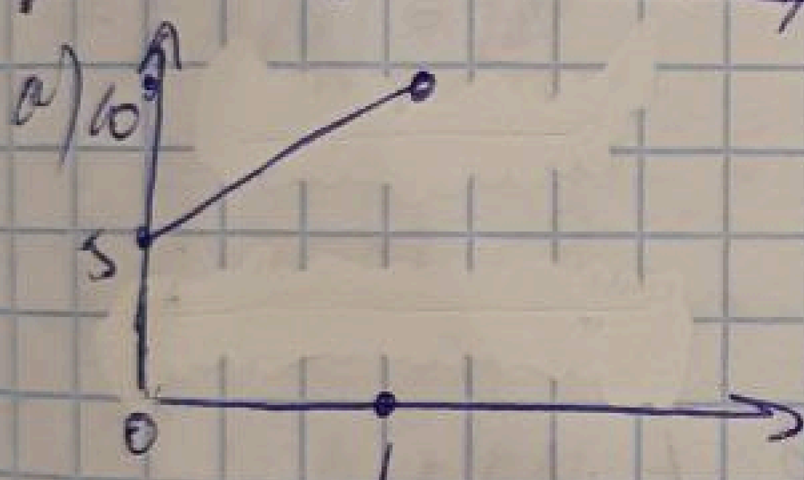
$|A| \leq |B| \Leftrightarrow f: A \to B$, f – инъективная ф.

$|\mathbb{N}|$ $\{a_0, a_1, a_2, a_3 \dots, a_n \dots\}$

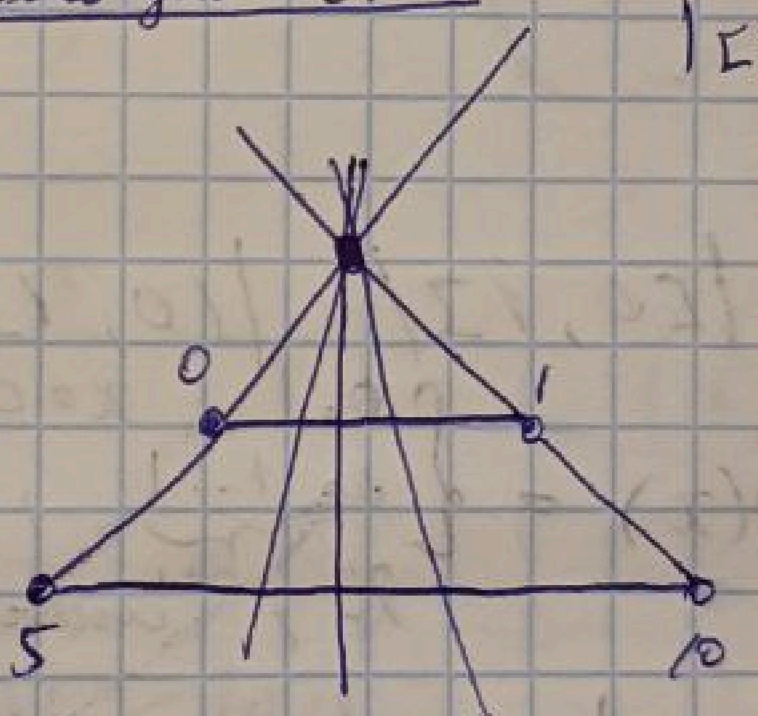
$|\mathbb{R}| = |2^{\mathbb{N}}| = |P(\mathbb{N})| \neq |\mathbb{N}|$

Как доказать равномощность!

а)



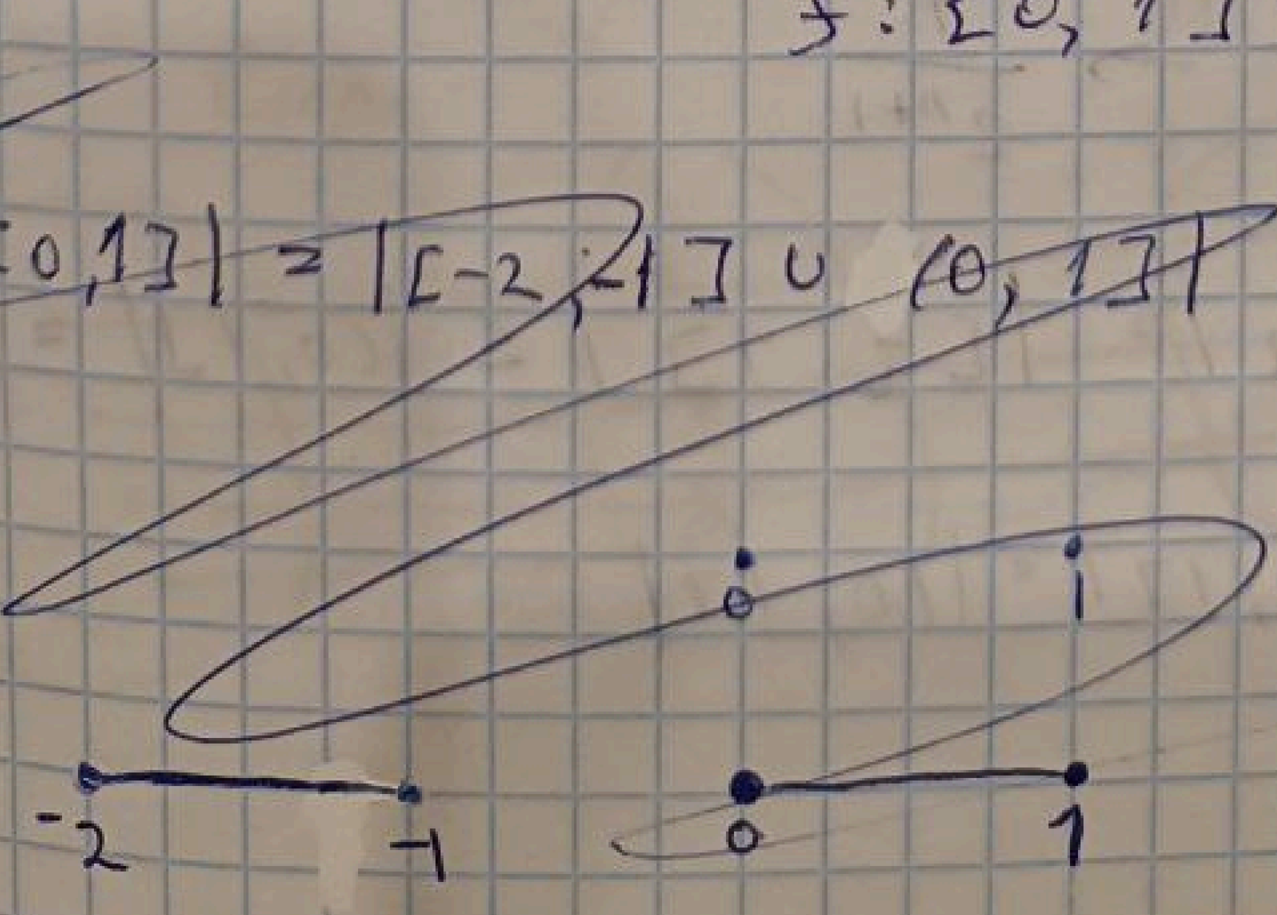
$|[0,1]| = |[5,10]|$



$f(x) = 5x + 5$

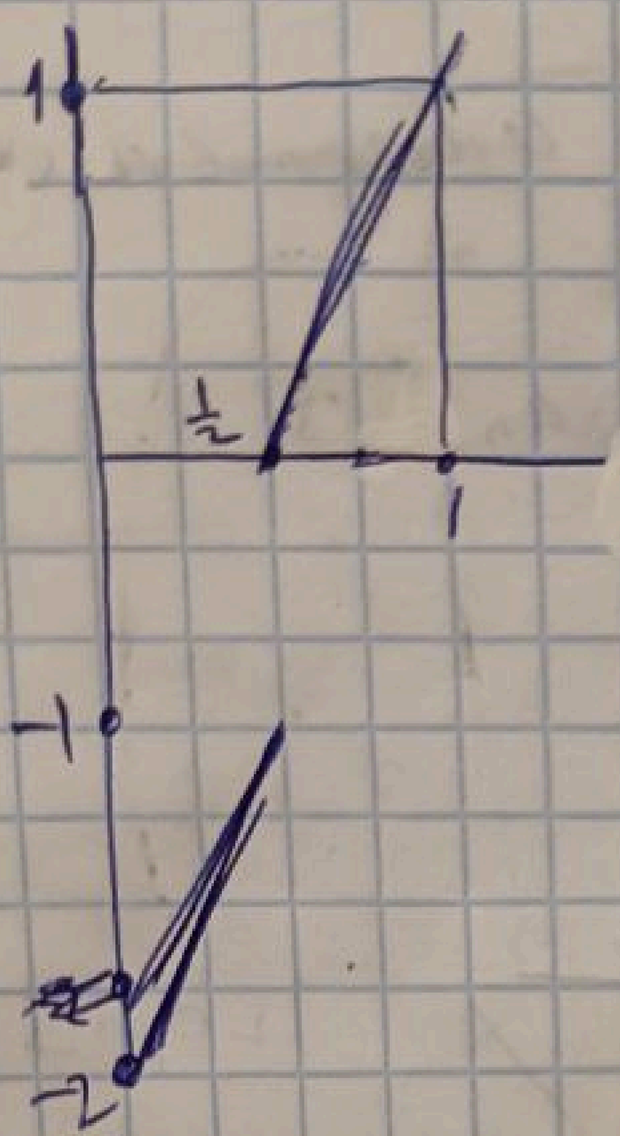
$f: [0;1] \to [5;10]$

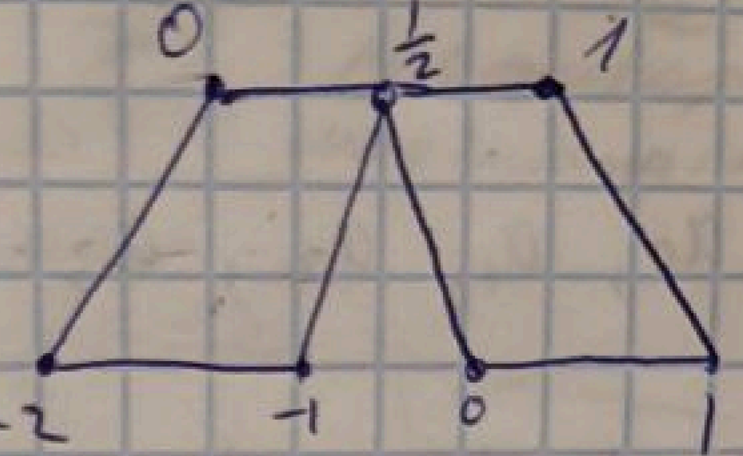
б) $|[0,1]| = |[-2,-1] \cup (0,1]|$



б) $|[0, 1]| = |[-2, -1] \cup (0, 1]|$

$$f(x) = \begin{cases} 2x-2, & x \in [0, \frac{1}{2}] \\ 2x-1, & x \in (\frac{1}{2}, 0] \end{cases}$$





в) $|[0, 1]| = |(0, 1]|$

$$f(x) = \begin{cases} 0, 1, & x = 0 \\ 0, \underbrace{1 \ldots 1}_{n+1}, & x = 0, \underbrace{1 \ldots 1}_{n} \\ x, & \text{иначе} \end{cases}$$

$$\ldots \frac{1}{n} \to \frac{1}{n+1} \ldots$$

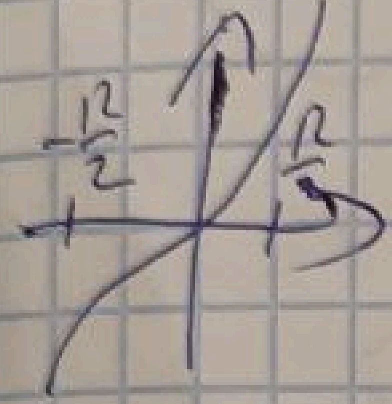
$$\ldots \frac{1}{2^n} \to \frac{1}{2^{n+1}}$$

$$|[0, 1]| = |[\tfrac{1}{2}, 1]| \leq |(0, 1]| \leq |[0, 1]| \Rightarrow$$

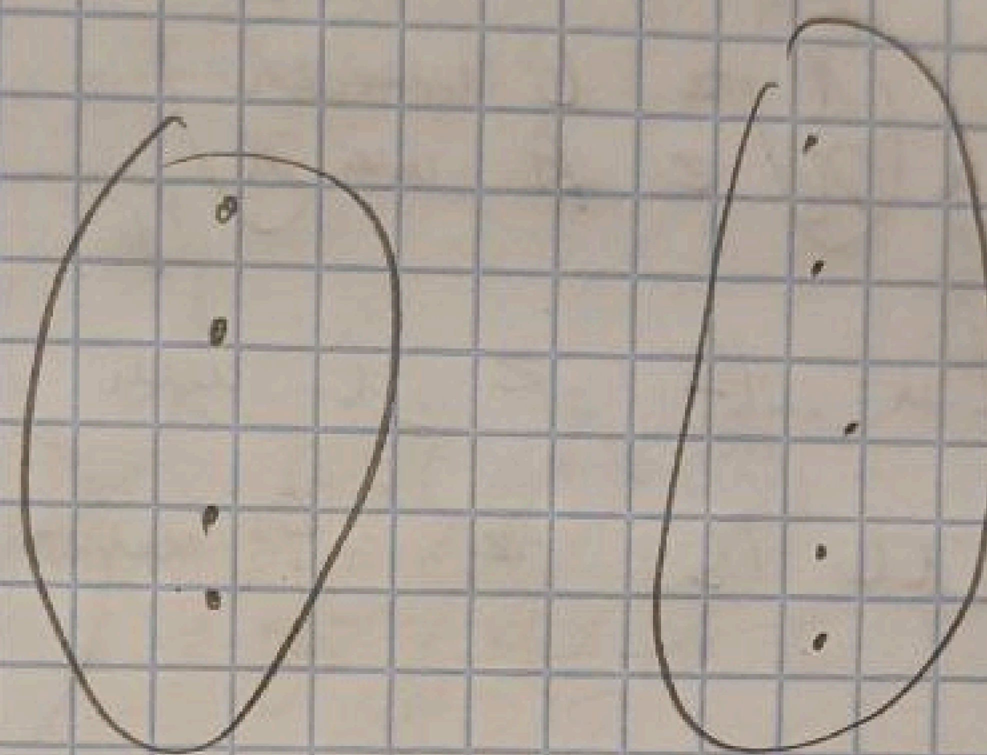
$$\Rightarrow |[0, 1]| = |(0, 1]|$$

г)

$$|(-\tfrac{\pi}{2};\ \tfrac{\pi}{2})| = |\mathbb{R}|$$

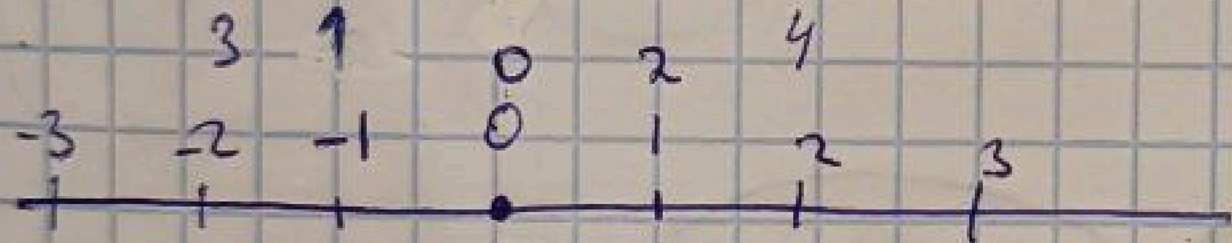


tg x



Верно ли?

а) $|\mathbb{N}| = |\mathbb{Z}|$



$(i, j) = 2^i 3^j$

$(-1)^n \left[\frac{n+1}{2}\right]$

$|\mathbb{N}| \le |\mathbb{Z}| \le |\mathbb{N} \times \mathbb{N}| \le |\mathbb{N}|$