№1 Доказать

б)

$\neg\varphi \vdash \exists x\neg\varphi$

$\neg\exists x\neg\varphi, \neg\varphi \vdash \Lambda$

$\neg\exists x\neg\varphi \vdash \varphi$

13 ---

$\neg\exists x\neg\varphi \vdash \forall x\varphi$

12 ---

$\neg\forall x\varphi, \neg\exists x\neg\varphi \vdash \forall x\varphi$

10 ---

$\neg\forall x\varphi, \neg\exists x\neg\varphi \vdash \Lambda$

9 ---

$\neg\forall x\varphi \vdash \exists x\neg\varphi$

$\neg\varphi \vdash (\neg\varphi)^x_x$

15 ---

$\neg\forall x\varphi \vdash \neg\forall x\varphi$

12 ---

$\forall x\varphi, \neg\forall x\varphi, \neg\exists x\neg\varphi$

x

$\neg\forall x\varphi$

$$\dfrac{\psi \vdash \varphi}{\neg\varphi, \psi \vdash \varphi,} \qquad \dfrac{\neg\varphi \vdash \neg\varphi}{\neg\varphi, \psi \vdash \neg\varphi}$$

$\neg\varphi, \psi \vdash \Lambda$

$$\dfrac{\psi \vdash \varphi}{\neg\varphi, \psi \vdash \Lambda} \qquad \dfrac{\psi, \neg\varphi \vdash \Lambda}{\psi \vdash \varphi}$$

б) обр.

$$\varphi \vdash \varphi$$

$$\neg\varphi, \varphi \vdash \Lambda$$

$$\neg\varphi, \varphi^x_x \vdash \Lambda$$

$$\forall x\varphi, \neg\varphi \vdash \Lambda$$

11, 16

$$\exists x\neg\varphi, \forall x\varphi \vdash \Lambda$$

9

$$\exists x\neg\varphi \vdash \neg\forall x\varphi$$

g)

$$\varphi \vdash \varphi \quad 4$$

$$\varphi \vdash \varphi \vee \psi \quad 15$$

$$\exists x\varphi \vdash \exists(\varphi \vee \psi) \quad 16$$

$$\psi \vdash \psi \quad 5$$

$$\psi \vdash \varphi \vee \psi$$

15

$$\psi \vdash \exists x(\varphi \vee \psi)$$

$$\exists x\varphi \vee \psi, \exists x\varphi \vdash \exists x(\varphi \vee \psi); \exists\varphi \vee \psi, \psi \vdash \exists x(\varphi \vee \psi);$$

6

$$\exists x\varphi \vee \psi \vdash \exists x(\varphi \vee \psi)$$

$$\exists x\varphi \vee \psi \vdash \exists x\varphi \vee \psi$$

д) обр.

$$\dfrac{\dfrac{\dfrac{15 \quad \varphi \vdash \varphi}{\varphi \vdash \exists x \varphi}\,4}{\varphi \vdash \exists x \varphi \vee \psi} \; ; \quad \dfrac{\psi \vdash \psi}{\psi \vdash \exists x \varphi \vee \psi}\,5}{\varphi \vee \psi, \varphi \vdash \exists x \varphi \vee \psi ; \; \varphi \vee \psi, \psi \vdash \exists x \varphi \vee \psi ; \; \varphi \vee \psi \vdash \varphi \vee \psi}\,11,12$$

$$6 \; \dfrac{\varphi \vee \psi, \varphi \vdash \exists x \varphi \vee \psi ; \; \varphi \vee \psi, \psi \vdash \exists x \varphi \vee \psi ; \; \varphi \vee \psi \vdash \varphi \vee \psi}{\varphi \vee \psi \vdash \exists x \varphi \vee \psi}$$

$$16 \; \dfrac{\varphi \vee \psi \vdash \exists x \varphi \vee \psi}{\exists x (\varphi \vee \psi)}$$

№2 ПНФ (пренексная нормальная форма)

а) $\neg \forall x \exists y \forall z \forall t (P(x,y) \wedge (Q(y) \to P(x,z))) \equiv$

$\equiv \neg \forall x \exists y \forall z \forall t (P(x,y) \wedge (\neg Q(y) \vee P(x,z))) \equiv \exists x \forall y \exists z \exists t \neg (P(x,y) \wedge (\neg Q(y) \vee P(x,z))) \equiv \underbrace{\exists x \forall y \exists z \exists t (\neg P(x,y) \vee (Q(y) \wedge \neg P(x,z)))}_{\text{ПНФ}}$

б) $\neg\forall x \exists y\, (P(x,y) \to P(y,z)) \wedge \forall x \exists z\, (Q(x) \to$

$\to P(z,x)) \equiv \neg\forall x \exists y\, (\neg P(x,y) \vee P(y,z)) \wedge$

$\wedge \forall x \exists z\, (\neg Q(x) \vee P(z,x)) \equiv$

$\equiv \exists x \forall y\, (P(x,y) \wedge \neg P(y,z)) \wedge \forall x \exists z\, (\neg Q(x) \vee$

$\vee P(z,x)) \equiv \exists x \forall y\, (P(x,y) \wedge \neg P(y,z)) \wedge$

~~$\wedge \forall x \exists z$~~ $\wedge \forall v \exists w\, (\neg Q(v) \vee P(w,v)) \equiv$

$\equiv$

Доп

$\sigma = (P, F, \mu)$

сигнатура

множество переменных

- $T(\sigma)$ – терм, $x \in V \to x \in T(\sigma)$
- $f^n \in F,\ t_1, \dots t_n \in T(\sigma) \Rightarrow f(t_1, \dots t_n) \in T(\sigma)$

Самые простые термы: $0, 1, x, y$

Постфиксная запись: $+(1, 0)$ $\quad +(1, +(x,y))$

Инфиксная запись: $1+0$ $\quad 1+(x+y)$

$\forall y((y+y=0) \Rightarrow (y=0))$

Формулы: $0=0,\ 1=0,\ x=(x+1)\quad x=y+(x\cdot z)$

$(1=0) \wedge (x+y=y+x) \quad \exists x(x=1)$