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1631. $\int \left(\frac{1-x}{x}\right)^2 dx =$

$\int x^m dx = \frac{x^{m+1}}{m+1} + C$

$\int \frac{dx}{x} = \ln|x| + C$

$= \int \frac{x^2 - 2x + 1}{x^2} dx =$

$= \int 1\, dx - \int \frac{2}{x^{x}} + \int \frac{1}{x^2} dx =$

$= x - 2\ln|x| - \frac{1}{x} + C$

1639. $\int \frac{x^2 dx}{1+x^2} = \int \frac{x^2+1-1}{1+x^2} dx =$

$= \int 1 dx + \int - \frac{1}{1+x^2} dx =$

$= x - \operatorname{arctg}|x| + C$

1650. $\int tg^2 x\, dx = \int \frac{\sin^2 x}{\cos^2 x} dx =$

$= \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx - \int 1 dx =$

$= tg\, x - x + C$

1655. $\int \frac{dx}{x+a} = \left\{ \begin{array}{l} x+a=t \\ x=t-a \\ dx=dt \end{array} \right\} = \int \frac{dt}{t} =$

$= \ln|t| + C = \ln|x+a| + C$

1656. $\int (2x-3)^{10} dx = \left\{ \begin{array}{l} 2x-3=t \\ x=\frac{t+3}{2} \\ dx=\frac{1}{2}dt \end{array} \right\} =$

$\int \frac{1}{2} t^{10} dt = \frac{1}{22} t^{11} + C$

$= \frac{1}{2} \cdot \frac{1}{10+1} \cdot (2x-3)^{10+1} + C =$

$= \frac{1}{2} \cdot \frac{1}{11} \cdot (2x-3)^{11} + C = \frac{(2x-3)^{11}}{22} + C$

ТРИГОНОМЕТРИЯ
(понижение степени)

1646. $\int \frac{e^{3x}+1}{e^x+1} dx = \left\{ \begin{array}{l} e^x = t \\ dt = e^x dx \\ dx = \frac{dt}{e^x} = \frac{dt}{t} \\ e^{3x} = t^3 \end{array} \right\} =$

$= \int \frac{t^3+1}{t+1} \frac{dt}{t} = \int \frac{(t+1)(t^2-t+1)}{t+1} \frac{dt}{t} =$

$= \int (t^2 - t + 1) \frac{dt}{t} =$

1668.

$$\int \frac{dx}{1+\cos x} = \int \frac{dx}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = tg\,\frac{x}{2} + C$$

1674.

$$\int \frac{x\,dx}{\sqrt{1-x^2}} = \left\{ \begin{array}{l} 1-x^2 = t \\ x = \sqrt{1-t} \\ dx = -\frac{dt}{2\sqrt{1-t}} \end{array} \right\} =$$

$$= \int \frac{\sqrt{1-t}}{\sqrt{t}} \cdot \left(-\frac{dt}{2\sqrt{1-t}}\right) =$$

$$= \int -\frac{dt}{2\sqrt{t}} = \int -\frac{1}{2}\cdot\frac{dt}{\sqrt{t}} =$$

$$= \int -\frac{t^{-\frac{1}{2}}}{2}\cdot dt = \frac{2}{2}\sqrt{t} + C =$$

$$= \sqrt{t} + C = \sqrt{1-x^2} + C$$

$$\frac{dy}{y} = d\ln|y|$$

1680. $\int \frac{dx}{(1+x)\sqrt{x}} = \left\{ \begin{array}{l} t = \sqrt{x} \\ dx = 2\sqrt{x}\, dt \end{array} \right\} =$

$= \int \frac{2dt}{1+t^2} = 2 \operatorname{arctg} \sqrt{x} + C$

1691. $\int \frac{dx}{e^x + e^{-x}} = \left\{ \begin{array}{l} u = e^x \\ du = e^x dx \end{array} \right\} =$

$dx = \frac{du}{e^x} = \frac{du}{u}$

$= \int \frac{1}{u + \frac{1}{u}} \frac{du}{u} = \int \frac{1}{(u^2+1)} du =$

$= \operatorname{arctg}(u) + C = \operatorname{arctg}(e^x) + C$

1692. $\int \frac{dx}{\sqrt{1+e^{2x}}} = \left\{ \begin{array}{l} t = e^{-x} \\ dt = -dx \cdot e^{-x} \\ dx = -\frac{dt}{e^x} = -\frac{dt}{t} \end{array} \right\} =$

$= \int \frac{dt}{\sqrt{1+\frac{1}{t^2}} \cdot t} = \int \frac{dt}{\sqrt{t^2+1}} = -\ln |e^{-x} + \sqrt{e^{-2x}+1}| + C$

$\int \frac{dx}{\sqrt{1+e^{2x}}} = \frac{dx}{\sqrt{e^{2x}(e^{-2x}+1)}} = \int \frac{dx}{e^x \sqrt{e^{-2x}+1}} = \int \frac{-d(e^{-x})}{\sqrt{(e^{-x})^2+1}} =$

1693.

$$\int \frac{\ln^2 x}{x}\,dx = \left| dy = d\,\frac{\ln y}{y}\cdot y \right.$$

$$= \int \ln^2 x \cdot d(\ln x) = \frac{\ln^3 x}{3} + C$$

$$\left/ \begin{aligned} t &= \ln x \\ x &= e^t \\ dx &= e^t dt \end{aligned} \right/ = \int \frac{t^2 e^t\,dt}{e^t} =$$

$$= \int t^2\,dt$$

1702.

$$\int \frac{dx}{\sin^2 x + 2\cos^2 x} =$$

$$= \int \frac{dx}{\cos^2 x\,(2 + tg^2 x)} = \int \frac{1}{2 + tg^2 x}\,d\,tg\,x =$$

$=$

$$\frac{1}{2}dx = d\left(\frac{x}{2}\right)$$

$$x\,dx = \frac{1}{2}\,dx^2$$

$$\frac{dy}{y} = d\ln|y|$$

$$\frac{dx}{\cos^2 x} = d(\mathrm{tg}\,x)$$

$$t = \frac{1}{(x+1)^2} \qquad dt = -\frac{dx}{x+1}$$

N1642

$$\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx + \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx =$$

$$= \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1+x^2}}\,dx =$$

$$= \arcsin x + C + \ln\left|x+\sqrt{x^2+1}\right|$$

N1660

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = \int \frac{\sqrt[5]{(1-x)^2}}{1-x}\,dx =$$

$$= \int (1-x)^{-\frac{3}{5}}\,dx = \left\{ \begin{aligned} t &= 1-x \\ x &= 1-t \\ dx &= \frac{1}{t'}dt = -1dt \end{aligned} \right\} =$$

$$= \int -t^{-\frac{3}{5}}\,dt = -\int t^{-\frac{3}{5}}\,dt = -\frac{5}{2}\cdot t^{\frac{2}{5}} + C =$$

$$= -\frac{5}{2}(1-x)^{\frac{2}{5}} + C$$

№1648 $\int\sqrt{1-\sin 2x}\,dx$, $0 \le x \le \pi$ =

$= \int\sqrt{1-2\sin x\cos x}\,dx = \int\sqrt{\sin^2 x - 2\sin x\cos x + \cos^2 x}\,dx =$

$= \int\sqrt{(\sin x - \cos x)^2}\,dx = \int(\sin x - \cos x)\,dx =$

$= \int\sin x\,dx - \int\cos x\,dx = -\cos x - \sin x + C$

№1657 $\int\sqrt[3]{1-3x}\,dx = \int(1-3x)^{\frac{1}{3}}\,dx = \left\{\begin{array}{l} t = 1-3x \\ x = \frac{1-t}{3} \\ dx = \frac{1}{t'}dt = -\frac{1}{3}dt \end{array}\right\} =$

$= \int -\frac{1}{3}t^{\frac{1}{3}}\,dt = -\frac{1}{3}\int t^{\frac{1}{3}}\,dt = -\frac{1}{3}\cdot\frac{3}{4}t^{\frac{4}{3}} + C =$

$= \frac{(1-3x)^{\frac{4}{3}}}{-4} + C = \frac{(1-3x)\sqrt[3]{1-3x}}{-4} + C$

N 1661

$$\int \frac{dx}{2+3x^2} = \int \frac{1}{3x^2+2}\,dx =$$

$$= \int \frac{1}{3(x^2+\frac{2}{3})}\,dx = \frac{1}{3}\int \frac{1}{x^2+\frac{2}{3}}\,dx =$$

$$\boxed{\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\cdot \operatorname{arctg}\left(\frac{x}{a}\right)+C}$$

$$= \frac{1}{3}\cdot\sqrt{\frac{3}{2}}\cdot \operatorname{arctg}\left(\frac{x}{\sqrt{\frac{2}{3}}}\right)+C =$$

$$= \frac{\sqrt{6}\cdot \operatorname{arctg}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}+C$$

N 1675

$$\int x^2\sqrt[3]{1+x^3}\,dx = \left\{\begin{matrix} t = 1+x^3 \\ dx = \frac{1}{t'}\,dt = \frac{1}{3x^2}\,dt\end{matrix}\right\} =$$

$$= \int x^2\sqrt[3]{1+x^3}\,\frac{1}{3x^2}\,dt = \frac{1}{3}\int (1+x^3)^{\frac{1}{3}}\,dt = \frac{1}{3}\int (t)^{\frac{1}{3}}\,dt =$$

$$= \frac{1}{3}\cdot\frac{3}{4}\cdot(1+x^3)^{\frac{4}{3}}+C = \frac{(1+x^3)\sqrt[3]{1+x^3}}{4}+C$$

N 1676

$$\int\frac{x\,dx}{3-2x^2} = \left\{\begin{matrix} t = 3-2x^2 \\ dx = \frac{1}{t'}\,dt = -\frac{1}{4}x\end{matrix}\right\} = \int\frac{x}{3-2x^2}\cdot\frac{1}{-4x}\,dt =$$

$\int\frac{1}{t}\,dt = \ln|t|+C$

$$= \int -\frac{1}{4(3-2x^2)}\,dt = -\frac{1}{4}\int\frac{1}{t}\,dt = -\frac{1}{4}\ln|3-2x^2|+C$$

№1677 $\int \frac{x\,dx}{(1+x^2)^2} = \left\{ \begin{array}{l} t = 1+x^2 \\ dx = \frac{1}{t'} dt = \frac{1}{2x} dt \end{array} \right\} =$

$= \int \frac{x}{(1+x^2)^2} \cdot \frac{1}{2x} dt = \frac{1}{2} \int \frac{1}{t^2} dt =$

$= \frac{1}{2} \int t^{-2} dt = \frac{1}{2} \cdot \frac{t^{-1}}{-1} + C = -\frac{1}{2(1+x^2)} + C$

№1678 $\int \frac{x\,dx}{4+x^4} = \left\{ \begin{array}{l} t = x^2 \\ dx = \frac{1}{t'} dt = \frac{1}{2x} dt \end{array} \right\} =$

$= \int \frac{x}{4+x^4} \cdot \frac{1}{2x} dt = \frac{1}{2} \int \frac{1}{4+t^2} dt =$

$= \left(\frac{1}{2}\right)^2 \cdot \operatorname{arctg} \frac{t}{2} + C = \frac{\operatorname{arctg}\left(\frac{x^2}{2}\right)}{4} + C$

№1681 $\frac{dx}{\sqrt{x}} = 2d(\sqrt{x})$ $\quad \int u\,dv = uv - \int v\,du$

$\int \sin\frac{1}{x} \cdot \frac{dx}{x^2} = \int \sin\frac{1}{x} \cdot \frac{1}{\sqrt{x^3}} \cdot \frac{dx}{\sqrt{x}} = \int \sin\frac{1}{x} \cdot \frac{1}{\sqrt{x^3}} \cdot 2d(\sqrt{x}) =$

$= 2\int \frac{\sin\frac{1}{x}}{\sqrt{x^3}} d(\sqrt{x}) = 2\int \sin\frac{1}{\sqrt{x}^2} \cdot \frac{1}{\sqrt{x}^3} d(\sqrt{x}) =$

$= 2\left(\sin\frac{1}{x} \cdot \frac{1}{\sqrt{x^3}} \cdot \sqrt{x} - \right.$

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№1703

$$\int \frac{dx}{\sin x} = \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} =$$

$$\frac{1}{2}\frac{dx}{\cos^2\frac{x}{2}} = \frac{d\frac{x}{2}}{\cos^2\frac{x}{2}} = d\left(tg\frac{x}{2}\right)$$

$$= \int \frac{dx}{2\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\cdot\cos^2\frac{x}{2}} = \int \frac{d\left(tg\frac{x}{2}\right)}{tg\frac{x}{2}} =$$

$$= \ln\left|tg\frac{x}{2}\right| + C$$

или

$$\int \frac{1 + tg^2\frac{x}{2}}{2\,tg\frac{x}{2}} = \int ctg\frac{x}{2}\,d\frac{x}{2} +$$

$$+ \int tg\frac{x}{2}\,d\frac{x}{2} = \ln\left|\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right| + C$$

$$d(tg\,t) = \frac{1}{\cos^2 t}\cdot dt$$

$$d[f(x)] = f'(x)\cdot dx$$