Интегрирование по частям

$$\int f(x)\, g'(x)\, dx = f(x)g(x) - \int f'(x)\, g(x)\, dx$$

$$\int u\, dv = uv - \int v\, du$$

$u = f(x) \qquad du = f'(x)\, dx$

$v = g(x) \qquad dv = g'(x)\, dx$

Пример:

① $\int x e^x dx = \left| \begin{matrix} u = e^x \\ dv = x\,dx \\ du = e^x dx \\ v = \frac{x^2}{2} \end{matrix} \right| =$

$= \frac{e^x x^2}{2} - \int \frac{x^2}{2} e^x dx$

② $\int x e^x dx = \left\{ \begin{matrix} u = x, & dv = e^x dx \\ du = dx, & v = e^x \end{matrix} \right\} =$

$= xe^x - \int e^x dx = xe^x - e^x + C$

Что брать за u и за v:

① Если в подынтегральном выражении есть $\ln(\varphi(x))$, arctg, $\arcsin x$ и т.д., то их удобно взять за u.

② Если —//— $P(x)e^{\alpha x}$, $P(x)\cos \alpha x$, $P(x)\sin \alpha x$, где $P(x)$ — полином от x, то удобно $\boxed{u = P(x)}$

③ Если $=/-//-$

$e^{\alpha x}\cos\beta x$, $e^{\alpha x}\sin\beta x$,

$\cos(\ln x)$, $\sin(\ln x)$,

тогда интеграл „зацикливается".

$$\boxed{I = \ldots\ldots + \alpha I}$$

$$\Downarrow$$

$$I = \frac{\ldots\ldots}{1-\alpha}$$

1792
1796
1799
1802
1807
1826
1829

Д/з $\int$
1816
1792
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$$\int u\,dv = uv - \int v\,du$$

№1792 $\int x^n \ln x \, dx, \ (n \neq -1) =$

$$= \cancel{\ln x \cdot x^n - \int \ln x \, d x^{n-1}}$$

$$= \left\{ \begin{aligned} u &= \ln x \\ du &= \frac{1}{x}dx \\ dv &= x^n \\ v &= \frac{x^{n+1}}{n+1} \end{aligned} \right\} = \ln x \cdot \frac{x^{n+1}}{n+1} -$$

$$- \int \frac{x^{n+1}}{n+1} \cdot \frac{1}{x}\,dx = \ln x \cdot \frac{x^{n+1}}{n+1} -$$

$$- \frac{1}{n+1}\int x^n dx = \ln x \cdot \frac{x^{n+1}}{n+1} -$$

$$- \frac{x^{n+1}}{(n+1)^2} + C$$

№1799

$$\int x^2 \sin 2x \, dx = \left\{ \begin{array}{l} u = x^2 \\ du = 2x\,dx \\ dv = \sin 2x\,dx \\ v = \dfrac{\cos 2x}{-2} \end{array} \right\} =$$

$$\int u\,dv = uv - \int v\,du$$

$$= x^2 \cdot \frac{\cos 2x}{-2} - \int \frac{\cos 2x}{-2} \cdot 2x\,dx =$$

$$= x^2 \cdot \frac{\cos 2x}{-2} + \int \cos 2x \cdot x\,dx =$$

$$= \left\{ \begin{array}{l} u = x \\ du = dx \\ dv = \cos 2x\,dx \\ v = \dfrac{-\sin 2x}{-2} \end{array} \right\} = x^2 \cdot \frac{\cos 2x}{-2} +$$

$$+ x \frac{\sin 2x}{2} - \int \frac{\sin 2x}{2}\,dx =$$

$$= \frac{x}{2}(\sin 2x - x\cos 2x) - \frac{1}{2}\int \sin 2x\,dx =$$

$$= \frac{x}{2}(\sin 2x - x\cos 2x) + \frac{1}{4}\cos x + C$$

$$df(x) = f'(x)\,dx$$

№1802

$$\int \operatorname{arctg} x\,dx = \left\{ \begin{array}{l} u = \operatorname{arctg} x \\ du = \dfrac{1}{1+x^2}\,dx \\ dv = dx \\ v = x \end{array} \right\} =$$

$$= \operatorname{arctg} x \cdot x - \int x \frac{1}{1+x^2}\,dx = \left\{ \begin{array}{l} t = 1+x^2 \\ dx = \dfrac{1}{t'}\,dt = \\ = \dfrac{1}{2x}\,dt \end{array} \right\} =$$

$$= (\operatorname{arctg} x) \cdot x - \int \frac{\cancel{x}}{1+x^2} \cdot \frac{1}{2\cancel{x}} \cdot dt =$$

$$= (\operatorname{arctg} x) \cdot x - \frac{1}{2} \int \frac{dt}{t} =$$

$$= (\operatorname{arctg} x) \cdot x - \frac{1}{2} \ln|1+x^2| + C$$

№ 1807 $\int \ln(x + \sqrt{1+x^2})\, dx =$

$$= \left\{ \begin{aligned} & u = \ln(x+\sqrt{1+x^2}) \\ & du = \frac{1 + \frac{2x}{2\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\, dx = \frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\, dx \\ & dv = dx \\ & v = x \end{aligned} \right\} =$$

$$= x \ln(x + \sqrt{1+x^2}) - \int x \cdot \frac{1 + \frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\, dx =$$

$$= x \ln(x+\sqrt{1+x^2}) - \int x \cdot \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \frac{x}{\sqrt{1+x^2}}\right) dx =$$

$$= x \ln(x+\sqrt{1+x^2}) - \int x \cdot \frac{1}{x+\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\, dx =$$

$$= x \ln(x+\sqrt{1+x^2}) - \int x \cdot \frac{1}{\sqrt{1+x^2}}\, dx = \left\{ \begin{aligned} & t = \sqrt{1+x^2} \\ & dx = \frac{1}{t'}\, dt = \frac{1}{\frac{1}{2\sqrt{1+x^2}} \cdot 2x}\, dt = \end{aligned} \right\} = \frac{\sqrt{1+x^2}}{x}\, dt$$

$$= x \ln(x+\sqrt{1+x^2}) - \int x \cdot \frac{1}{\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}}{x}\, dt =$$

$$= x \ln(x+\sqrt{1+x^2}) - \int 1 \cdot dt = x \ln(x+\sqrt{1+x^2}) - t + C =$$

$$= x \ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + C$$

N 1816

$$\int \frac{x \quad 2x}{(1+x^2)^2} dx = \frac{1}{2} \int \underset{u}{x}\, d\underbrace{\left(\frac{1}{x^2+1}\right)}_{v} =$$

$$= \left\{\begin{array}{l} u = x \\ du = dx \end{array}\right\} -\left(\frac{1}{2}\right)\frac{x}{x^2+1}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

N 1826

$$\underbrace{\int \sin(\ln x)\,dx}_{A} = \left\{\begin{array}{l} u = \sin(\ln x) \\ du = \frac{\cos(\ln x)}{x}dx \\ dv = dx \\ v = x \end{array}\right\} =$$

$$= x\sin(\ln x) - \int x \frac{\cos(\ln x)}{x} dx =$$

$$= x\sin(\ln x) - \int \cos(\ln x)\,dx = \left\{\begin{array}{l} u = \cos(\ln x) \\ du = -\frac{\sin(\ln x)}{x}dx \\ dv = dx \\ v = x \end{array}\right\} =$$

$$= x\sin(\ln x) - \left(x\cos(\ln x) - \int x \left(-\frac{\sin(\ln x)}{x}\right)dx\right) =$$

$$= x\sin(\ln x) - x\cos(\ln x) + \int \sin(\ln x)\,dx$$

$$2A = x(\sin(\ln x) - \cos(\ln x)) \Rightarrow$$

$$\Rightarrow A = \frac{x(\sin(\ln x) - \cos(\ln x))}{2} + C$$

N1829

$$\overbrace{\int e^{ax}\sin bx\,dx}^{A} = \left\{\begin{array}{l} u = \sin bx \\ du = b\cos bx\,dx \\ dv = e^{ax}dx \\ v = \frac{e^{ax}}{a} \end{array}\right\} =$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$= \sin bx \cdot \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} b\cos bx\,dx =$$

$$= \frac{e^{ax}\sin bx}{a} - \frac{b}{a}\int e^{ax}\cos bx\,dx = \left\{\begin{array}{l} u = \cos bx \\ du = -b\sin bx\,dx \\ dv = e^{ax}dx \\ v = \frac{e^{ax}}{a} \end{array}\right\} =$$

$$= \frac{e^{ax}\sin bx}{a} - \frac{b}{a}\left(\frac{e^{ax}\cos bx}{a} + \int \frac{e^{ax}}{a} b\sin bx\,dx\right) =$$

$$= \frac{e^{ax}\sin bx}{a} - \frac{b}{a}\left(\frac{e^{ax}\cos bx}{a} + \frac{b}{a}\int e^{ax}\sin bx\,dx\right) =$$

$$\Rightarrow A = \frac{e^{ax}\sin bx}{a} - \frac{b}{a}\left(\frac{e^{ax}\cos bx}{a} + \frac{b}{a}A\right)$$

$$A = \frac{e^{ax}\sin bx}{a} - \frac{b\,e^{ax}\cos bx}{a^2} - \frac{b^2}{a^2}A$$

$$A\left(1 + \frac{b^2}{a^2}\right) = \frac{e^{ax}}{a}\left(\sin bx - \frac{b\cos bx}{a}\right)$$

$$A = \frac{\frac{e^{ax}}{a}\left(\sin bx - \frac{b\cos bx}{a}\right)}{1 + \frac{b^2}{a^2}}$$