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~~Найти $y^{(n)}$~~

~~$y = \frac{ax+b}{cx+d}$~~

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$$

13/9 $\lim\limits_{x\to 0} \frac{\operatorname{ch} x - \cos x}{x^2}$

Правило Лопиталя

Работает для неопределённостей $\boxed{\frac{0}{0}}$, $\frac{\infty}{\infty}$

$f(x), g(x)$ — опр. и непр. в $U_\varepsilon(a)$, $a \in \overline{\mathbb{R}}$

Если 1) $f, g \xrightarrow[x\to a]{} 0$

2) $\exists f', g'$ в $\mathring{U}_\varepsilon(a)$, $(f')^2 + (g')^2 \neq 0$

3) $\exists \lim\limits_{x\to a} \frac{f'}{g'}$ — конечный или бесконечный

Тогда: $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$

$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$

$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$

$\operatorname{ch}' x = \operatorname{sh} x$

$\operatorname{sh}' x = \operatorname{ch} x$

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$$\lim_{x\to 0} \frac{\operatorname{ch} x - \cos x}{x^2} = \lim_{x\to 0} \frac{\operatorname{sh} x + \sin x}{2x} = \lim_{x\to 0} \frac{\operatorname{ch} x + \cos x}{2} = 1$$

1336 $\lim\limits_{x\to+\infty} \frac{\ln x}{x^{e}}$ $(\varepsilon > 0)$

$$\lim_{x\to+\infty} = \frac{\frac{1}{x}}{e x^{e-1}} = \lim_{x\to+\infty} \frac{1}{e x^{e}} = 0$$

1354 $\lim\limits_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^{x}-1}\right) = \lim\limits_{x\to 0} \left(\frac{e^{x}-1-x}{x(e^{x}-1)}\right) =$

$$= \lim_{x\to 0} \frac{e^{x}-1}{e^{x}-1+x\cdot e^{x}} = \lim_{x\to 0} \frac{e^{x}}{e^{x}+e^{x}+xe^{x}} = \frac{1}{2}$$

Формула Тейлора

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^{k} + o\left((x-x_0)^{k}\right)$$

x_0 $U_{\varepsilon}(x_0)$

1376 $P(x) = 1 + 3x + 5x^{2} - 2x^{3}$

↳ Разложить по степени $(x+1)$

$x_0 = -1$

$P(x) = 5$

$P'(x) = 3 + 10x - 6x^{2}$ $3 + 10(-1) - 6(-1)^{2} = -13$

$P''(x) = 10 - 12x$ 22

$P'''(x) = -12$ -12

$P''''(x) = 0$ 0

$$P(x) = \frac{5}{0!}(x+1)^0 + \frac{-13}{1!}(x+1)^1 + \frac{22}{2!}\cdot$$

$$\cdot(x+1)^2 + \frac{-12}{3!}(x+1)^3 =$$

$$= 5 - 13(x+1) + 11(x+1)^2 - 2(x+1)^3$$

<u>1377</u>

$$f(x) = \frac{1+x+x^2}{1-x+x^2} = 1 \qquad x_0 = 0$$

$$\boxed{(1+x)^m = 1 + mx + \frac{m(m-1)}{2}x^2 + \ldots}$$

$$\frac{1+x+x^2}{1-x+x^2} = \frac{(1+x)(1+x+x^2)}{(1+x)(1-x+x^2)} =$$

$$=$$

Формула Тейлора

$$f(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + a_3(x-x_0)^3 + \dots$$

$$a_k = \frac{f^{(k)}(x_0)}{k!}, \quad k = 0, 1, \dots$$

1380

$$f(x) = \sqrt{1-2x+x^3} - \sqrt[3]{1-3x+x^2} =$$

$$= (1-2x+x^3)^{\frac{1}{2}} - (1-3x+x^2)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{2}(1-2x+x^3)^{-\frac{1}{2}} \cdot (-2+3x^2) -$$

$$- \frac{1}{3}(1-3x+x^2)^{-\frac{2}{3}} \cdot (-3+2x)$$

$$f''(x) = -\frac{1}{2^2}(1-2x+x^3)^{-\frac{3}{2}}(2+3x^2)^2 +$$

$$+ \frac{1}{2}(1-2x+x^3)^{\frac{1}{2}} \cdot 6x + \frac{2}{9}(1-3x+x^3)^{-\frac{5}{3}} \cdot (-3+2x)^2$$

$$- \frac{1}{3}(1-3x+x^2)^{-\frac{2}{3}} \cdot 2$$

$$f(0) = 0$$

$$f'(0) = \frac{1}{2}(-2) - \frac{1}{3}(-3) = 0$$

$$f''(0) = \frac{1}{2^2} \cdot 2^2 + \frac{2}{9} \cdot 9 - \frac{2}{3} = \frac{1}{3}$$

$$f(x) = f(0) + f'(0)(x - \underset{\downarrow\, 0}{x_0}) + \frac{f''(0)}{2} \cdot x^2 + o(x^2) =$$

$$= \frac{1}{6}x^2 + o(x^2)$$

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2} \cdot x^2 + \dots$$

$$(1+(x^3-2x))^{\frac{1}{2}} - (1+(x^2-3x))^{\frac{1}{3}} =$$

$$= 1 + \frac{1}{2}(x^3-2x) + \frac{1}{2}\cdot\frac{1}{2}\cdot\left(-\frac{1}{2}\right)(x^3-2x)^2 -$$

$$- \left[1 + \frac{1}{3}(x^2-3x) + \frac{1}{2}\cdot\frac{1}{3}\cdot\left(-\frac{2}{3}\right)\cdot(x^2-3x)^2\right] +$$

$$+ o(x^3)] = -\frac{1}{2^3}\cdot 2^2 x^2 - \frac{1}{3}x^2 + \frac{1}{9}9x^2 +$$

$$+ o(x^2) = \frac{1}{6}x^2 + o(x^2)$$

1377

$$f(x) = \frac{1+x+x^2}{1-x+x^2}$$

$$(1+x)^m = 1 + mx + \frac{m(m-1)x^2}{2} + \frac{m(m-1)(m-2)}{2\cdot 3}x^3$$

$$(1+(x^2-x))^{-1} = 1-(x^2-x)+(x^2-x)^2-(x^2-$$

$$-x)^3 = (1+x+x^2)\left[1-x^2+x+x^4-2x^3\right.$$

$$\left.+x^2+x^6-3x^5+3x^4-x^3\right] =$$

$$= 1-x^2+x+x^4-2x^3+x^2+3x^4-x^3+x-x^3+$$

$$+x^2-2x^4+x^3-x^4+x^2-x^4+x^3+x^4 =$$

$$= 1+2x+2x^2-2x^3$$

1382 $f(x) = \frac{x}{e^x - 1} = \frac{1}{\frac{e^x-1}{x}}$

$\left(\frac{e^x-1}{x}\right)^{-1} = (1+\alpha)^{-1} = 1 - \alpha + \alpha^2 - \alpha^3 + \alpha^4 + o(\alpha^4) =$

$e^x - 1 = x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$ | $e^x = 1 + x + \frac{x^2}{2!} + \dots$

$\frac{e^x-1}{x} = 1 + \underbrace{\frac{x}{2} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots}_{\alpha}$

$= 1 - \frac{x}{2} + \frac{x^2}{3!} + \frac{x^3}{4!} + \frac{x^4}{5!} + \dots$

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + o(x^4)$

1384 $\ln \cos x = \frac{1}{2} \ln(1 - \sin^2 x)$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} -$ | $\sin x = x - \frac{x^3}{3!} + o($

$\cos^2 x = 1 - \sin^2 x$

$= -\frac{1}{2}\left(\left(x - \frac{x^3}{x!}\right)^2 + \left(x - \frac{x^3}{3!}\right)^4 + \left(x - \frac{x^3}{3!}\right)^4\right) =$

$= -\frac{1}{2}x^2 - \frac{1}{12}x^4 - \frac{1}{45}x^6 + o(x^6)$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$o(\sin^6 x) \sim o(x^6)$