Семинар. Нормальный вид.

а) $\left(\overline{\left((p \to q) \to (r \to \bar{p})\right)}\right) \to (\bar{q} \to \bar{r})$

$\equiv \left(\overline{(\bar{p} \vee q) \vee (\bar{r} \vee \bar{p})}\right) \vee (q \vee \bar{r}) \equiv$

$\equiv \left((\bar{p} \vee q) \wedge (r \wedge p)\right) \vee q \vee \bar{r} \equiv$

$\equiv (\bar{p} \wedge r \wedge q) \vee (q \wedge r \wedge p) \vee q \vee \bar{r} \equiv$

$\equiv q \wedge r \wedge p \vee q \vee \bar{r} \equiv q \vee \bar{r} \equiv$

$\varphi \wedge \psi \vee \varphi \equiv \varphi$

$(\varphi \vee \psi) \wedge \varphi \equiv \varphi$

$\varphi \equiv \varphi \wedge (x \vee \bar{x}) \equiv (\varphi \wedge x) \vee (\varphi \wedge \bar{x})$

$\equiv q(p \vee \bar{p})(r \vee \bar{r}) \vee (q \vee \bar{q})(p \vee \bar{p}) \wedge \bar{r} \equiv$

$\equiv qpr \vee q\bar{p}r \vee qp\bar{r} \vee q\bar{p}\bar{r} \vee qp\bar{r} \vee$
$\vee q\bar{p}\bar{r} \vee \bar{q}p\bar{r} \vee \bar{q}\bar{p}\bar{r} \equiv$

$\equiv qpr \vee q\bar{p}r \vee qp\bar{r} \vee q\bar{p}\bar{r} \vee \bar{q}p\bar{r} \vee \bar{q}\bar{p}\bar{r}$

$\wedge, \vee, \neg$ — для СДНФ

$\wedge, \oplus, 1$ — для полинома Жегалкина

$\bar{x} = 1 \oplus x$

$x \vee y = x \oplus y \oplus xy$

$x \oplus x = 0$

$x \wedge x = x$

$q \vee \bar{r} \equiv q \oplus \bar{r} \oplus q\bar{r} \equiv$

$\equiv q \oplus (1 \oplus r) \oplus q(1 \oplus r) \equiv$

$\equiv q \oplus 1 \oplus r \oplus q \oplus qr \equiv 1 \oplus r \oplus qr$

→ Для любой формулы существует единственный полином Жегалкина, который ей эквивалентен (т.е. полином Жегалкина – каноническая форма)

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1) б) $((((p \overset{1}{\rightarrow} q) \overset{3}{\rightarrow} \overset{2}{\bar{p}}) \overset{5}{\rightarrow} \overset{4}{\bar{q}}) \overset{7}{\rightarrow} \overset{6}{\bar{r}}) \overset{8}{\rightarrow} r$

p	q	r	1	2	3	4	5	6	7	8
0	0	0	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	0	0	1
0	1	0	1	1	1	0	0	1	1	0
0	1	1	1	1	1	0	0	0	1	1
1	0	0	0	0	1	1	1	1	1	0
1	0	1	0	0	1	1	1	0	0	1
1	1	0	1	0	0	0	1	1	1	0
1	1	1	1	0	0	0	1	0	0	1

$(\bar{p} \wedge \bar{q} \wedge r) \vee (\bar{p} \wedge q \wedge r) \vee (p \wedge \bar{q} \wedge r) \vee (p \vee q \vee r)$

r – ПН

в) $\overline{(p \to (q \to r))} \to (\overline{(p \to \bar r)} \to (p \to \bar q)) \equiv$

$\equiv \overline{(\bar p \vee (\bar q \vee r))} \vee (\overline{(\bar p \vee \bar r)} \vee (\bar p \vee \bar q)) \equiv$

$\equiv (p \wedge (q \wedge \bar r)) \vee ((p \wedge r) \vee (\bar p \vee \bar q)) \equiv$

$\equiv pq\bar r \vee pr(q \vee \bar q) \vee \bar p (q \vee \bar q)(r \vee \bar r) \vee$

$\vee \bar q (p \vee \bar p)(r \vee \bar r) \equiv pq\bar r \vee prq \vee pr\bar q \vee$

$\vee \bar p q r \vee \bar p q \bar r \vee \bar p \bar q r \vee \bar p \bar q \bar r \vee \bar q p r \vee \bar q p \bar r \vee$

$\vee \bar q \bar p r \vee \bar q \bar p \bar r \equiv pq\bar r \vee pqr \vee \cancel{p\bar q r} \vee$

$\vee \bar p q r \vee \bar p q \bar r \vee \bar p \bar q r \vee \bar p \bar q \bar r \vee$

$\vee p\bar q r \vee p\bar q \bar r \vee \cancel{\bar p \bar q r} \vee \cancel{\bar p \bar q \bar r} \equiv$

$\equiv pq\bar r \vee pqr \vee p\bar q r \vee p\bar q \bar r \vee \bar p q r \vee \bar p q \bar r \vee$

$\vee \bar p \bar q r \vee \bar p \bar q \bar r$ СДНФ

ДНФ: $pq\bar r \vee pr \vee \bar p \vee \bar q$

$(p \wedge q \wedge \bar r) \vee (p \wedge r) \vee (\bar p) \vee (\bar q) \equiv$

$(p \wedge q \wedge \bar r) \oplus (p \wedge r) \oplus (p \wedge q \wedge 0) \oplus (1 \oplus q) \equiv$

$\equiv (p \wedge q \wedge \bar r) \oplus (p \wedge r) \oplus 1 \oplus q$

▷ $\varphi, \neg\psi \vdash \neg(\varphi \vdash \psi)$

$$\cfrac{\cfrac{\varphi, \neg\psi, (\varphi \vdash \psi) \vdash \ ; \ \varphi, \neg\psi, (\varphi \vdash \psi) \vdash \neg}{\varphi, \neg\psi, (\varphi \vdash \psi) \vdash \bot}}{\varphi, \neg\psi \vdash \neg(\varphi \vdash \psi)}$$

(7)

1б) $(p \overset{2}{\to} (q \overset{1}{\to} r)) \overset{8}{\to} ((p \overset{4}{\to} \bar{r}^{3}) \overset{7}{\to} (p \overset{6}{\to} \bar{q}^{5}))$

p	q	r	1	2	3	4	5	6	7	8
0	0	0	1	1	1	1	1	1	1	1
0	0	1	1	1	0	1	1	1	1	1
0	1	0	0	1	1	1	0	1	1	1
0	1	1	1	1	0	1	0	1	1	1
1	0	0	1	1	1	1	1	1	1	1
1	0	1	1	1	0	0	1	1	1	1
1	1	0	0	0	1	1	0	0	0	1
1	1	1	1	1	0	0	0	0	1	1

<u>СДНФ</u>: $(\bar{p} \wedge \bar{q} \wedge \bar{r}) \vee (\bar{p} \wedge \bar{q} \wedge r) \vee (\bar{p} \wedge q \wedge \bar{r}) \vee$
$\vee (\bar{p} \wedge q \wedge r) \vee (p \wedge \bar{q} \wedge \bar{r}) \vee (p \wedge \bar{q} \wedge r) \vee$
$\vee (p \wedge q \wedge \bar{r}) \vee (p \wedge q \wedge r)$

Полином Жегалкина:

$f(0,0,0) = a_{000} = 1$

$f(0,0,1) = a_{000} \oplus a_{001} = 1 \oplus 1 = 0$

$f(0,1,0) = a_{000} \oplus a_{010} = 1 \oplus 1 = 0$

$f(1,0,0) = a_{000} \oplus a_{100} = 1 \oplus 1 = 0$

$f(0,1,1) = a_{000} \oplus a_{001} \oplus a_{010} \oplus a_{011} = 1 \oplus 0 \oplus 0 \oplus 1 = 0$

$1 \quad 1$

$f(1,0,1) = a_{000} \oplus a_{001} \oplus a_{100} \oplus a_{101} = 1 \oplus 0 \oplus 0 \oplus 1 = 0$

$f(1,1,0) = a_{000} \oplus a_{011} \oplus a_{100} \oplus a_{110} = 1 \oplus 0 \oplus 0 \oplus 1 = 0$

$f(1,1,1) = a_{000} \oplus a_{001} \oplus a_{010} \oplus a_{100} \oplus a_{101} \oplus a_{110} \oplus a_{011} \oplus$
$\oplus a_{111} = 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 1 = 0$

$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0$

$f = 1$:
$f_{000} \to 1$

ПЖ: 1

4) $((r \xrightarrow{1} p) \xrightarrow{5} (\overset{2}{\neg} (q \overset{3}{\vee} r) \xrightarrow{4} p))$

p	q	r	1	2	3	4	5
0	0	0	1	0	1	0	0
0	0	1	0	1	0	1	1
0	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1
1	0	0	1	0	1	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	0	1	1

СДНФ:
$\bar{p}\bar{q}r \vee \bar{p}q\bar{r} \vee \bar{p}qr \vee$
$\vee p\bar{q}\bar{r} \vee p\bar{q}r \vee pq\bar{r} \vee pqr$

$f = 1$:

$f(0,0,1) = r$

$f(0,1,0) = q$

$f(1,0,0) = p$

$f(0,1,1) = qr$

$f(1,0,1) = pr$

$f(1,1,0) = pq$

$f(1,1,1) = pqr$

Полином Жегалкина:
$r \oplus q \oplus p \oplus qr \oplus pr \oplus pq \oplus$
$\oplus pqr$

$f(0,0,0) = a_{000} = 0$

$f(0,0,1) = a_{000} \oplus a_{001} = 0 \oplus 1 = 1$

$f(0,1,0) = a_{000} \oplus a_{010} = 0 \oplus 1 = 1$

$f(1,0,0) = a_{000} \oplus a_{100} = 0 \oplus 1 = 1$

$f(0,1,1) = a_{000} \oplus a_{001} \oplus a_{010} \oplus a_{011} = 0 \oplus 1 \oplus 1 \oplus 1 = 1$

$1 \quad 0 \quad 1$

$f(1,0,1) = a_{000} \oplus a_{001} \oplus a_{100} \oplus a_{101} = 0 \oplus 1 \oplus 1 \oplus 1 = 1$

$f(1,1,0) = a_{000} \oplus a_{010} \oplus a_{100} \oplus a_{110} = 0 \oplus 1 \oplus 1 \oplus 1 = 1$

$f(1,1,1) = a_{000} \oplus a_{001} \oplus a_{010} \oplus a_{100} \oplus a_{011} \oplus a_{101} \oplus a_{110} \oplus a_{111} = 0 \oplus 1 \oplus$
$\oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 = 1$

9) $\left(\overline{((p \underset{1}{\wedge} q) \underset{2}{\rightarrow} p)}_{3} \underset{6}{\vee} (p \underset{5}{\wedge} (q \underset{4}{\vee} r)) \right)$

p	q	r	1	2	3	4	5	6
0	0	0	0	1	0	0	0	0
0	0	1	0	1	0	1	0	0
0	1	0	0	1	0	1	0	0
0	1	1	0	1	0	1	0	0
1	0	0	0	1	0	0	0	0
1	0	1	0	1	0	1	1	1
1	1	0	1	1	0	1	1	1
1	1	1	1	1	0	1	1	1

СДНФ:

$p\bar{q}r \vee pq\bar{r} \vee pqr$

СКНФ:

$(p \vee q \vee r) \wedge (p \vee q \vee \bar{r}) \wedge$
$\wedge (p \vee \bar{q} \vee r) \wedge (p \vee \bar{q} \vee \bar{r}) \wedge$
$\wedge (p \vee \bar{q} \vee \bar{r})$

$f(0,0,0) = a_{000} = 0$

$f(0,0,1) = a_{000} \oplus a_{001} = 0 \oplus 0 = 0$

$f(0,1,0) = a_{000} \oplus a_{010} = 0 \oplus 0 = 0$

$f(1,0,0) = a_{000} \oplus a_{100} = 0 \oplus 0 = 0$

$f(0,1,1) = a_{000} \oplus a_{001} \oplus a_{010} \oplus a_{011} = \underbrace{0 \oplus 0 \oplus 0 \oplus 0}_{0 \quad 0 \quad 0} = 0$

$f(1,0,1) = a_{000} \oplus a_{001} \oplus a_{100} \oplus a_{101} = 0 \oplus 0 \oplus 0 \oplus 1 = 1$

$f(1,1,0) = a_{000} \oplus a_{010} \oplus a_{100} \oplus a_{110} = 0 \oplus 0 \oplus 0 \oplus 1 = 1$

$f(1,1,1) = a_{000} \oplus a_{001} \oplus a_{010} \oplus a_{100} \oplus a_{101} \oplus a_{011} \oplus a_{110} \oplus$
$\oplus a_{111} = 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 1 = 0$

$f = 1:$

$f(1,0,1) = pr$
$f(1,1,0) = pq$

ПЖ:

$pr \oplus pq$