Док

G - группа, $g \in G$

$\varphi_g(h) = g^{-1}hg$

$\varphi_g : G \to G$

$(\varphi_{g^{-1}} \circ \varphi_g)(h) = \varphi_g(\varphi_{g^{-1}}(h)) =$

$= \varphi_g(ghg^{-1}) = g^{-1}ghg^{-1}g = h$

$(g^{-1})^{-1} = g$

Док-во

$(g^{-1})^{-1} \cdot g^{-1} = e$

$g \cdot g^{-1} = e$

$\varphi_{g^{-1}} \circ \varphi_g = id_G$

$\varphi_g \circ \varphi_{g^{-1}} = id_G$

$(g^{-1})^{-1} \circ g^{-1} = e \quad / \cdot g$

$(g^{-1})^{-1} g^{-1} g = g$

$(g^{-1})^{-1} = g$

Проверить гомоморфизм:

1) $\varphi_g(h_1 h_2) = g^{-1} h_1 h_2 g$

$\varphi_g(h_1)\varphi_g(h_2) = g^{-1} h_1 g \, g^{-1} h_2 g$

$\cancel{\varphi: G_1 \to G_2}$

2) $\varphi(e \cdot e) = \varphi(e)$

$\varphi(e) \cdot \varphi(e)$

$\varphi(e) = \varphi(e)\varphi(e)$

$e = \varphi(e)$

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3) $e = \varphi(e) = \varphi(hh^{-1}) = \varphi(h)\varphi(h^{-1}) =$

$= \cancel{\varphi(h)\,\varphi(h^{-1})} = (\varphi(h))^{-1}$

$a_{1\sigma(1)}\ a_{2\sigma(2)}\ a_{3\sigma(3)}$

$$\begin{vmatrix} a_{12} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{13}a_{21}a_{32} +$$

$$+ a_{12}a_{23}a_{31} - a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31} -$$

$$- a_{12}a_{21}a_{33}$$

$a \in G$

$\langle a \rangle = \{ a^n \mid n \in \mathbb{Z} \}$

$>0 \quad a^n = \underbrace{a \cdot \ldots a}_{n \text{ раз}}$

$n = 0 \quad a^n = e$

$n < 0 \quad a^n = \underbrace{a^{-1} \ldots a^{-1}}_{-n \text{ раз}} = (a^{-n})^{-1}$

Доказать: $\langle a \rangle < G$

$\mathfrak{A} = \langle A, \Sigma^{\mathfrak{A}} \rangle$

$\mathfrak{B} = \langle B, \Sigma^{\mathfrak{B}} \rangle$

$\mathfrak{A} \subseteq \mathfrak{B}$ $\quad$ $A \subseteq B$

$A \subseteq B \quad \Leftrightarrow \quad f^{\mathfrak{B}} \upharpoonright_{(A)} = f^{\mathfrak{A}}$

$P^{\mathfrak{B}} \cap A^{\nu(F)} = P^{\mathfrak{A}}$

$a^n \cdot a^m = a^{n+m} \in \langle a \rangle$

$(a^n)^{-1} = a^{-n} \in \langle a \rangle$

$a^0 = e \in \langle a \rangle$

$\langle +^2, -^1, 0^0, \cdot^2, 1^0 \rangle$

$a + 0 = 0 + a = a$

$a \cdot 1 = 1 \cdot a = a$

$\vdash (x=x)$

$x=y\ ,\ (\varphi)^z_x = (\varphi)^z_y$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\varphi\vdash\varphi}{\varphi\vdash(\varphi)^x_x}\,15}{\varphi\vdash\exists x\varphi} \qquad \dfrac{}{\neg\exists x\varphi,\varphi\vdash\neg\exists x\varphi}\,12}{\neg\exists x\varphi,\varphi\vdash\exists x\varphi,\quad \neg\exists x\varphi,\varphi\vdash\neg\exists x\varphi}\,11,12}{\neg\exists x\varphi,\varphi\vdash\Lambda}\,10}{\neg\exists x\varphi\vdash\neg\varphi}\,9}{\neg\exists x\varphi\vdash\forall x\neg\varphi}\,13$$

$$\dfrac{\Gamma\vdash\varphi}{\Gamma,\vdash\psi}$$

$$\dfrac{\Gamma,\varphi\vdash\psi,\quad \Gamma,\neg\varphi\vdash\psi,\quad \vdash\varphi\vee\neg\varphi}{\Gamma\vdash\psi}$$

13. $\dfrac{\Gamma\vdash\varphi}{\Gamma\vdash\forall x\varphi}\ ,\ x\notin Fv(\Gamma)$

15. $\dfrac{\Gamma\vdash(\varphi)^x_t}{\Gamma\vdash\exists x\varphi}$

14. $\dfrac{\Gamma,(\varphi)^x_t\vdash\psi}{\Gamma,\forall x\varphi\vdash\psi}$

16. $\dfrac{\Gamma,\varphi\vdash\psi}{\Gamma,\exists x\varphi\vdash\psi}\ x\notin Fv(\Gamma,\psi$