

1-1.9-21

EE24BTECH11062 - Homa Harshitha Vuddanti

Question:

Given vertices of a parallelogram $\mathbf{A} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 4 \\ b \end{pmatrix}$, and $\mathbf{D} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find the values of a and b . Hence, find the lengths of its sides.

Solution:

Given, the sides of the parallelogram.

Variable	Description
a	x -coordinate of point B
b	y -coordinate of point C

TABLE 0: Variables Used

In a parallelogram $ABCD$, since AB is parallel to CD ,

From (1.1.3.1),

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \quad (0.1)$$

$$\begin{pmatrix} a - (-2) \\ 0 - 1 \end{pmatrix} = \begin{pmatrix} 4 - 1 \\ b - 2 \end{pmatrix} \quad (0.2)$$

$$\begin{pmatrix} a + 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ b - 2 \end{pmatrix} \quad (0.3)$$

$$a + 2 = 3 \quad (0.4)$$

$$b - 2 = -1 \quad (0.5)$$

From equations (0.4) and (0.5),

$$a = 1, \quad (0.6)$$

$$b = 1 \quad (0.7)$$

To find lengths of sides,

$$AB = CD = \sqrt{(A - B)^\top (A - B)} \quad (0.8)$$

$$= \sqrt{A^\top A - A^\top B - B^\top A + B^\top B} \quad (0.9)$$

$$AD = BC = \sqrt{(A - D)^\top (A - D)} \quad (0.10)$$

$$= \sqrt{A^\top A - A^\top D - D^\top A + D^\top D} \quad (0.11)$$

$$(0.12)$$

Substituting values,

$$AB = \sqrt{\begin{pmatrix} -3 \\ 1 \end{pmatrix}^T \begin{pmatrix} -3 \\ 1 \end{pmatrix}} = \sqrt{(-3)(-3) + (1)(1)} \quad (0.13)$$

$$AB = CD = \sqrt{10} \quad (0.14)$$

$$AD = \sqrt{\begin{pmatrix} -3 \\ -1 \end{pmatrix}^T \begin{pmatrix} -3 \\ -1 \end{pmatrix}} = \sqrt{(-3)(-3) + (-1)(-1)} \quad (0.15)$$

$$AD = BC = \sqrt{10} \quad (0.16)$$

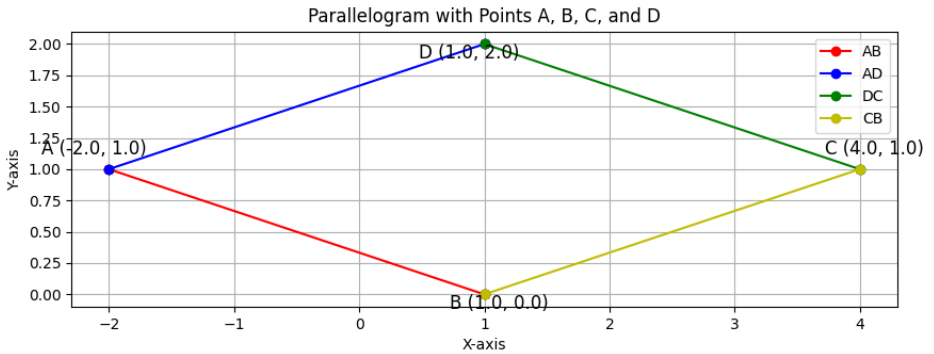


Fig. 0.1: Plot