1-1.9-21

EE24BTECH11062 - Homa Harshitha Vuddanti

Question:

Given vertices of a parallelogram $\mathbf{A} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ b \end{pmatrix}$, and $\mathbf{D} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find the values of a and b. Hence, find the lengths of its sides.

Solution:

Given, the sides of the parallelogram.

Variable	**Description**
a	x-coordinate of point B
b	y-coordinate of point C

TABLE 0: Variables Used

In a parallelogram $ABCD$, since AB is parallel to CD,

From (1.1.3.1),

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{0.1}$$

$$\begin{pmatrix} a - (-2) \\ 0 - 1 \end{pmatrix} = \begin{pmatrix} 4 - 1 \\ b - 2 \end{pmatrix} \tag{0.2}$$

$$\begin{pmatrix} a + 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ b - 2 \end{pmatrix} \tag{0.3}$$

$$a + 2 = 3 \tag{0.4}$$

$$b - 2 = -1 \tag{0.5}$$

From equations (0.4) and (0.5),

$$a = 1, \tag{0.6}$$

$$b = 1 \tag{0.7}$$

To find lengths of sides,

$$AB = CD = \sqrt{(A - B)^\top (A - B)} \tag{0.8}$$

$$= \sqrt{A^\top A - A^\top B - B^\top A + B^\top B} \tag{0.9}$$

$$AD = BC = \sqrt{(A - D)^\top (A - D)} \tag{0.10}$$

$$= \sqrt{A^\top A - A^\top D - D^\top A + D^\top D} \tag{0.11}$$

$$\tag{0.12}$$

Substituting values,

$$AB = \sqrt{\begin{pmatrix} -3 \\ 1 \end{pmatrix}^\top \begin{pmatrix} -3 \\ 1 \end{pmatrix}} = \sqrt{(-3)(-3) + (1)(1)} \tag{0.13}$$

$$AB = CD = \sqrt{10} \tag{0.14}$$

$$AD = \sqrt{\begin{pmatrix} -3 \\ -1 \end{pmatrix}^\top \begin{pmatrix} -3 \\ -1 \end{pmatrix}} = \sqrt{(-3)(-3) + (-1)(-1)} \tag{0.15}$$

$$AD = BC = \sqrt{10} \tag{0.16}$$

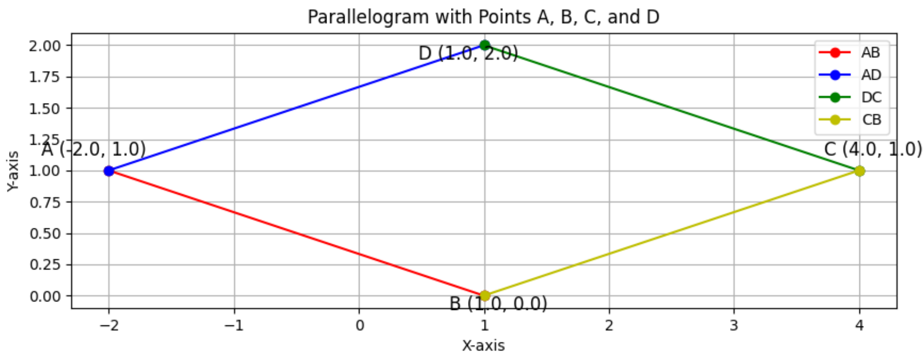

Fig. 0.1: Plot