3-3.3-8

EE24BTECH11062 - Homa Harshitha Vuddanti

Question:
Construct a triangle ABC with side $BC = 7cm, \angle B = 45°, \angle A = 105°$.
Solution:
Given,

Variable	Description
a	$7cm$
$\angle B$	$45°$
$\angle A$	$105°$

TABLE 0: Given variables

By angle sum property,

$$\angle A + \angle B + \angle C = \pi \tag{0.1}$$

$$\angle C = \pi - \left(\frac{\pi}{4} + \frac{7\pi}{12} \right) \tag{0.2}$$

$$\angle C = \frac{\pi}{6} \tag{0.3}$$

Using projection rule,

$$a = b \cos C + c \cos B \tag{0.4}$$

Using Sine formula, in $\triangle ABC$,

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} \tag{0.5}$$

$$b \sin C - c \sin B = 0 \tag{0.6}$$

Solving equations (0.4)and (0.6),

$$\begin{pmatrix} \cos C & \cos B \\ \sin C & -\sin B \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{0.7}$$

$$\begin{pmatrix} \cos \frac{\pi}{6} & \cos \frac{\pi}{4} \\ \sin \frac{\pi}{6} & -\sin \frac{\pi}{4} \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{0.8}$$

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{0.9}$$

Solving for b and c,

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} & 7 \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \xrightarrow{R1=R1+R2} \begin{pmatrix} \frac{\sqrt{3}+1}{2} & 0 & 7 \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \xrightarrow{R2=\frac{R1}{\sqrt{3}+1}-R2} \tag{0.10}$$

$$\begin{pmatrix} \frac{\sqrt{3}+1}{2} & 0 & 7 \\ 0 & \frac{1}{\sqrt{2}} & \frac{7}{\sqrt{3}+1} \end{pmatrix} \xrightarrow{R1=\frac{2}{\sqrt{3}+1}R1, R2=\sqrt{2}R2} \begin{pmatrix} 1 & 0 & \frac{14}{\sqrt{3}+1} \\ 0 & 1 & \frac{7\sqrt{2}}{\sqrt{3}+1} \end{pmatrix} \tag{0.11}$$

$$b = \frac{14}{\sqrt{3}+1} \tag{0.12}$$

$$c = \frac{7\sqrt{2}}{\sqrt{3}+1} \tag{0.13}$$

From (3.1.1.3),

$$\mathbf{A} = c \begin{pmatrix} \cos B \\ \sin B \end{pmatrix} = \begin{pmatrix} \frac{7}{\sqrt{3}+1} \\ \frac{7}{\sqrt{3}+1} \end{pmatrix} \tag{0.14}$$

$$\mathbf{B} = 0 \tag{0.15}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{0.16}$$

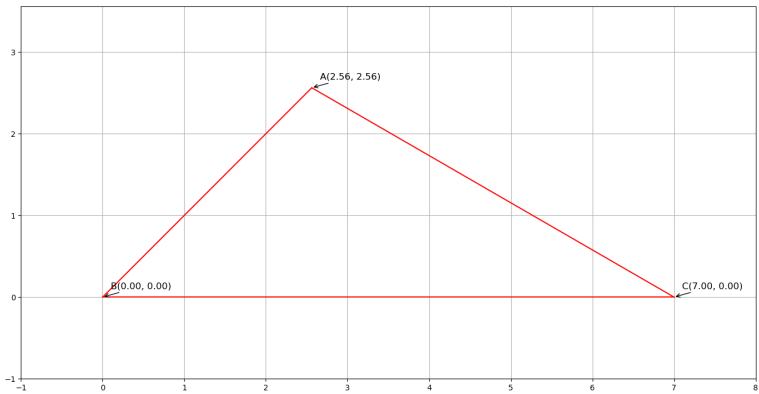Fig. 0.1: Plot