

# JEE MAINS

EE1030

MARCH 17 - SHIFT - 2

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QUESTIONS- 16 TO 30

## SECTION A

- 1) If the sides  $AB, BC$ , and  $CA$  of a triangle  $ABC$  have, 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices is equal to:
  - a) 360
  - b) 240
  - c) 333
  - d) 364
- 2) The value of  $\lim_{x \rightarrow \infty} [r] + [2r] + \dots [nr] / [n^2]$ , where  $r$  is a non-zero number and  $[r]$  denotes the greatest integer less than or equal to  $r$ , is equal to :
  - a) 0
  - b)  $r$
  - c)  $r/2$
  - d)  $2r$
- 3) The value of  $\sum_{r=0}^6 ({}^6C_r \cdot {}^6C_{6-r})$  is equal to :
  - a) 1124
  - b) 924
  - c) 1324
  - d) 1024
- 4) Two tangents are drawn from a point  $\mathbf{P}$  to the circle  $x^2 + y^2 - 2x - 4y + 4 = 0$ , such that the angle between these tangents is  $\tan^{-1}(12/5)$ , where  $\tan^{-1}(12/5) \in (0, \pi)$ . If the centre of the circle is denoted by  $\mathbf{C}$  and these tangents touch the circle at points  $\mathbf{A}$  and  $\mathbf{B}$ , then the ratio of the areas of  $\triangle PAB$  and  $\triangle CAB$  is:
  - a) 11:4
  - b) 9:4
  - c) 2:1
  - d) 3:1
- 5) The number of solutions of the equation  $\sin^{-1} [x^2 + (1/3)] + \cos^{-1} [x^2 - (2/3)] = x^2$ , for  $x \in [-1, 1]$ , and  $[x]$  denotes the greatest integer less than or equal to  $x$ , is:
  - a) 0
  - b) 2
  - c) 4

d) infinite

### SECTION B

- 6) Let the coefficients of third, fourth and fifth terms in the expansion of  $\left[x + \left(a/x^2\right)\right]^n$ ,  $x \neq 0$  be in the ratio 12:8:3. Then the term independent of  $x$  in the expansion is equal to
- 7) Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and  $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$  such that  $AB = B$  and  $a + d = 2021$ , then the value of  $ad - bc$  is equal to
- 8) Let  $f : [-1, 1] \mapsto R$  be defined as  $f(x) = ax^2 + bx + c$  for all  $x \in [-1, 1]$ , where  $a, b, c \in R$  such that  $f(-1) = 2$ ,  $f'(-1) = 1$  and for  $x \in [-1, 1]$  the maximum value of  $f''(x)$  is  $1/2$ . If  $f(x) \leq \alpha$ ,  $x \in [-1, 1]$ , then least value of  $\alpha$  is equal to
- 9) Let  $I_n = \int_1^e x^{19} (\log |x|)^n dx$ , where  $n \in N$ . If  $20(I_{10}) = \alpha I_9 + \beta I_8$ , for natural numbers  $\alpha$  and  $\beta$ , then  $\alpha - \beta$  is equal to
- 10) Let  $f : [-3, 1] \mapsto R$  be given as  $f(x) = \begin{cases} \min \left[ (x + 6, x^2) \right], & -3 \leq x \leq 0 \\ \max \left[ \sqrt{x}, x^2 \right], & 0 \leq x \leq 1 \end{cases}$  If the area bounded by  $y = f(x)$  and x-axis is  $A$ , then the value of  $6A$  is equal to
- 11) Let  $\mathbf{x}$  be a vector in the plane containing vectors  $a = 2i - j + k$  and  $b = i + 2j - k$ . If the vector  $\mathbf{x}$  is perpendicular to  $(3i + 2j - k)$  and its projection on  $a$  is  $17\sqrt{6}/2$ , then the value of  $x^2$  is equal to
- 12) Consider a set of  $3n$  numbers having variance 4. In this set, the mean of the first  $2n$  numbers is 6 and the mean of the remaining  $n$  numbers is 3. A new set is constructed by adding 1 into each of the first  $2n$  numbers and subtracting 1 from each of the remaining  $n$  numbers. If the variance of the new set is  $k$ , then  $9k$  is equal to
- 13) If  $1, \log_{10}(4^x - 2)$  and  $\log_{10}(4^x + (18/5))$  are in arithmetic progression for a real number  $x$ , then the value of the determinant  $\begin{vmatrix} 2[x - (1/2)] & x - 1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$  is equal to
- 14) Let  $\mathbf{P}$  be an arbitrary point having the sum of squares of the distances from the planes  $x + y + z = 0$ ,  $lx - nz = 0$  and  $x - 2y + z = 0$ , equal to 9. If the locus of the point  $\mathbf{P}$  is  $x^2 + y^2 + z^2 = 9$  then the value of  $l - n$  is equal to
- 15) Let  $\tan \alpha, \tan \beta$  and  $\tan \gamma$ ;  $\alpha, \beta, \gamma \neq [2n - 1]\pi/2, n \in N$  be the slopes of three-line segment  $OA, OB$  and  $OC$ , respectively, where  $\mathbf{O}$  is origin. If the circumcentre of triangle  $ABC$  coincides with the origin and its orthocentre lies on y-axis, then the value of  $[(\cos 3\alpha + \cos 3\beta + \cos 3\gamma) / (\cos \alpha * \cos \beta * \cos \gamma)]^2$  is equal to