JEE MAINS

EE1030

MARCH 17 - SHIFT - 2

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QUESTIONS- 16 TO 30

SECTION A

1) If the sides AB, BC, and CA of a triangle ABC have, 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices is equal to:

   a) 360 b) 240 c) 333 d) 364

2) The value of $\lim_{r\to\infty} [r] + [2r] + \dots [nr] / \left[n^2\right]$, where r is a non-zero number and $[r]$ denotes the greatest integer less than or equal to r, is equal to :

   a) 0 b) r c) $r/2$ d) $2r$

3) The value of $\sum_{r=0}^{6} \left({}^6C_r . {}^6C_{6-r}\right)$ is equal to :

   a) 1124 b) 924 c) 1324 d) 1024

4) Two tangents are drawn from a point **P** to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}(12/5)$, where $\tan^{-1}(12/5) \in (0, \pi)$. If the centre of the circle is denoted by **C** and these tangents touch the circle at points **A** and **B**, then the ratio of the areas of $\triangle PAB$ and $\triangle CAB$ is:

   a) 11:4 b) 9:4 c) 2:1 d) 3:1

5) The number of solutions of the equation $\sin^{-1}\left[x^2 + (1/3)\right] + \cos^{-1}\left[x^2 - (2/3)\right] = x^2$, for $x \in [-1, 1]$, and $[x]$ denotes the greatest integer less than or equal to x, is:

   a) 0 b) 2 c) 4 d) infinite

SECTION B

6) Let the coefficients of third, fourth and fifth terms in the expansion of $\left[x + \left(a/x^2\right)\right]^n, x \neq 0$ be in the ratio 12:8:3. Then the term independent of x in the expansion is equal to

7) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that $AB = B$ and $a + d = 2021$, then the value of $ad - bc$ is equal to

8) Let $f : [-1, 1] \mapsto R$ be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where $a, b, c \in R$ such that $f(-1) = 2, f'(-1) = 1$ and for $x \in [-1, 1]$ the maximum value of $f''(x)$ is $1/2$. If $f(x) \leq \alpha, x \in [-1, 1]$, then least value of α is equal to

9) Let $I_n = \int_1^e x^{19} (log\,|x|)^n\ dx$, where $n \in N$. If $20\,(I_{10}) = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ is equal to

10) Let $f : [-3, 1] \mapsto R$ be given as $f(x) = \begin{cases} min\left[\left(x + 6, x^2\right)\right], -3 \leq x \leq 0 \\ max\left[\sqrt{x}, x^2\right], 0 \leq x \leq 1 \end{cases}$ If the area bounded by $y = f(x)$ and x-axis is A, then the value of $6A$ is equal to

11) Let $\mathbf{x}$ be a vector in the plane containing vectors $a = 2i - j + k$ and $b = i + 2j - k$. If the vector $\mathbf{x}$ is perpendicular to $(3i + 2j - k)$ and its projection on a is $17\sqrt{6}/2$, then the value of x^2 is equal to

12) Consider a set of $3n$ numbers having variance 4. In this set, the mean of the first $2n$ numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of the first $2n$ numbers and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then $9k$ is equal to

13) If 1, $log_{10}(4^x - 2)$ and $log_{10}(4^x + (18/5))$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{bmatrix} 2\,[x - (1/2)] & x - 1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{bmatrix}$ is equal to

14) Let $\mathbf{P}$ be an arbitary point having the sum of squares of the distances from the planes $x + y + z = 0, lx - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point $\mathbf{P}$ is $x^2 + y^2 + z^2 = 9$ then the value of $l - n$ is equal to

15) Let $\tan\alpha, \tan\beta$ and $\tan\gamma; \alpha, \beta, \gamma \neq [2n - 1]\,\pi/2, n \in N$ be the slopes of three-line segment OA, OB and OC, respectively, where $\mathbf{O}$ is origin. If the circumcentre of triangle ABC coincides with the origin and its orthocentre lies on y-axis, then the value of $[(\cos 3\alpha + \cos 3\beta + \cos 3\gamma) / (\cos\alpha * \cos\beta * \cos\gamma)]^2$ is equal to