JEE MAINS

EE1030

APRIL 08, 2023 - SHIFT - 2

Homa Harshitha Vuddanti
(EE24BTECH11062)

QUESTIONS- 16 TO 30

SECTION A

1) The integral $\int\left(\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x\right)log_2 x\ dx$ is equal to

   a) $\left(\frac{x}{2}\right)^x log_2\left(\frac{2}{x}\right)+C$ b) $\left(\frac{x}{2}\right)^x-\left(\frac{2}{x}\right)^x+C$ c) $\left(\frac{x}{2}\right)^x log_2\left(\frac{x}{2}\right)+C$ d) $\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x+C$

2) The value of $36\left(4\cos^2 9^\circ-1\right)\left(4\cos^2 27^\circ-1\right)\left(4\cos^2 81^\circ\right)\left(4\cos^2 243^\circ-1\right)$ is

   a) 27 b) 54 c) 18 d) 36

3) Let $\mathbf{A}(0,1), \mathbf{B}(1,1)$ and $\mathbf{C}(1,0)$ be the midpoints of the sides of a triangle with incentre at the point $\mathbf{D}$. If the focus of the parabola $y^2=4ax$ passing through $\mathbf{D}$ is $\left(\alpha+\beta\sqrt{3},0\right)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

   a) 6 b) 8 c) $\frac{9}{2}$ d) 12

4) The negation of $(p\wedge(\sim q))\vee(\sim p)$ is equivalent to

   a) $p\wedge(\sim q)$ b) $p\wedge(q\wedge(\sim p))$ c) $p\vee(q\vee(\sim p))$ d) $p\wedge q$

5) Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are co-prime, then $m+n$ is equal to

   a) 316 b) 317 c) 315 d) 314

SECTION B

6) Let $R=\{a,b,c,d,e\}$ and $S=\{1,2,3,4\}$. Total number of onto functions $f: R\mapsto S$ such that $f(a)\neq 1$, is equal to
7) Let m and n be the number of real roots of the quadratic equations $x^2-12x+[x]+31=0$ and $x^2-5|x+2|-4=0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then m^2+mn+n^2 is equal to
8) Let P_1 be the plane $3x-y-7z=11$ and P_2 be the plane passing through points $(2,-1,0)$,$(2,0,-1)$, and $(5,1,1)$. If the foot of the perpendicular drawn from the point

$(7,4,-1)$ on the line of intersection of the planes P_1 and P_2 is (α,β,γ), then $\alpha+\beta+\gamma$ is equal to

9) If the domain of the function $\ln\left(\frac{6x^2+5x+1}{2x-1}\right)+\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha,\beta)\cup(\gamma,\delta]$, then , $18\left(\alpha^2+\beta^2+\gamma^2+\delta^2\right)$ is equal to

10) Let the area enclosed by the lines $x+y=2, y=0, x=0$ and the curve $f(x)=min\{x^2+\frac{3}{4}, 1+[x]\}$ where $[x]$ denotes the greatest integer $\leq x$, be A, then the value of $12A$ is

11) Let $0<z<y<x$ be three real numbers such that $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy+yz+zx=\frac{3}{\sqrt{2}}xyz$, then $3(x+y+z)^2$ is equal to

12) Let the solution curve $x=x(y), 0<y<\frac{\pi}{2}$, of the differential equation $(\ln(\cos y))^2\cos y\,dx-(1+3x\ln(\cos y))\sin y\,dy=0$ satisfy $x\left(\frac{\pi}{3}\right)=\frac{1}{2\ln 2}$. If $x\left(\frac{\pi}{6}\right)=\frac{1}{\ln m-\ln n}$, where m and n are co-prime, then mn is equal to

13) Let $[t]$ denote the greatest integer function. If $\int_0^{2.4}\left[x^2\right]dx=\alpha+\beta\sqrt{2}+\gamma\sqrt{3}+\delta\sqrt{5}$, then $\alpha+\beta+\gamma+\delta$ is equal to

14) The ordinates of the points **P** and **Q** on the parabola with focus $(3,0)$ and directrix $x=-3$ are in the ratio $3:1$. If $\mathbf{R}(\alpha,\beta)$ is the point of intersection of the tangents to the parabola at **P** and **Q**, then $\frac{\beta^2}{\alpha}$ is equal to

15) Let k and m be positive real numbers such that the function

$$f(x)=\begin{cases}3x^2+k\sqrt{x+1}, 0<x<1\\ mx^2+k^2, x\geq 1\end{cases}$$

is differentiable for all $x>0$. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to