JEE MAINS

EE1030

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QUESTIONS- 16 TO 30

SECTION A

1) Let **A** be the point of intersection of the lines $3x + 2y = 14, 5x - y = 6$ and **B** be the point of intersection of the lines $4x + 3y = 8, 6x + y = 5$. The distance of the point $\mathbf{P}(5, -2)$ from the line AB is

a) 8 b) $\frac{5}{2}$ c) 2 d) $\frac{13}{2}$

2) The function $f(x) = \frac{x}{x^2-6x-16}$, $x \in R - \{-2, 8\}$
   a) decreases in $(-\infty, -2)$ and increases in $(8, \infty)$
   b) decreases in $(-2, 8)$ and increases in $(-\infty, -2) \cup (8, \infty)$
   c) decreases in $(-\infty, -2) \cup (-2, \infty) \cup (8, \infty)$
   d) increases in $(-\infty, -2) \cup (-2, \infty) \cup (8, \infty)$

3) If $\sin\left(\frac{y}{x}\right) = \ln|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to

a) 9 b) 4 c) 12 d) 3

4) If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of the first four observations is $\frac{7}{2}$, then the variance of the first four observations is equal to

a) $\frac{77}{12}$ b) $\frac{105}{4}$ c) $\frac{5}{4}$ d) $\frac{4}{5}$

5) Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to

a) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$ b) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$ c) $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$ d) $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

SECTION B

6) Let the slope of the line $45x + 5y + 3 = 0$ be $27r_1 + \frac{9r_2}{2}$ for some $r_1, r_2 \in R$ then $\lim_{x\to 3}\left(\int_3^x \frac{8t^2}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}\,dt\right)$ is equal to
7) Let the area of the region $\left\{(x, y) : 0 \le x \le 3, 0 \le y \le min\left\{x^2 + 2, 2x + 2\right\}\right\}$ be A. Then $12A$ is equal to
8) Let $f(x) = \sqrt{\lim_{r\to x}\left\{\frac{2r^2\left[(f(r))^2 - f(x)f(r)\right]}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right\}}$ be differentiable in $(-\infty, 0) \cup (0, \infty)$ and $f(1) = 1$. Then the value of ea, such that $f(a) = 0$, is equal to
9) Let for any three distinct consecutive terms a, b, c of an A.P, the lines $ax + by + c = 0$ be concurrent at the point **P** and **Q** (α, β) be a point such that the system of equations
$x + y + z = 6$
$2x + 5y + \alpha z = \beta$
$x + 2y + 3z = 4$, has infinitely many solutions. Then $(PQ)^2$ is equal to
10) Let **P** (α, β) be a point on the parabola $y^2 = 4x$. If **P** lies on the chord of the parabola $x^2 = 8y$ whose midpoint id $\left(1, \frac{5}{4}\right)$, then $(\alpha - 28)(\beta - 8)$ is equal to
11) Let the set $C - \left\{(x, y) \mid x^2 - 2^y = 2023, x, y \in N\right\}$. Then $\sum_{(x,y)\in C}(x + y)$ is equal to
12) If $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 - \sin 2x}\,dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β and γ are rational numbers, then $3\alpha + 4\beta - gamma$ is equal to
13) Let **O** be the origin, and **M** and **N** be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM} \cdot \overrightarrow{ON}$ is equal to
14) Remainder when $64^{32^{32}}$ is divided by 9 is equal to
15) Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that $Im(\alpha) > Im(\beta)$. Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta} + \alpha^{99} = 3^n(a + ib), i = \sqrt{-1}$. Then $n + a + b$ is equal to