JEE ADVANCED

EE1030

Chapter 16: APPLICATIONS OF DERIVATIVES

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SECTION A

E. SUBJECTIVE SKILLS

1) Suppose $p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$. If $|p(x)| \le \left|e^{x-1} - 1\right|$ for all $x \ge 0$, prove that $|a_1 + 2a_2 + \cdots + na_n| \le 1$.
(2000 - 5 Marks)

2) Let $-1 \le p \le 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval $[1/2, 1]$ and identify it.
(2003 - 2 Marks)

3) Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line $x + y = 7$, is minimum.
(2003 - 4 Marks)

4) Using the relation $2(1 - \cos x) < x^2, x \neq 0$ or otherwise, prove that $\sin(\tan x) \ge x, \forall x \in \left[0, \frac{\pi}{4}\right]$.
(2003 - 4 Marks)

5) If the function $f : [0, 4] \mapsto R$ is differentiable then show that
   a) For $a, b \in (0, 4), (f(4))^2 - (f(0))^2 = 8f'(a) f(b)$
   b) $\int_0^4 f(t)\, dt = 2\left[\alpha f\left(\alpha^2\right) + \beta f\left(\beta^2\right)\right] \forall 0 < \alpha, \beta < 2$

(2003 - 4 Marks)

6) If $P(1) = 0$ and $\frac{dP(x)}{dx} > P(x)$ for all $x \ge 1$ then prove that $P(x) > 0$ for all $x > 1$.
(2003 - 4 Marks)

7) Using Rolle's theorem, prove that there is at least one root in $\left(45^{1/100}, 46\right)$ of the polynomial $P(x) = 51x^{101} - 2323(x)^{100} - 45x + 1035$.
(2004 - 2 Marks)

8) Prove that for $x \in \left[0, \frac{\pi}{2}\right], \sin x + 2x \ge \frac{3x(x+1)}{\pi}$. Explain the identity if any used in the proof.

(2004 - 4 Marks)

9) If $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$, for all $x_1, x_2 \in R$. Find the equation of tangent to the curve $y = f(x)$ at the point $(1, 2)$.

(2005 - 2 Marks)

10) If $p(x)$ be a polynomial of degree 3 satisfying $p(-1) = 10, p(1) = -6$ and $p(x)$ has maxima at $x = -1$ and $p'(x)$ has minima at $x = 1$. Find the distance between the local maxima and local minima of the curve.

(2005 - 4 Marks)

11) For a twice differentiable function $f(x), g(x)$ is defined as $g(x) = \left(f'(x)^2 + f''(x)\right) f(x)$ on $[a, e]$. If for $a < b < c < d < e, f(a) = 0, f(b) = 2, f(c) = -1, f(d) = 2, f(e) = 0$ then find the minimum number of zeros of $g(x)$.

(2006 - 6 Marks)

F. MATCH THE FOLLOWING

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A,B,C and D, while the statements in Column-II are labelled p,q,r,s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example. If the correct matches are A-p, s and ; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

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1) In this questions there are entries in column I and II. Each entry in column I is related to exactly one entry in column II. Write the correct letter from column II against the entry number in column I in your answer book.

(1992 - 2 Marks)

Let the functions defined in column I have domain $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Column 1	**Column 2**
(A) $x + \sin x$	(p) increasing
(B) $\sec x$	(q) decreasing
	(r) neither increasing nor decreasing

(Qs. 2-4): By appropriately matching the information given in the three columns of the following table. Let $f(x) = x + \ln x - x \ln x, x \in (0, \infty)$

Column 1 contains information about zeroes of $f(x), f'(x)$ and $f''(x)$.

Column 2 contains information about the limiting behaviour of $f(x), f'(x)$ and $f''(x)$ at infinity.

Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	**Column 2**	**Column 3**
(I) $f(x) = 0$ for some $x \in \left(1, e^2\right)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in $(0, 1)$
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is increasing in $\left(e, e^2\right)$
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in $(0, 1)$
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in $\left(e, e^2\right)$

2) Which of the following options is the only correct combination?
   a) $(I)(i)(P)$
   b) $(II)(ii)(Q)$
   c) $(III)(iii)(R)$
   d) $(IV)(iv)(S)$
3) Which of the following options is the only correct combination?
   a) $(I)(ii)(R)$
   b) $(II)(iii)(S)$
   c) $(III)(iv)(P)$
   d) $(IV)(i)(S)$
4) Which of the following options is the only correct combination?
   a) $(I)(iii)(P)$
   b) $(II)(iv)(Q)$
   c) $(III)(i)(R)$
   d) $(II)(iii)(P)$