2.32) a) $S(\beta_0,\beta_1)=\sum (y_i - \beta_0 - \beta_1 x_i)^2$ with β_0 known. We need to take the derivative of this with respect to β_1 and set it equal to zero. This gives

$$-2\sum_{i=1}^{n}(y_i - \beta_0 - \widehat{\beta}_1 x_i)x_i = 0$$

$$\widehat{\beta}_1 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n}(y_i - \beta_0)x_i$$

$$\widehat{\beta}_1 = \frac{\sum_{i=1}^{n}(y_i - \beta_0)x_i}{\sum_{i=1}^{n} x_i^2}$$

This equation satisfies the basic format of β_1, so it is reasonable.

b)

$$\begin{aligned} Var(\widehat{\beta}_1) &= \frac{1}{(\sum_{i=1}^{n} x_i^2)^2} Var(\sum_{i=1}^{n} y_i x_i) \\ &= \frac{1}{(\sum_{i=1}^{n} x_i^2)^2}(\sum_{i=1}^{n} x_i^2)\sigma^2 \\ &= \frac{\sigma^2}{\sum_{i=1}^{n} x_i^2} \end{aligned}$$

c)

$\frac{\widehat{\beta}_1 - \beta_1}{\sqrt{MS_E/\sum x_i^2}} \sim t_{n-2}$ so we get $\widehat{\beta}_1 \pm t_{\alpha/2,n-2}\sqrt{MS_E/\sum x_i^2}$ which is narrower than when both are unknown.