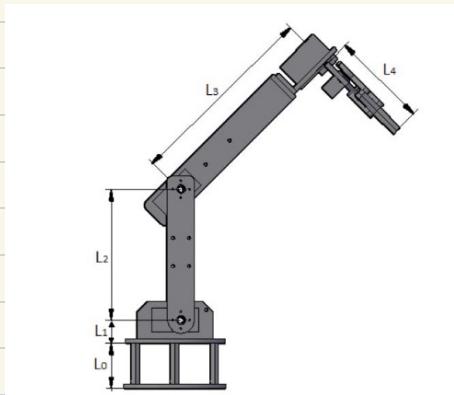


# Robotic Arm FBD

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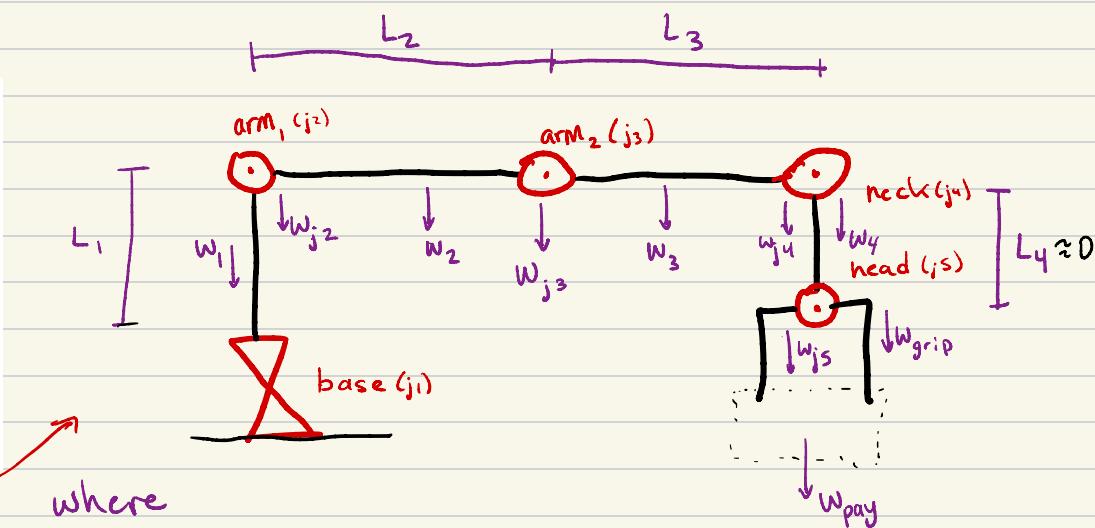
5 motors, 5-DOF



Similar Approach

where

reference FBD



$L_1$  = length from base to arm<sub>1</sub> motor

$L_2$  = length of arm 1

$L_3$  = Length of arm 2 (to the neck)

$L_4$  = Length from neck motor to head

$w_{j1}$  = Weight of joint 1 (motor and components)

$w_{j2}$  = Weight of joint 2

$w_{j3}$  = Weight of joint 3

$w_{j4}$  = Weight of joint 4

$w_{j5}$  = Weight of joint 5

$w_1$  = Weight link between base and arm 1

$w_2$  = Weight of arm 1

$w_3$  = Weight of arm 2

$w_4$  = Weight of components between neck and head

$w_{grip}$  = Weight of gripper apparatus

$w_{pay}$  = Weight of the payload

We need to calculate:

$T_{j1}$  = Torque needed at base joint (angular torque)

$T_{j2}$  = Torque needed at arm 1 joint (gravitational torque)

$T_{j3}$  = Torque needed at arm 2 joint (gravitational torque)

$T_{j4}$  = Torque needed at neck joint "

$T_{j5}$  = Torque needed at head joint "

# Robotic Arm Calculations

$T_{j1}$  = Torque of base motor (rotational torque)

$T_{j2}$  = Torque of arm 1 motor

$T_{j3}$  = Torque of arm 2 motor

$T_{j4}$  = Torque of neck motor needed

For  $T_{j1}$ , we want to rotate the entire cobot

For rotational torque, we need to find the moment of inertia of system

$$T_{j1} = I_T \times \alpha$$

$I_T = I_1 + I_2 + I_{grip} + I_{pay}$  where  $I_{1,2,3}$  is inertia about each arm and  $I_{grip}$  and  $I_{pay}$  is the inertia of the head relative to the base

For  $T_{j2}$ , this motor lifts weight of all components and payload

$$T_{j2} = W_2 \left( \frac{L_2}{2} \right) + W_{j3} L_2 + W_3 \left( L_2 + \frac{L_3}{2} \right) + (W_{j4} + W_4 + W_{j5} + W_{grip} + W_{pay}) (L_2 + L_3)$$

For  $T_{j3}$ , this motor lifts the weight from arm 2 and onward

$$T_{j3} = W_3 \left( \frac{L_3}{2} \right) + (W_{j4} + W_4 + W_{j5} + W_{grip} + W_{pay}) L_3$$

For  $T_{j4}$ , this motor will control the rotational movement of our grabber apparatus and payload.

So, there is no gravitational force needed since motor 5 (grabber motor) will take care of it.

Thus,  $T_{j4}$  can be found as

$T_{j4} = I_{cm} \cdot \alpha_{neck}$  where  $I_{cm}$  is inertia of grabber and payload about joint 4 and  $\alpha_{neck}$  is the angular acceleration we want to achieve

For  $T_{j5}$ , the motor will already come with the grabber apparatus and its

Motor/servo is the MG996R servo with rated pulling force:

9.4 kg/cm (4.8V)

11 kg/cm (6V)

# Component Specifications

When calculating the torque, I have to estimate the weight at joints and for the arms. I also have to estimate the different lengths used throughout. I will do three ranges of calculations:

- Low: Lower weight and length estimates
  - Medium: Expected/theorized weight and length estimates
  - High: Higher range of weight and length estimations

The gripper and payload specifications are:

$$\text{Length gripper } (L_{\text{grip}}) = 83 \text{ mm} = 0.083 \text{ m}$$

$$\text{Width grip } (W_{\text{grip}}) = 150 \text{ mm} = 0.15 \text{ m } (\text{worst case})$$

$$\text{Weight gripper } (m_{\text{grip}}) = 40 \text{ g} + m_{\text{servo}} = 40 \text{ g} + 55 \text{ g} = 95 \text{ g} = 0.095 \text{ kg}$$

$$\text{Payload Weight (M}_{\text{pay}}\text{)} = 2 \text{ lbs} \approx 0.91 \text{ kg}$$

The values I have chosen are as follows:

- Weight of arms considered as  $\sim 2.7 \text{ g/cm}^3$ ,  $\sim 3.5 \text{ g/cm}^3$ ,  $\sim 4.5 \text{ g/cm}^3$
  - Cross sectional area for weight of each arm assumed  $.00021 \text{ m}^2$ ,  $.001 \text{ m}^2$ ,  $.0015 \text{ m}^2$
  - $W_1$  will be close to zero, as the neck motor and gripper will be attached. Thus  $W_{15}$  is weight of gripper
  - We estimate the weight of associated PCBs to be  $\approx 4 \text{ g}$ . Let's use  $0.6 \text{ g}$  to account for other things as well

## Low Range:

$$\alpha_{base} = 1.5 \text{ rad/s}^2 \quad L_4 = 0.083 \text{ m} \quad m_{j5} = 0.095 \text{ kg} \quad m_{j2} \approx 1.4 \text{ kg} + 0.6 \text{ kg} \approx 1.4 \text{ kg}$$

$$L_1 = 5 \text{ mm} = 0.005 \text{ m} \quad W_1 = (0.0024 \text{ m}^2 \cdot 0.005 \text{ m}) 2700 \text{ kg/m}^3 = 0.0324 \text{ kg} \quad W_{j3} \approx 0.8 \text{ kg}$$

$$L_2 = 5 \text{ inches} = 0.127 \text{ m} \quad W_2 = 0.0822 \text{ kg} \quad W_{\text{pay}} = 2 \text{ lb} = 0.91 \text{ kg} \quad W_{j4} \approx 0.25 \text{ kg}$$

$$L_3 = 4 \text{ inches} = 0.1016 \text{ m} \quad w_3 = 0.06583 \text{ kg}$$

## Mid Range:

$$\alpha_{base} = 2.25 \text{ rad/s}^2 \quad L_y = 0.083 \text{ m} \quad W_{js} = 0.095 \text{ kg} \quad W_{j2} \approx 1.4 \text{ kg}$$

$$L_1 = 10 \text{ mm} = 0.01 \text{ m} \quad w_1 = (0.001 \text{ m}^2 \cdot 0.1 \text{ m}) 3500 \text{ kg/m}^3 = 0.35 \text{ kg} \quad w_{j3} \approx 0.8 \text{ kg}$$

$$L_2 = 8 \text{ inches} = 0.2032 \text{ m} \quad W_2 = 0.7112 \text{ kg} \quad W_{\text{pay}} = 0.91 \text{ kg} \quad W_{\text{ju}} \approx 0.25 \text{ kg}$$

$$z = b \cdot 5 \text{ inches} = 0.1651 \text{ m } W_3 = 0.5778 \text{ kg}$$

## High Range :

$$\omega_{base} = 3 \text{ rad/s}^2 \quad L_y = 0.083 \text{ m} \quad W_{\text{sc}} = 0.095 \text{ kg} \quad W_{j2} \approx 1.4 \text{ kg}$$

$$W_{i3} \approx 1.4 \text{ kg}$$

(W) = 20 kg

W<sub>IV</sub> ≈ 0.25 kg

$$z = 1 \text{ foot} = 0.3048 \text{ m} \quad W_2 = 2.0574 \text{ kg}$$

$$-3 = 10 \text{ inches} = 0.254 \text{ m } W_2 = 1.7145 \text{ kg}$$

# Actual Calculations

For the moment of inertia of the gripper and payload (not in relation to the base)

$$I_{cm} = I_{grip} + I_{payload}$$

Treat gripper as rectangular beam rotating about one end

$$I_{grip} = \frac{1}{3} M_{grip} \cdot L_{grip}^2 = \frac{1}{3} (0.095\text{kg}) (0.083\text{m})^2 = 0.00021815 = 2.1815 \times 10^{-5} \text{kg}\cdot\text{m}^2$$

Treat payload as point mass at the end of gripper

where  $L_{grip}$  is the distance from the axis of rotation

$$I_{pay} = M_{pay} \cdot L_{grip}^2 = (0.91\text{kg}) (0.083\text{m})^2 = .006269 \text{kg}\cdot\text{m}^2$$

$$\text{So, } I_{cm} = I_{grip} + I_{pay} = 2.1815 \cdot 10^{-5} + .006269 = \boxed{.006487 \text{ kg}\cdot\text{m}^2}$$

Now for inertia about arm, and  $arm_2$  and the gripper/payload relative to the base

Both arms are treated as rods rotating about one end

$I_{arm} = \frac{1}{3} M L^2$ . As we move further from the base, we must use the Parallel Axis Theorem thus,

$$I_{arm_1} = \frac{1}{3} M_{arm_1} L_{arm_1}^2 = \frac{1}{3} W_2 L_2^2$$

$$I_{arm_2} = \frac{1}{3} M_{arm_2} L_{arm_2}^2 + M_{arm_2} L_{arm_1}^2 = \frac{1}{3} W_3 L_3^2 + W_3 L_2^2$$

$$I_{grip} = \frac{1}{3} M_{grip} L_{grip}^2 + M_{grip} (L_{arm_1} + L_{arm_2})^2 = \frac{1}{3} W_{grip} L_{grip}^2 + W_{grip} (L_2 + L_3)^2$$

$$I_{pay} = M_{pay} (L_{arm_1} + L_{arm_2} + L_{grip})^2 = W_{grip} (L_2 + L_3 + L_{grip})^2$$

Now for low, med, and high, find the torque needed for base motor

Low

$$\omega_{base} = 1.5 \text{ rad/s}^2 \quad L_4 = 0.083\text{m} \quad W_{j_1} = 0.095\text{kg} \quad W_{j_2} \approx 1.4\text{kg} + 0.8\text{kg} \approx 2.2\text{kg}$$

$$L_1 = 5\text{mm} = 0.005\text{m} \quad W_1 = (0.005\text{m}^2 \cdot 0.095\text{kg}) (2000\text{kg/m}^3) = 0.0025\text{kg} \quad W_{j_3} \approx 0.8\text{kg}$$

$$L_2 = 5\text{ inches} = 0.127\text{m} \quad W_2 = 0.0822\text{kg} \quad W_{j_4} = 2.16 \cdot 0.91\text{kg} \approx 1.91\text{kg}$$

$$L_3 = 4\text{ inches} = 0.1016\text{m} \quad W_3 = 0.06983\text{kg} \quad L_{grip} = 0.083\text{m}$$

$$I_{arm_1} = \frac{1}{3} W_2 L_2^2 = \frac{1}{3} (0.0822\text{kg}) (0.127\text{m})^2 = \boxed{.00044193 \text{ kg}\cdot\text{m}^2}$$

$$I_{arm_2} = \frac{1}{3} W_3 L_3^2 + W_3 L_2^2 = \frac{1}{3} (0.06983\text{kg}) (0.1016\text{m})^2 + (0.06983\text{kg}) (0.127\text{m})^2 = \boxed{.0012871 \text{ kg}\cdot\text{m}^2}$$

$$I_{grip} = \frac{1}{3} W_{grip} L_{grip}^2 + W_{grip} (L_2 + L_3)^2 =$$

$$= \frac{1}{3} (0.095\text{kg}) (0.083\text{m})^2 + (0.095\text{kg}) (0.127\text{m} + 0.1016\text{m})^2 = \boxed{.000518 \text{ kg}\cdot\text{m}^2}$$

$$I_{pay} = W_{pay} (L_2 + L_3 + L_{grip}) = (0.91\text{kg}) (0.127\text{m} + 0.1016\text{m} + 0.083\text{m})^2 = \boxed{0.0883 \text{ kg}\cdot\text{m}^2}$$

$$I_{total} = I_{arm_1} + I_{arm_2} + I_{grip} + I_{pay} = \boxed{0.0952 \text{ kg}\cdot\text{m}^2}$$

Med

$$\omega_{base} = 2.25 \text{ rad/s}^2 \quad L_4 = 0.083\text{m} \quad W_{j_1} = 0.095\text{kg} \quad W_{j_2} \approx 1.4\text{kg}$$

$$L_1 = 10\text{mm} = 0.01\text{m} \quad W_1 = (0.01\text{m}^2 \cdot 0.095\text{kg}) (2000\text{kg/m}^3) = 0.35\text{kg}$$

$$L_2 = 8\text{ inches} = 0.2032\text{m} \quad W_2 = 0.7112\text{kg} \quad W_{j_3} \approx 0.8\text{kg}$$

$$L_3 = 6.5\text{ inches} = 0.1651\text{m} \quad W_3 = 0.57785\text{kg} \quad L_{grip} = 0.083\text{m}$$

$$I_{arm_1} = \frac{1}{3} W_2 L_2^2 = \frac{1}{3} (0.7112\text{kg}) (0.2032\text{m})^2 = \boxed{.009788 \text{ kg}\cdot\text{m}^2}$$

$$I_{arm_2} = \frac{1}{3} W_3 L_3^2 + W_3 L_2^2 = \frac{1}{3} (0.57785\text{kg}) (0.1651\text{m})^2 + (0.57785\text{kg}) (0.2032\text{m})^2 = \boxed{.0291 \text{ kg}\cdot\text{m}^2}$$

$$I_{grip} = \frac{1}{3} W_{grip} L_{grip}^2 + W_{grip} (L_2 + L_3)^2 =$$

$$= \frac{1}{3} (0.095\text{kg}) (0.083\text{m})^2 + (0.095\text{kg}) (0.2032\text{m} + 0.1651\text{m})^2 = \boxed{0.0131 \text{ kg}\cdot\text{m}^2}$$

$$I_{pay} = W_{pay} (L_2 + L_3 + L_{grip}) =$$

$$= (.91\text{kg}) (0.2032\text{m} + 0.1651\text{m} + 0.083\text{m})^2 = \boxed{0.1853 \text{ kg}\cdot\text{m}^2}$$

High

$$\omega_{base} = 3 \text{ rad/s}^2 \quad L_4 = 0.083\text{m} \quad W_{j_1} = 0.095\text{kg} \quad W_{j_2} \approx 2.2\text{kg}$$

$$L_1 = 13\text{mm} = 0.013\text{m} \quad W_1 = (0.013\text{m}^2 \cdot 0.095\text{kg}) (4500\text{kg/m}^3) = 0.177\text{kg}$$

$$L_2 = 1\text{ foot} = 0.3048\text{m} \quad W_2 = 2.0574\text{kg} \quad W_{j_3} \approx 0.8\text{kg}$$

$$L_3 = 10\text{ inches} = 0.254\text{m} \quad W_3 = 1.7145\text{kg} \quad L_{grip} = 0.083\text{m}$$

$$I_{arm_1} = \frac{1}{3} W_2 L_2^2 = \frac{1}{3} (2.0574\text{kg}) (0.3048\text{m})^2 = \boxed{.0637 \text{ kg}\cdot\text{m}^2}$$

$$I_{arm_2} = \frac{1}{3} W_3 L_3^2 + W_3 L_2^2 = \frac{1}{3} (1.7145\text{kg}) (0.254\text{m})^2 + (1.7145\text{kg}) (0.3048\text{m})^2 = \boxed{0.19615 \text{ kg}\cdot\text{m}^2}$$

$$I_{grip} = \frac{1}{3} W_{grip} L_{grip}^2 + W_{grip} (L_2 + L_3)^2 =$$

$$= \frac{1}{3} (0.095\text{kg}) (0.083\text{m})^2 + (0.095\text{kg}) (0.3048\text{m} + 0.254\text{m})^2 = \boxed{0.02988 \text{ kg}\cdot\text{m}^2}$$

$$I_{pay} = W_{pay} (L_2 + L_3 + L_{grip}) =$$

$$= (.91\text{kg}) (0.3048\text{m} + 0.254\text{m} + 0.083\text{m})^2 = \boxed{0.3748 \text{ kg}\cdot\text{m}^2}$$

To achieve  $\omega = 1.5 \text{ rad/s}^2$

$$N = \frac{1}{\text{kg}\cdot\text{m}}$$

$$T = 0.0952 \text{ kg}\cdot\text{m}^2 \cdot 1.5 \text{ rad/s}^2$$

$$= 0.1428 \frac{\text{kg}\cdot\text{rad}}{\text{s}^2}$$

$$= 0.1428 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = \boxed{0.1428 \text{ N/m}}$$

$$I_{total} = I_{arm_1} + I_{arm_2} + I_{grip} + I_{pay} = \boxed{0.0952 \text{ kg}\cdot\text{m}^2}$$

like before,

$$T = 0.2373 \text{ kg}\cdot\text{m}^2 \cdot 2.25 \text{ rad/s}^2$$

$$T = .533925 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = \boxed{.533925 \text{ N/m}}$$

$$I_{total} = I_{arm_1} + I_{arm_2} + I_{grip} + I_{pay} = \boxed{0.0952 \text{ kg}\cdot\text{m}^2}$$

like before,

$$T = .6645 \text{ kg}\cdot\text{m}^2 \cdot 3 \text{ rad/s}^2$$

$$= 1.9935 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = \boxed{1.9935 \text{ N/m}}$$

# Continued Calculations

Given our specifications:

## Low Range:

$$\alpha_{base} = 1.5 \text{ rad/s}^2 \quad L_4 = 0.083 \text{ m} \quad W_{j_3} = 0.095 \text{ kg} \quad W_{j_2} \approx 1.4 \text{ kg} + 0.6 \text{ kg} \approx 1.4 \text{ kg}$$

$$L_1 = 5 \text{ mm} = 0.005 \text{ m} \quad W_1 = (.0002 \text{ m}^2 \cdot .005 \text{ m}) 2700 \text{ kg/m}^3 = 0.0324 \text{ kg} \quad W_{j_3} \approx 0.8 \text{ kg}$$

$$L_2 = 5 \text{ inches} = 0.127 \text{ m} \quad W_2 = 0.0822 \text{ kg} \quad W_{pay} = 2 \text{ lb} = 0.91 \text{ kg} \quad W_{j_4} \approx 0.25 \text{ kg}$$

$$L_3 = 4 \text{ inches} = 0.1016 \text{ m} \quad W_3 = 0.06583 \text{ kg}$$

## Mid Range:

$$\alpha_{base} = 2.25 \text{ rad/s}^2 \quad L_4 = 0.083 \text{ m} \quad W_{j_3} = 0.095 \text{ kg} \quad W_{j_2} \approx 1.4 \text{ kg}$$

$$L_1 = 10 \text{ mm} = 0.01 \text{ m} \quad W_1 = (.001 \text{ m}^2 \cdot .01 \text{ m}) 2500 \text{ kg/m}^3 = 0.35 \text{ kg} \quad W_{j_3} \approx 0.8 \text{ kg}$$

$$L_2 = 8 \text{ inches} = 0.2032 \text{ m} \quad W_2 = 0.7112 \text{ kg} \quad W_{pay} = 0.91 \text{ kg} \quad W_{j_4} \approx 0.25 \text{ kg}$$

$$L_3 = 6.5 \text{ inches} = 0.1651 \text{ m} \quad W_3 = 0.57785 \text{ kg}$$

## High Range:

$$\alpha_{base} = 3 \text{ rad/s}^2 \quad L_4 = 0.083 \text{ m} \quad W_{j_3} = 0.095 \text{ kg} \quad W_{j_2} \approx 1.4 \text{ kg}$$

$$L_1 = 13 \text{ mm} = 0.013 \text{ m} \quad W_1 = (.0013 \text{ m}^2 \cdot .013 \text{ m}) 4500 \text{ kg/m}^3 = 0.17 \text{ kg} \quad W_{j_3} \approx 0.8 \text{ kg}$$

$$L_2 = 1 \text{ foot} = 0.3048 \text{ m} \quad W_2 = 2.0574 \text{ kg}$$

$$L_3 = 10 \text{ inches} = 0.254 \text{ m} \quad W_3 = 1.7145 \text{ kg}$$

Find the torque at the first arm joint ( $T_{j_2}$ ) for low, med, high cases where:

$$T_{j_2} = W_2 \left( \frac{L_2}{2} \right) + W_{j_3} L_2 + W_3 \left( L_2 + \frac{L_3}{2} \right) + (W_{j_4} + W_4 + W_{j_3} + W_{grip} + W_{pay})(L_2 + L_3)$$

$$T_{j_2L} = (0.0822 \text{ kg}) \left( \frac{.127 \text{ m}}{2} \right) + (.8 \text{ kg})(.127 \text{ m}) + (.06583 \text{ kg}) \left( .127 \text{ m} + \frac{.1016 \text{ m}}{2} \right) + (.25 \text{ kg} + .095 \text{ kg} + .91 \text{ kg}) \left( .127 \text{ m} + .1016 \text{ m} \right) =$$

$$T_{j_2L} = 0.40542 \text{ kg} \cdot \text{m} = \underline{563 \text{ oz} \cdot \text{in}}$$

$$T_{j_2M} = (.7112 \text{ kg}) \left( \frac{.2032 \text{ m}}{2} \right) + (.8 \text{ kg})(.2032 \text{ m}) + (.57785 \text{ kg}) \left( .2032 \text{ m} + \frac{.1651 \text{ m}}{2} \right) + (.25 \text{ kg} + .095 \text{ kg} + .91 \text{ kg}) \left( .2032 \text{ m} + .1651 \text{ m} \right) =$$

$$T_{j_2M} = 0.86215 \text{ kg} \cdot \text{m} = \underline{1197.3 \text{ oz} \cdot \text{in}}$$

$$T_{j_2H} = (2.0574 \text{ kg}) \left( \frac{.3048 \text{ m}}{2} \right) + (.8 \text{ kg})(.3048 \text{ m}) + (1.7145 \text{ kg}) \left( .3048 \text{ m} + \frac{.254 \text{ m}}{2} \right) + (.25 \text{ kg} + .095 \text{ kg} + .91 \text{ kg}) \left( .3048 \text{ m} + .254 \text{ m} \right) =$$

$$T_{j_2H} = 1.99 \text{ kg} \cdot \text{m} = \underline{2763 \text{ oz} \cdot \text{in}}$$

# Continued Calculations

For reference:

**Low Range:**

$$\alpha_{base} = 1.5 \text{ rad/s}^2 \quad L_4 = 0.083 \text{ m} \quad W_{j_3} = 0.095 \text{ kg} \quad W_{j_2} \approx 1.4 \text{ kg} + 0.6 \text{ kg} \approx 1.4 \text{ kg}$$

$$L_1 = 5 \text{ mm} = 0.005 \text{ m} \quad W_1 = (0.005 \text{ m}^2 \cdot 0.005 \text{ m}) 2700 \text{ kg/m}^3 = 0.00324 \text{ kg} \quad W_{j_3} \approx 0.8 \text{ kg}$$

$$L_2 = 5 \text{ inches} = 0.127 \text{ m} \quad W_2 = 0.0822 \text{ kg} \quad W_{pay} = 2 \text{ lb} = 0.91 \text{ kg} \quad W_{j_4} \approx 0.25 \text{ kg}$$

$$L_3 = 4 \text{ inches} = 0.1016 \text{ m} \quad W_3 = 0.06583 \text{ kg}$$

**Mid Range:**

$$\alpha_{base} = 2.25 \text{ rad/s}^2 \quad L_4 = 0.083 \text{ m} \quad W_{j_3} = 0.095 \text{ kg} \quad W_{j_2} \approx 1.4 \text{ kg}$$

$$L_1 = 10 \text{ mm} = 0.01 \text{ m} \quad W_1 = (0.01 \text{ m}^2 \cdot 0.01 \text{ m}) 2500 \text{ kg/m}^3 = 0.035 \text{ kg} \quad W_{j_3} \approx 0.8 \text{ kg}$$

$$L_2 = 8 \text{ inches} = 0.2032 \text{ m} \quad W_2 = 0.7112 \text{ kg} \quad W_{pay} = 0.91 \text{ kg} \quad W_{j_4} \approx 0.25 \text{ kg}$$

$$L_3 = 6.5 \text{ inches} = 0.1651 \text{ m} \quad W_3 = 0.57785 \text{ kg}$$

**High Range:**

$$\alpha_{base} = 3 \text{ rad/s}^2 \quad L_4 = 0.083 \text{ m} \quad W_{j_3} = 0.095 \text{ kg} \quad W_{j_2} \approx 1.4 \text{ kg}$$

$$W_{j_3} \approx 0.8 \text{ kg}$$

$$L_1 = 13 \text{ mm} = 0.013 \text{ m} \quad W_1 = (0.013 \text{ m}^2 \cdot 0.013 \text{ m}) 4500 \text{ kg/m}^3 = 0.117 \text{ kg} \quad W_{j_3} \approx 0.25 \text{ kg}$$

$$L_2 = 1 \text{ foot} = 0.3048 \text{ m} \quad W_2 = 2.0574 \text{ kg}$$

$$L_3 = 10 \text{ inches} = 0.254 \text{ m} \quad W_3 = 1.7145 \text{ kg}$$

Let's now find the torque needed for the second arm joint ( $T_{j_3}$ ) where:

$$T_{j_3} = W_3 \left( \frac{L_3}{2} \right) + (W_{j_4} + W_4 + W_{j_3} + W_{grip} + W_{pay}) L_3$$

$L_4 = 0 \quad W_{j_3} = W_{grip}$

$$T_{j_3L} = (0.06583 \text{ kg}) \left( \frac{0.1016 \text{ m}}{2} \right) + (0.25 \text{ kg} + 0.095 \text{ kg} + 0.91 \text{ kg}) \cdot 0.1016 \text{ m} =$$

$$T_{j_3L} = 0.13085 \text{ kg} \cdot \text{m} = \underline{181.72 \text{ oz} \cdot \text{in}}$$

$$T_{j_3M} = (0.57785 \text{ kg}) \left( \frac{0.1651 \text{ m}}{2} \right) + (0.25 \text{ kg} + 0.095 \text{ kg} + 0.91 \text{ kg}) \cdot 0.1651 \text{ m} =$$

$$T_{j_3M} = 0.2549 \text{ kg} \cdot \text{m} = \underline{354 \text{ oz} \cdot \text{in}}$$

$$T_{j_3H} = (1.7145 \text{ kg}) \left( \frac{0.254 \text{ m}}{2} \right) + (0.25 \text{ kg} + 0.095 \text{ kg} + 0.91 \text{ kg}) \cdot 0.254 \text{ m} =$$

$$T_{j_3H} = 0.5365 \text{ kg} \cdot \text{m} = \underline{745 \text{ oz} \cdot \text{in}}$$

# Continued Calculations

Now for the torque of the neck motor ( $T_{jy}$ ), we need  $I_{cm}$ , which we found above, and an estimate for how fast we want to spin

$$T_{jy} = I_{cm} \times \alpha_{neck} \quad I_{cm} = 0.006487 \text{ kg}\cdot\text{m}^2$$

For low, med, high, lets assume different accelerations

$$T_{jyL} = 0.006487 \text{ kg}\cdot\text{m}^2 \cdot 1 \text{ rad/s}^2 = .006487 \text{ N}\cdot\text{m}$$

$$T_{jyM} = 0.006487 \text{ kg}\cdot\text{m}^2 \cdot 1.5 \text{ rad/s}^2 = .0097305 \text{ N}\cdot\text{m}$$

$$T_{jyH} = 0.006487 \text{ kg}\cdot\text{m}^2 \cdot 2 \text{ rad/s}^2 = .01297 \text{ N}\cdot\text{m}$$

With all calculations,

Motor/joint #	Rated torque LOW	Rated torque MED	Rated Torque HIGH
Base motor ( $T_{j1}$ ) <i>rotational</i>	.1428 N/m	.5339 N/m	1.9935 N·m
Arm 1 Motor ( $T_{j2}$ )	563 oz·in	1197 oz·in	2763 oz·in
Arm 2 Motor ( $T_{j3}$ )	181.72 oz·in	354 oz·in	745 oz·in
Neck Motor ( $T_{jy}$ ) <i>rotational</i>	.006487 N·m	.0097305 N·m	.01297 N·m
Gripper Motor ( $T_{j5}$ ) <i>(stall)</i>	11 kg/cm	11 kg/cm	11 kg/cm