Robotic Arm FBD

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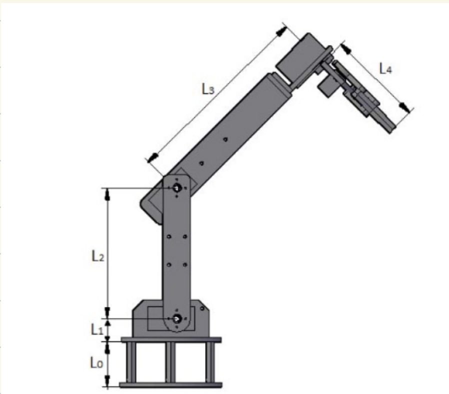
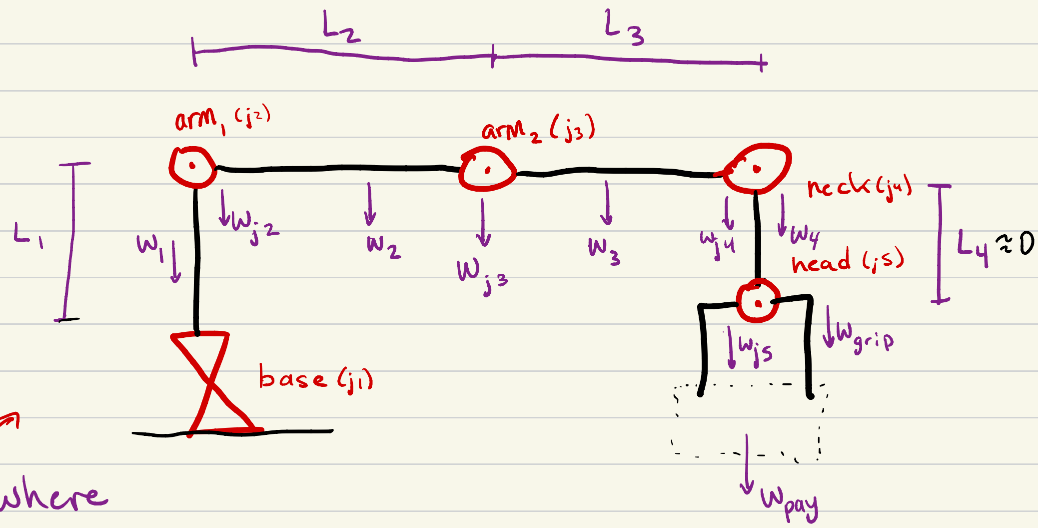
5 motors, 5-DOF



Similar Approach

reference FBD



where

L_1 = length from base to Arm1 motor
L_2 = length of arm 1
L_3 = Length of arm 2 (to the neck)
L_4 = Length from neck motor to head

W_1 = Weight link between base and arm 1
W_2 = Weight of arm 1
W_3 = Weight of arm 2
W_4 = Weight of components between neck and head

W_{grip} = Weight of gripper apparatus
W_{pay} = Weight of the payload

W_{j1} = Weight of joint 1 (motor and components)
W_{j2} = Weight of joint 2
W_{j3} = Weight of joint 3
W_{j4} = Weight of joint 4
W_{j5} = Weight of joint 5

We need to calculate:

T_{j1} = Torque needed at base joint (angular torque)
T_{j2} = Torque needed at arm 1 joint (gravitational torque)
T_{j3} = Torque needed at arm 2 joint (gravitational torque)
T_{j4} = Torque needed at neck joint "
T_{j5} = Torque needed at head joint "

Robotic Arm Calculations

T_{j1} = Torque of base motor (Rotational torque)

T_{j2} = Torque of arm 1 motor

T_{j3} = Torque of arm 2 motor

T_{j4} = Torque of neck motor needed

For T_{j1}, we want to rotate the entire cobot

For rotational torque, we need to find the moment of inertia of system

$T_{j1} = I_T \times \alpha$

$I_T = I_1 + I_2 + I_{grip} + I_{pay}$ where $I_{1,2,3}$ is Inertia about each arm and I_{grip} and I_{pay} is the inertia of the head relative to the base

For T_{j2}, this motor lifts weight of all components and payload

$$T_{j2} = W_2\left(\frac{L_2}{2}\right) + W_{j3}L_2 + W_3\left(L_2 + \frac{L_3}{2}\right) + \left(W_{j4} + W_4 + W_{j5} + W_{grip} + W_{pay}\right)\left(L_2 + L_3\right)$$

For T_{j3}, this motor lifs the weight from arm 2 and onward

$$T_{j3} = W_3\left(\frac{L_3}{2}\right) + \left(W_{j4} + W_4 + W_{j5} + W_{grip} + W_{pay}\right)L_3$$

For T_{j4}, this motor will control the rotational movement of our grabber apparatus and payload.

So, there is no gravitational force needed since motor 5 (grabber motor) will take care of it.

Thus, T_{j4} can be found as

$T_{j4} = I_{cm} \cdot \alpha_{neck}$ where I_{cm} is inertia of grabber and payload about joint 4 and α_{neck} is the angular acceleration we want to achieve

For T_{j5}, the motor will already come with the grabber apparatus and its

Motor/servo is the MG996R servo with rated pulling force:

9.4 kg/cm (4.8V)

11 kg/cm (6V)

Component Specifications

When calculating the torque, I have to estimate the weight at joints and for the arms. I also have to estimate the different lengths used throughout. I will do three ranges of calculations:

- **Low:** Lower weight and length estimates
- **Medium:** Expected/theorized weight and length estimates
- **High:** Higher range of weight and length estimations

The gripper and payload specifications are:

Length gripper (L_{grip}) = 83 mm = 0.083 m

Width grip (W_{grip}) = 150 mm = 0.15 m (worst case)

Weight gripper (m_{grip}) = 40g + m_{servo} = 40g + 55g = 95g = 0.095 kg

Payload weight (m_{pay}) = 2 lbs ≈ 0.91 kg

The values I have chosen are as follows:

- Weight of arms considered as low: ~2.7 g/cm³ (2700 kg/m³), mid: ~3.5 g/cm³, high: ~4.5 g/cm³
- Cross sectional area for weight of each arm assumed low: .00024 m², mid: .001 m², high: .0015 m²
- W_4 will be close to zero, as the neck motor and gripper will be attached. Thus W_{j5} is weight of gripper
- We estimate the weight of associated pcbs to be ≈ .4g. Lets use 0.6g to account for other things as well

Low Range:

$\alpha_{base} = 1.5\ rad/s^2$ $L_4 = 0.083\,m$ $W_{j5} = 0.095\,kg$ $W_{j2} \approx 1.4\,kg + 0.6g \approx 1.4\,kg$

$L_1 = 5\,mm = 0.005\,m$ $W_1 = (.00024\,m^2 \cdot .005\,m)\,2700\,kg/m^3 = .00324\,kg$ $W_{j3} \approx 0.8\,kg$

$L_2 = 5\ inches = 0.127\,m$ $W_2 = 0.0822\,kg$ $W_{pay} = 2\,lb = 0.91\,kg$ $W_{j4} \approx 0.25\,kg$

$L_3 = 4\ inches = 0.1016\,m$ $W_3 = 0.06583\,kg$

Mid Range:

$\alpha_{base} = 2.25\ rad/s^2$ $L_4 = 0.083\,m$ $W_{j5} = 0.095\,kg$ $W_{j2} \approx 1.4\,kg$

$L_1 = 10\,mm = 0.01\,m$ $W_1 = (.001\,m^2 \cdot .01\,m)\,3500\,kg/m^3 = .035\,kg$ $W_{j3} \approx 0.8\,kg$

$L_2 = 8\ inches = 0.2032\,m$ $W_2 = 0.7112\,kg$ $W_{pay} = 0.91\,kg$ $W_{j4} \approx 0.25\,kg$

$L_3 = 6.5\ inches = 0.1651\,m$ $W_3 = 0.57785\,kg$

High Range:

$\alpha_{base} = 3\ rad/s^2$ $L_4 = 0.083\,m$ $W_{j5} = 0.095\,kg$ $W_{j2} \approx 1.4\,kg$

$W_{j3} \approx 0.8\,kg$

$L_1 = 13\,mm = 0.013\,m$ $W_1 = (.0015\,m^2 \cdot .013\,m)\,4500\,kg/m^3 = 0.117\,kg$ $W_{j4} \approx 0.25\,kg$

$L_2 = 1\ foot = 0.3048\,m$ $W_2 = 2.0574\,kg$

$L_3 = 10\ inches = 0.254\,m$ $W_3 = 1.7145\,kg$

Actual Calculations

For the moment of inertia of the gripper and payload (not in relation to the base)

$$I_{cm} = I_{grip} + I_{payload}$$

Treat gripper as rectangular beam rotating about one end

$$I_{grip} = \frac{1}{3} M_{grip} \cdot L_{grip}^2 = \frac{1}{3}(0.095\,kg)(0.083\,m)^2 = 0.00021815 = 2.1815 \times 10^{-5}\,kg \cdot m^2$$

Treat payload as point mass at the end of gripper

Where L_{grip} is the distance from the axis of rotation

$$I_{pay} = M_{pay} \cdot L_{grip}^2 = (0.91\,kg)(0.083\,m)^2 = .006269\,kg \cdot m^2$$

$$\text{So, } I_{cm} = I_{grip} + I_{pay} = 2.1815 \cdot 10^{-5} + .006269 = \boxed{.006487\,kg \cdot m^2}$$

Now for inertia about arm_1 and arm_2 and the gripper/payload relative to the base

Both arms are treated as rods rotating about one end

$I_{arm} = \frac{1}{3} m L^2$. As we move further from the base, we must use the Parallel Axis Theorem

thus,

$$I_{arm_1} = \frac{1}{3} M_{arm_1} L_{arm_1}^2 = \frac{1}{3} W_2 L_2^2$$

$$I_{arm_2} = \frac{1}{3} M_{arm_2} L_{arm_2} + M_{arm_2} L_{arm_1}^2 = \frac{1}{3} W_3 L_3^2 + W_3 L_2^2$$

$$I_{grip} = \frac{1}{3} M_{grip} L_{grip}^2 + M_{grip}(L_{arm_1} + L_{arm_2})^2 = \frac{1}{3} W_{grip} L_{grip}^2 + W_{grip}(L_2 + L_3)^2$$

$$I_{pay} = M_{pay}(L_{arm_1} + L_{arm_2} + L_{grip})^2 = W_{grip}(L_2 + L_3 + L_{grip})^2$$

Now for low, med, and high, find the torque needed for base motor

Low

$\alpha_{base} = 1.5\,rad/s^2$ $\quad L_4 = 0.083\,m$ $\quad W_{j5} = 0.095\,kg$ (〃 W_{grip}) $\quad W_{j2} \approx 1.4\,kg + 0.6\,g \approx 1.4\,kg$

$L_1 = 5\,mm = 0.005\,m$ $\quad W_1 = (.00024\,m^2 \cdot .005\,m)\,2700\,kg/m^3 = .00324\,kg$ $\quad W_{j3} \approx 0.8\,kg$

$L_2 = 5$ inches $= 0.127\,m$ $\quad W_2 = 0.0822\,kg$ $\quad W_{pay} = 2\,lb = 0.91\,kg$ $\quad W_{j4} \approx 0.25\,kg$

$L_3 = 4$ inches $= 0.1016\,m$ $\quad W_3 = 0.06583\,kg$ $\quad L_{grip} = .083\,m$

$$I_{arm_1} = \frac{1}{3} W_2 L_2^2 = \frac{1}{3}(.0822\,kg)(.127\,m)^2 = .00044193\,kg \cdot m^2$$

$$I_{arm_2} = \frac{1}{3} W_3 L_3^2 + W_3 L_2^2 = \frac{1}{3}(.0658\,kg)(.1016\,m)^2 + (.0658\,kg)(.127\,m)^2 = 0.001287\,kg\,m^2$$

$$I_{grip} = \frac{1}{3} W_{grip} L_{grip}^2 + W_{grip}(L_2 + L_3)^2 =$$

$$= \frac{1}{3}(.095\,kg)(.083\,m)^2 + (.095\,kg)(.127\,m + .1016\,m)^2 = 0.00518\,kg \cdot m^2$$

$$I_{pay} = W_{pay}(L_2 + L_3 + L_{grip}) = (0.91\,kg)(.127\,m + .1016\,m + .083\,m)^2 = 0.0883\,kg \cdot m^2$$

$$I_{total} = I_{arm_1} + I_{arm_2} + I_{grip} + I_{pay} = 0.0952\,kg \cdot m^2$$

To achieve $\alpha = 1.5\,rad/s^2$ $\qquad$ $\#\, N = 1\,\frac{kg \cdot m}{s^2}$

$$T = 0.0952\,kg \cdot m^2 \cdot 1.5\,rad/s^2$$

$$= 0.1428\,\frac{kg \cdot rad \cdot m^2}{s^2}$$

$$= 0.1428\,\frac{kg \cdot m^2}{s^2} = \boxed{0.1428\,N/m}$$

Med

$\alpha_{base} = 2.25\,rad/s^2$ $\quad L_4 = 0.083\,m$ $\quad W_{j5} = 0.095\,kg$ (〃 W_{grip}) $\quad W_{j2} \approx 1.4\,kg$

$L_1 = 10\,mm = 0.01\,m$ $\quad W_1 = (.001\,m^2 \cdot .01\,m)\,3500\,kg/m^3 = .035\,kg$ $\quad W_{j3} \approx 0.8\,kg$

$L_2 = 8$ inches $= 0.2032\,m$ $\quad W_2 = 0.7112\,kg$ $\quad W_{pay} = 0.91\,kg$ $\quad W_{j4} \approx 0.25\,kg$

$L_3 = 6.5$ inches $= 0.1651\,m$ $\quad W_3 = 0.57785\,kg$ $\quad L_{grip} = .083\,m$

$$I_{arm_1} = \frac{1}{3} W_2 L_2^2 = \frac{1}{3}(.7112\,kg)(.2032\,m)^2 = .009788\,kg \cdot m^2$$

$$I_{arm_2} = \frac{1}{3} W_3 L_3^2 + W_3 L_2^2 = \frac{1}{3}(.5778\,kg)(.1651)^2 + (.5778\,kg)(.2032\,m)^2$$

$$= 0.0291\,kg \cdot m^2$$

$$I_{grip} = \frac{1}{3} W_{grip} L_{grip}^2 + W_{grip}(L_2 + L_3)^2 =$$

$$= \frac{1}{3}(.095\,kg)(.083\,m)^2 + (.095)(.2032\,m + .1651\,m)^2 = 0.0131\,kg \cdot m^2$$

$$I_{pay} = W_{pay}(L_2 + L_3 + L_{grip}) =$$

$$= (.91\,kg)(.2032\,m + .1651\,m + .083\,m)^2 = 0.1853\,kg \cdot m^2$$

$$I_{total} = I_{arm_1} + I_{arm_2} + I_{grip} + I_{pay} = 0.2373\,kg \cdot m^2$$

like before,

$$T = 0.2373\,kg \cdot m^2 \cdot 2.25\,rad/s^2$$

$$T = .533925\,\frac{kg \cdot m^2}{s^2} = \boxed{.533925\,N/m}$$

High

$\alpha_{base} = 3\,rad/s^2$ $\quad L_4 = 0.083\,m$ $\quad W_{j5} = 0.095\,kg$ (〃 W_{grip}) $\quad W_{j2} \approx 1.4\,kg$

$W_{j3} \approx 0.8\,kg$

$L_1 = 13\,mm = 0.013\,m$ $\quad W_1 = (.0015\,m^2 \cdot .013\,m)\,4500\,kg/m^3 = 0.117\,kg$ $\quad W_{j4} \approx 0.25\,kg$

$L_2 = 1$ foot $= 0.3048\,m$ $\quad W_2 = 2.0574\,kg$ $\quad W_{pay} = .091\,kg$

$L_3 = 10$ inches $= 0.254\,m$ $\quad W_3 = 1.7145\,kg$ $\quad L_{grip} = .083\,m$

$$I_{arm_1} = \frac{1}{3} W_2 L_2^2 = \frac{1}{3}(2.057\,kg)(.3048\,m)^2 = .0637\,kg \cdot m^2$$

$$I_{arm_2} = \frac{1}{3} W_3 L_3^2 + W_3 L_2^2 = \frac{1}{3}(1.7145\,kg)(.254\,m)^2 + (1.7145\,kg)(.3048\,m)^2$$

$$= 0.19615\,kg \cdot m^2$$

$$I_{grip} = \frac{1}{3} W_{grip} L_{grip}^2 + W_{grip}(L_2 + L_3)^2 =$$

$$= \frac{1}{3}(.095\,kg)(.083\,m)^2 + (.095)(.3048\,m + .254\,m)^2 = 0.02988\,kg \cdot m^2$$

$$I_{pay} = W_{pay}(L_2 + L_3 + L_{grip}) =$$

$$= (.91\,kg)(.3048\,m + .254\,m + .083\,m)^2 = 0.3748\,kg \cdot m^2$$

$$I_{total} = I_{arm_1} + I_{arm_2} + I_{grip} + I_{pay} = 0.6645\,kg \cdot m^2$$

like before,

$$T = .6645\,kg \cdot m^2 \cdot 3\,rad/s^2$$

$$= 1.9935\,\frac{kg \cdot m^2}{s^2} = \boxed{1.9935\,N/m}$$

Continued Calculations

Given our specifications:

Low Range:

$\alpha_{base} = 1.5\ rad/s^2$ $\quad L_4 = 0.083\,m$ $\quad W_{js} = 0.095\,kg$ $\quad W_{j2} \approx 1.4\,kg + 0.6\,g \approx 1.4\,kg$

$L_1 = 5\,mm = 0.005\,m$ $\quad W_1 = (.00024\,m^2 \cdot .005\,m)\,2700\,kg/m^3 = .00324\,kg$ $\quad W_{j3} \approx 0.8\,kg$

$L_2 = 5\text{ inches} = 0.127\,m$ $\quad W_2 = 0.0822\,kg$ $\quad W_{pay} = 2\,lb = 0.91\,kg$ $\quad W_{j4} \approx 0.25\,kg$

$L_3 = 4\text{ inches} = 0.1016\,m$ $\quad W_3 = 0.06583\,kg$

Mid Range:

$\alpha_{base} = 2.25\ rad/s^2$ $\quad L_4 = 0.083\,m$ $\quad W_{js} = 0.095\,kg$ $\quad W_{j2} \approx 1.4\,kg$

$L_1 = 10\,mm = 0.01\,m$ $\quad W_1 = (.001\,m^2 \cdot .01\,m)\,3500\,kg/m^3 = .035\,kg$ $\quad W_{j3} \approx 0.8\,kg$

$L_2 = 8\text{ inches} = 0.2032\,m$ $\quad W_2 = 0.7112\,kg$ $\quad W_{pay} = 0.91\,kg$ $\quad W_{j4} \approx 0.25\,kg$

$L_3 = 6.5\text{ inches} = 0.1651\,m$ $\quad W_3 = 0.57785\,kg$

High Range:

$\alpha_{base} = 3\ rad/s^2$ $\quad L_4 = 0.083\,m$ $\quad W_{js} = 0.095\,kg$ $\quad W_{j2} \approx 1.4\,kg$

$L_1 = 13\,mm = 0.013\,m$ $\quad W_1 = (.0015\,m^2 \cdot .013\,m)\,4500\,kg/m^3 = 0.117\,kg$ $\quad W_{j3} \approx 0.8\,kg$

$L_2 = 1\text{ foot} = 0.3048\,m$ $\quad W_2 = 2.0574\,kg$ $\quad W_{j4} \approx 0.25\,kg$

$L_3 = 10\text{ inches} = 0.254\,m$ $\quad W_3 = 1.7145\,kg$

Find the torque at the first arm joint (T_{j2}) for low, med, high cases where:

$$T_{j2} = W_2\left(\frac{L_2}{2}\right) + W_{j3}L_2 + W_3\left(L_2 + \frac{L_3}{2}\right) + \left(W_{j4} + \overset{L_4=0}{\cancel{W_4}} + \overset{W_{js}=W_{grip}}{\cancel{W_{js}}} + W_{grip} + W_{pay}\right)(L_2 + L_3)$$

$$T_{j2L} = (.0822\,kg)\left(\frac{.127\,m}{2}\right) + (.8\,kg)(.127\,m) + (.06583\,kg)\left(.127\,m + \frac{.1016\,m}{2}\right) + (.25\,kg + .095\,kg + .91\,kg)(.127\,kg + .1016\,kg) =$$

$$T_{j2L} = 0.40542\ kg\cdot m = \underline{563\ oz\cdot in}$$

$$T_{j2M} = (.7112\,kg)\left(\frac{.2032\,m}{2}\right) + (.8\,kg)(.2032\,m) + (.57785\,kg)\left(.2032\,m + \frac{.1651\,m}{2}\right) + (.25\,kg + .095\,kg + .91\,kg)(.2032\,m + .1651\,m) =$$

$$T_{j2m} = 0.86215\ kg\cdot m = \underline{1197.3\ oz\cdot in}$$

$$T_{j2H} = (2.0574\,kg)\left(\frac{.3048\,m}{2}\right) + (.8\,kg)(.3048\,m) + (1.7145\,kg)\left(.3048\,m + \frac{.254\,m}{2}\right) + (.25\,kg + .095\,kg + .91\,kg)(.3048\,m + .254\,m) =$$

$$T_{j2H} = 1.99\ kg\cdot m = \underline{2763\ oz\cdot in}$$

Continued Calculations

For reference:

Low Range:

$\alpha_{base} = 1.5\ rad/s^2$ $L_4 = 0.083m$ $W_{j5} = 0.095kg$ $W_{j2} \approx 1.4kg + 0.6g \approx 1.4kg$

$L_1 = 5\ mm = 0.005m$ $W_1 = (.00024m^2 \cdot .005m)2700\ kg/m^3 = .00324kg$ $W_{j3} \approx 0.8\ kg$

$L_2 = 5\ inches = 0.127m$ $W_2 = 0.0822kg$ $W_{pay} = 2\ lb = 0.91kg$ $W_{j4} \approx 0.25\ kg$

$L_3 = 4\ inches = 0.1016m$ $W_3 = 0.06583kg$

Mid Range:

$\alpha_{base} = 2.25\ rad/s^2$ $L_4 = 0.083m$ $W_{j5} = 0.095kg$ $W_{j2} \approx 1.4kg$

$L_1 = 10mm = 0.01m$ $W_1 = (.001m^2 \cdot .01m)3500\ kg/m^3 = .035kg$ $W_{j3} \approx 0.8kg$

$L_2 = 8\ inches = 0.2032m$ $W_2 = 0.7112kg$ $W_{pay} = 0.91kg$ $W_{j4} \approx 0.25kg$

$L_3 = 6.5\ inches = 0.1651m$ $W_3 = 0.57785kg$

High Range:

$\alpha_{base} = 3\ rad/s^2$ $L_4 = 0.083m$ $W_{j5} = 0.095kg$ $W_{j2} \approx 1.4kg$

$W_{j3} \approx 0.8kg$

$L_1 = 13mm = 0.013m$ $W_1 = (.0015m^2 \cdot .013m)4500\ kg/m^3 = 0.117kg$ $W_{j4} \approx 0.25kg$

$L_2 = 1\ foot = 0.3048m$ $W_2 = 2.0574kg$

$L_3 = 10\ inches = 0.254m$ $W_3 = 1.7145kg$

Lets now find the torque needed for the second arm joint (T_{j3}) where:

$$T_{j3} = W_3\left(\frac{L_3}{2}\right) + \left(W_{j4} + \overset{L_4=0}{\cancel{W_4}} + \overset{W_{j5}=W_{grip}}{\cancel{W_{j5}}} + W_{grip} + W_{pay}\right)L_3$$

$$T_{j3L} = (0.06583kg)\left(\frac{.1016m}{2}\right) + (.25kg + .095kg + .91kg).1016m =$$

$$T_{j3L} = 0.13085\ kg \cdot m = \underline{181.72\ oz \cdot in}$$

$$T_{j3M} = (.57785kg)\left(\frac{.1651}{2}\right) + (.25kg + .095kg + .91kg).1651m =$$

$$T_{j3M} = 0.2549\ kg \cdot m = \underline{354\ oz \cdot in}$$

$$T_{j3H} = (1.7145kg)\left(\frac{.254}{2}\right) + (.25kg + 0.95kg + .91kg).254m =$$

$$T_{j3H} = .5365\ kg \cdot m = \underline{745\ oz \cdot in}$$

Continued Calculations

Now for the torque of the neck motor (T_{j4}), we need I_{cm}, which we found above, and an estimate for how fast we want to spin

$$T_{j4} = I_{cm} \times \alpha_{neck} \qquad I_{cm} = 0.006487\ kg \cdot m^2$$

For low, med, high, lets assume different accelerations

$$T_{j4L} = 0.006487\ kg \cdot m^2 \cdot 1\ rad/s^2 = .006487\ N \cdot m$$

$$T_{j4M} = 0.006487\ kg \cdot m^2 \cdot 1.5\ rad/s^2 = .0097305\ N \cdot m$$

$$T_{j4H} = 0.006487\ kg \cdot m^2 \cdot 2\ rad/s^2 = .01297\ N \cdot m$$

With all calculations,

Motor joint #	Rated torque LOW	Rated torque MED	Rated Torque HIGH
Base motor (T_{j1}) rotational	.1428 N/m	.5339 N/m	1.9935 N·m
Arm 1 Motor (T_{j2})	563 oz·in	1197 oz·in	2763 oz·in
Arm 2 Motor (T_{j3})	181.72 oz·in	354 oz·in	745 oz·in
Neck motor (T_{j4}) rotational	.006487 N·m	.0097305 N·m	.01297 N·m
Gripper Motor (T_{j5}) 11 kg/cm (stall)	11 kg/cm	11 kg/cm	11 kg/cm