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Engineering Notes

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Adaptive Kalman Filter for Tracking Maneuvering Targets

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Introduction

THE classical Kalman filter, when applied to target tracking, experiences a divergence problem when a maneuver is exercised. In this Note, a simple procedure, which is mathematically tractable and easily amenable to real-time implementation, is introduced to avert this divergence.

A Kalman filter has been a very successful tool for providing a track file of flying targets. In a typical situation, the filter processes a set of measurements composed of radar range and angle rates from two rate gyros to give an optimal estimate of the desired parameters.

The target state dynamics of the filter are usually designed to model a straight, level, and constant-velocity target. Target acceleration cannot be modeled due to the arbitrary and unpredictable nature of target maneuverability. The filter, with this design, performs well under the assumed environment; however, it behaves poorly when a maneuver occurs, since the dynamics of the filter are unaware of the target acceleration. This leads to a degraded estimate of the position and velocity, and the large residuals will eventually lead to a filter divergence. The sequence of events that leads to this divergence can be averted if the filter is adapted to operate under target maneuver situations.

Problem Definition and Methods of Approach

In the discrete time case, the system, assumed to be linear, is governed by the following equations.

Plant model:

$$X_{k+1} = \phi_{k+1,k} X_k + G_k U_{k+1}$$

Measurement equation:

$$Y_k = H_k X_k + v_k$$

where U_k and v_k are assumed to be additive Gaussian white noise. The Kalman filter equations are standard and summarized below:

$$K = PH^T (HPH^T + R)^{-1}$$

$$\bar{X}_k = X_k + K(y_k - HX_k)$$

$$\bar{P}_k = (I - KH) P_k$$

$$X_{k+1} = \phi_{k+1,k} \bar{X}_k$$

$$P_{k+1} = \phi_{k+1,k} \bar{P}_k \phi^T_{k+1,k} + Q$$

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The above system of equations will generally provide a reliable estimate of the position and velocity for a non-maneuvering target. As soon as a target starts a maneuver, the filter estimate of the state will deviate from the true path and will lead to a filter divergence.

Several approaches have been studied to overcome this situation. These approaches can be grouped into two basic classifications: 1) non-Bayesian methods, in which the residuals are processed according to a presumed criterion and the process noise matrix is adjusted accordingly, and 2) Bayesian methods, for which the measurements are processed by a bank of Kalman filters. The state estimate is then the weighted sum of all the estimates of these filters.[1-3]

The first approach is more appealing due to its simplicity. Then, a deviation of the monitored system from the modeled one will be compensated for by enlarging the uncertainty in the state model, represented by the process noise matrix "Q."

In earlier designs,[4] the Kalman filter process was initiated using a large "Q," regardless of whether or not a maneuverability option would be exercised. While some improvement may be observed, on the average, a serious degradation in the filter performance will occur for non-maneuvering circumstances. Jazwiniski,[5] introduced a Q matrix augmentation method based on processing the measurement residuals.

In this Note, an algorithm for adapting the Kalman filter to maneuvering targets is presented, which first detects the presence and the amount of any maneuver that might be assumed by the target and then augments the Q matrix accordingly. In this manner, the filter performance for non-maneuvering targets will not be affected and a satisfactory estimate of the true path will be assured.

The Algorithm

The algorithm consists of two parts: 1) a maneuver detector to indicate when the Q matrix should be adjusted and 2) a measure of the maneuver, to indicate how the Q matrix will be adjusted.

The analysis will be based on a typical target-tracking problem where the measurements are: 1) slant range, R, 2) azimuth rate, $\dot{G}\cos S$, and 3) elevation rate, $\dot{S}$ where G and S are the azimuth and elevation angles, respectively.

Consider

$$Z = Y - \hat{Y}$$

where Y is the measurement vector and $\hat{Y}$ is the predicted measurement vector and a function of the state. For a linear Kalman filter, Z is a Gaussian white noise vector whose covariance is given by[6]

$$C = EZZ^T = HPH^T + R$$

Fortunately, this is a byproduct of the Kalman filter process and the diagonal of this matrix will be the variance of the components of Z. By monitoring the residuals Z, one should be able to determine if the diagonal elements of C are true representatives of the monitored components of Z. If this is not the case, one may assume that a deviation of the system from the model has occurred or, equivalently, a target maneuver has occurred.

plant model. If Z exceeds its limits, then one may assume the occurrence of a maneuver. It would, of course, be better to utilize $L>1$ observations to judge the occurrence of a target maneuver.

Then with probability 0.95,

$$\left| S=\sum_{i=K-L}^{L} Z(i) \right| \leq 2\sqrt{LC}$$

if the system is following the designated path. In the above inequality, S is the sum of the residues at time KT over the preceding L observations.

In a multimeasurement situation, a maneuver detection criterion would be

$$|S_m| > J\sqrt{LC_m} \quad \text{maneuver detected}$$

$$|S_m| < J\sqrt{LC_m} \quad \text{no maneuver detected}$$

where the subscript m indicates the mth measurement. The number J will indicate the degree of confidence in the maneuver detection criterion. (For more details about statistics, see Ref. 7.) As an example, suppose $J=1.5$ and two measurements are available. Then for one measurement, the probability of a false alarm is $P_{FA}=P$ (detecting a maneuver | no maneuver has occurred) = 0.1336. That is, the probability P_D

$$P_D = P\,(\text{detecting a maneuver} \mid \text{no maneuver has occurred})$$

$$= 1-0.01785 = 0.98215$$

which implies a high degree of confidence for target tracking.

The second part of the algorithm addresses the modification of the process noise matrix when a maneuver is detected. Since the uncertainty in the model would be due to target maneuver, the acceleration components must be part of the state. Therefore, one choice is a nine-dimensional state vector whose elements are target position, velocity, and acceleration in Cartesian coordinates.

It is desired that the augmentation process be applied properly. That is, if an acceleration occurs along the y axis, then the uncertainty in a_y is the only element that should be augmented. In general, the augmentation of the elements of the Q matrix should be proportional to the amount of acceleration along each axis. This will require some knowledge of the magnitude and direction of the assumed acceleration which can be provided by the residuals. For example, in a spherical coordinate system, let Z_1, Z_2, and Z_3 denote the Kalman filter residues in the three measurements, respectively, and T be the time interval over which these residues are observed. Then

$$a_{S1} = Z_1/(T^2/2)$$

$$a_{S2} = R\cdot Z_2/(T^2/2)$$

$$a_{S3} = R\cdot Z_3/(T^2/2)$$

would constitute a reasonable estimate of the acceleration components in the spherical coordinate system.

A better scheme would utilize the average sum of the residues over an assigned time interval which would effectively reduce the effects of random noise on the residues. For this purpose, suppose that the residues are observed over n time intervals and assume, arbitrarily, that z is varying linearly with time. Then the sum of these z's would be

$$S_n = \sum_{i=1}^{n} Z_i = \frac{n+1}{2} Z_n$$

or

$$Z_n = \frac{2}{n+1} S_n \quad \text{for} \quad n=10 \qquad Z_n = \frac{S_n}{5.5}$$

and the average spherical Cartesian components will be

$$\bar{a}_{S1} = S_1/(5.5\,T^2/2)$$

$$\bar{a}_{S2} = R\cdot S_2/(5.5\,T^2/2)$$

$$\bar{a}_{S3} = R\cdot S_3/(5.5\,T^2/2)$$

By a coordinate transformation, the Cartesian acceleration components

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos G & -\sin G & 0 \\ \sin G & \cos G & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos S & 0 & -\sin S \\ 0 & 1 & 0 \\ \sin S & 0 & \cos S \end{bmatrix} \begin{bmatrix} \bar{a}_{S1} \\ \bar{a}_{S2} \\ \bar{a}_{S3} \end{bmatrix}$$

and can now be used to modify the Q matrix by adding the square of the components of the acceleration vector to the diagonal elements of the Q matrix.

Kalman filter estimates can largely be enhanced by avoiding large or unnecessary Q-matrix augmentation. Conservative measures can be taken to avoid unnecessary augmentation. For example, since a 1σ of 2g's on the target maneuverability is very reasonable, 20 m/s^2 should be used as a limit on the acceleration estimate. Therefore, we shall require

$$P_{7,7} = P_{7,7} + a_x C$$

$$P_{8,8} = P_{8,8} + a_y C$$

$$P_{9,9} = P_{9,9} + a_z C$$

where

$$C = \min\left(1, \frac{400}{a^2}\right) \quad \text{and} \quad a^2 = a_x^2 + a_y^2 + a_z^2$$

Then, to avoid unnecessary augmentation of the Q matrix, the following statements are implemented as a part of the algorithm:

$$P_{7,7} = \max(P_{7,7}, a^2 C)$$

$$P_{8,8} = \max(P_{8,8}, a^2 C)$$

$$P_{9,9} = \max(P_{9,9}, a^2 C)$$

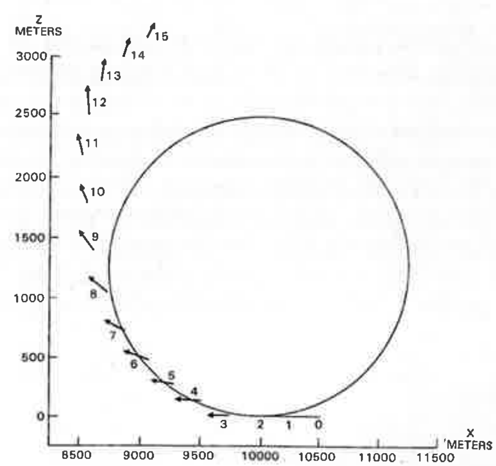

Fig. 1 Kalman filter estimate of the position and velocity without adaptation algorithm.

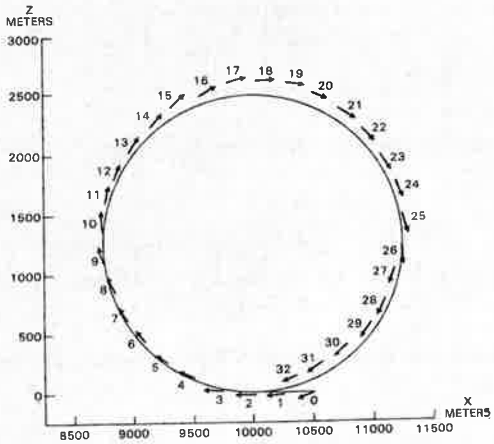

Fig. 2 Kalman filter estimate of the position and velocity with adaptation algorithm.

Results

A simulation program was made to test the validity of the adaptation process. A target flight path is coded so that at $t=0$: $X=10{,}500$ m, $Y=0$, $Z=500$ m, $V=-250$ m/s (parallel to the X axis).

At $t=2$ s, the target starts a $5g$ maneuver (50 m/s^2) in the horizontal plane. The measurements and its noise statistics are: 1) radar range, R, $1\sigma=10$ m; 2) azimuth rate, $\dot{G}\cos S$, $1\ \sigma=3$ mrad/s; and 3) elevation rate, $\dot{S}$, $1\sigma=3$ mrad/s. The noise on all the measurements are Gaussian and colored with bandwith = 1.2 Hz. The Kalman filter process interval is 0.05 s.

The projection of the estimated position and velocity are plotted in the following two graphs. Figure 1 shows the filter estimate without using the adaptation process, while Fig. 2 shows the filter estimates when the adaptation process is implemented. The results are plotted for even seconds.

Figure 2 shows that the filter was able to produce a reliable estimate for both a maneuvering and nonmaneuvering situation.

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