Set Theory

* Set →

An unordered collection of distinct objects, called as elements or members of the set.

* Generally, we use captial letters to denote a set and lowercase to denote the elements of sets.

Example:

$$A = \{a, b, c, d\} \quad \text{———} \quad (i)$$

$$A = \{a, d, c, b\} \quad \text{———} \quad (ii)$$

* The statement "p is an element of A" or equivalently, "p belongs to A" is written as —

$$p \in A \quad \text{———} \quad (iii)$$

The statement "p is not an element of A", is written as —

$$p \notin A. \quad \text{———} \quad (iv)$$

* How to specify a set?

- There are two ways to specify a particular set.

i) Tabular form of a set

If a set is defined by actually listing its members, then it is expressed as —

$$P = \{a, b, c, d\}$$

$$Q = \{1, 2, 3, 4, 5\}$$

This is called as Tabular form of a set.

ii) Builder form of a Set

If a set is defined by the properties which its elements must satisfy e.g.,

$$A = \{x : x \text{ is a letter in the English alphabet, } x \text{ is a vowel}\} \quad \text{— (v)}$$

$$A = \{a, e, i, o, u\} \quad \text{— (vi)}$$

$$P = \{x : x \in \underset{\substack{\uparrow \\ \text{set of natural numbers}}}{N}, x \text{ is a multiple of } 3\}$$

$$P = \{3, 6, 9, 12, \ldots\ldots\}$$

Finite Set –

If a set consists of specific number of different elements, then the set is called as finite set.

e.g. ⇒ $P = \{x : x \in N,\ 3 < x < 9\} = \{4, 5, 6, 7, 8\}$

$Q = \{2, 4, 6, 8\}$

R = { ~~set~~ months of year }

Infinite Set –

If a set consist of infinite number of different elements.

e.g. $I = \{$set of all integers$\}$

$E = \{x : x \in N,\ x$ is a multiple of $2\}$

Null Set or Empty Set –

The set that contains no element is called a null set ~~of~~ or Empty set.

⇒ denoted by → ϕ .

e.g. → $P = \{x : x^2 = 4,\ x$ is odd$\}$

$Q = \{x : x^2 = 9,\ x$ is even$\}$

* The set $A = \{\phi\}$ is not a null set because ϕ is an element belonging to A.

* The set $\phi = \{0\}$ is not a null set because 0 is the element of the set.

Equality of Sets →

→ Two sets A and B are said to be equal and written as $A = B$ if both have the same elements.

→ Every element which belongs to A is also an element of the set B and every element which belongs to the set B is also an element of the set A.

$$A = B \Leftrightarrow (x \in A \Leftrightarrow x \in B).$$

→ If there is some element in a set A that does not belong to set B or vice-versa then $A \neq B$.

* A set does not change if one or more of its elements are repeated.

* A set does not change if we change the order in which its elements are tabulated.

• i) $A = \{x : x < 10 \text{ and } x \text{ is even}\}$
$B = \{2, 4, 6, 8\}$
$C = \{x : x > 1 \text{ and } x < 10 \text{ and } x \text{ is even}\}$
All three sets are equal.

ii) $P = \{r, s, t\}$
$Q = \{r, r, s, t\}$ → All Equal
$R = \{s, r, t\}$

iii) $A = \{a, b, c\}$
$B = \{b, a, c\}$
$C = \{b, a\}$

$A = B$, but $C \neq A$
$C \neq B$

<u>Disjoint Sets</u> →

Two sets A and B are said to be disjoint if no element of A is in B and no element of B is in A. e.g.,

$A = \{a, b, c\}$, $B = \{1, 2, 3\}$

A and B are disjoint.

<u>Subset of a set</u> →

If every element of a set A is also an element of a set B then A is called subset of B and is written as $A \subseteq B$.

B is called superset of A.

$$A \subseteq B = \{x : x \in A \Rightarrow x \in B\}$$

(i) **Proper Subset →**

If A is subset of B and $A \neq B$, then A is said to be proper subset of B.

⇒ If A is a proper subset of B then B is not subset of A i.e., there is atleast one element in B which is not in A.

e.g →

(i) Let $A = \{2, 3, 4\}$

$B = \{2, 3, 4, 5\}$

A is proper subset of B that is $A \subset B$.

ii) The null set ϕ is a subset of every set.

⇒ Let us suppose that –

$\phi \not\subset A$

Then $\exists$ an element $x \in \phi$ s.t. $x \notin A$

But ϕ is null set. Therefore for every x, $x \notin \phi$ (By definition)

From above, $x \in \phi$ and $x \notin \phi$ which is a contradiction.

Hence proved.