Date:
Page No.

UNIT - 2

Proposition :→ It is a declarative sentence which is either true or false - but not both.

Primitive :→ When a proposition can't be broken down into further / simpler propositions.

BASIC LOGICAL OPERATIONS :-

① Conjunction

→ $p \wedge q$
→ "and"

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

② Disjunction

→ $p \vee q$
→ "or"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Date:
Page No.

③ Negation

→ $\neg p$

→ "not"

p	$\neg p$
T	F
F	T

TAUTOLOGIES AND CONTRADICTIONS :-

Some propositions $P(p,q,\dots)$ contain only T in the last column of their truth tables or, in other words, they are true for any truth values of their variables.
Such propositions are called "TAUTOLOGIES".

A proposition is called "CONTRADICTION" if it contains only F in the last column of it's truth table.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

(Tautology)

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

(Contradiction)

Date:
Page No.

LOGICAL EQUIVALENCE :-

Two propositions $P(p,q,\ldots)$ & $Q(p,q,\ldots)$ are said to be logically equivalent, or simply equivalent or equal, denoted by:

$$P(p,q,\ldots) \equiv Q(p,q,\ldots)$$

if they have identical truth tables.

eg: truth tables of

① $\neg(p \wedge q)$

② $\neg p \vee \neg q$

CONDITIONAL AND BICONDITIONAL STATEMENTS :-

If a statement is in the form:

"if p then q"

it is called conditional statements denoted by:

$$p \rightarrow q.$$

If a statement is in the form:

"p if and only if q"

it is called biconditional statement. denoted by:

$$p \leftrightarrow q.$$

Date:
Page No.

$\neg p \vee q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

SEMIGROUPS :- (AND MONOID)

Let S be a non-empty set with an operation.
Then 'S' is called semi-group if the operation is associative.

If the operation also has identity element, then 'S' is called monoid.

Suppose * is the operation on S.
then;

Associative law :-

$(a * b) * c = a * (b * c)$, $a, b, c \in S$.

Let an element 'e' in S called identity element for * if, for any element 'a' in S;

$a * e = e * a = a$.

Date:
Page No.

GROUP :-

Let G be a non-empty set with a binary operation.
Then G is called the group if the following AXIOMS holds :-

i) Associative law :-
For any a, b, c in G
we have $(ab)c = a(bc)$

ii) Identity element :-
There exists an element 'e' in G such that $ae = ea = a$ for $\forall a \in G$.

iii) Inverses :-
For each a in G there exists an element a^{-1} in G such that
$aa^{-1} = a^{-1}a = e$.

A Group is said to be ABELIAN if the commutative law holds i.e
if $ab = ba$ for every $a, b \in G$.

Date:
Page No.

SUBGROUPS :-

Let H be a subset of group G. Then H is called the subgroup of G if H itself is a group under the operation of G.

A subset H of a group G is a subgroup of G if:

1. the identity element $e \in H$
2. if $a, b \in H$ then $ab \in H$.
3. if $a \in H$, then $a^{-1} \in H$.

COSETS :-

If H is a subgroup of G and $a \in G$, then the set

$$Ha = \{ha : h \in H\}$$

is called the right coset of H.

aH is called the left coset of H.

NORMAL SUBGROUPS :-

A subgroup H of G is a normal subgroup if $a^{-1}Ha \subseteq H$ for every $a \in G$.

Equivalently, H is normal if $aH = Ha$ for every $a \in G$.

Date:
Page No.

CYCLIC SUBGROUP :-

Let G be any group G' and 'a' be any element of G.

$\Rightarrow a^0 = e$

$a^{n+1} = a^n \cdot a$

$a^m \cdot a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$\therefore$ All powers of a,

$\ldots, a^{-3}, a^{-2}, a^{-1}, e, a, a^2, a^3, \ldots$

form a cyclic subgroup

denoted by gp(a)

Smallest +ve integer m, such that $a^m = e$

is called order of a

denoted by |a|

If $|a| = m$, then

$gp(a) = \{e, a, a^2, a^3, \ldots, a^{m-1}\}$

HOMOMORPHISM :- (AND ISOMORPHISM)

A mapping 'f' from group G to G' is called homomorphism if

$f(ab) = f(a) f(b)$

$\forall a, b \in G$

If f is one-to-one & onto

$\Rightarrow$ f is ISOMORPHISM & G & G' are isomorphic. $G \simeq G'$.

Date:
Page No.

If $f: G \to G'$ is a homomorphism, then the kernel of f, written $\ker f$, is the set of elements whose image is the identity element e' of G'; i.e,

$$\ker f = \{a, \in G : f(a) = e'\}$$

LAGRANGE'S THEORUM :-

Theorum :- The order of each subgroup of a finite group is a discover of the order of the group.

If G is a group, H is a subset of G then H is a subgroup if

1. $x, y \in G$ for all $x, y \in G$
2. $x^{-1} \in G$ for all $x \in G$
3. $1 \in G$.