Equating equation (*ii*) with R.H.S. of equation (*i*), we get

$$(Z^2 - 3Z + 2)A = 1$$

or
$$A = \frac{1}{Z^2 - 3Z + 2} = \frac{1}{(Z-1)(Z-2)} \quad (Z \neq 1, Z \neq 2)$$

Therefore, the particular solution is $\frac{Z^r}{(Z-1)(Z-2)}$.

**Example 14.** *Find the particular solution of the difference equation*

$$a_{r+2} - 5a_{r+1} + 6a_r = 5^r.$$

**Sol.** Let us assume the general form of the solution $= A \cdot 5^r$. ...(*i*)

Now to find the value of A, put this solution on L.H.S. of the equation (*i*), then this becomes

$$= A.\, 5^{r+2} - 5 \cdot A5^{r+1} + 6 \cdot A5^r$$
$$= 25A \cdot 5^r - 25A \cdot 5^r + 6A \cdot 5^r$$
$$= 6A \cdot 5^r$$
...(*ii*)

Equating equation (*ii*) to R.H.S. of equation (*i*), we get

$$A = \frac{1}{6}$$

Therefore, the particular solution of the difference equation is $= \frac{1}{6} \cdot 5^r$.

**Example 15.** *Find the particular solution of the difference equation*

$$a_{r+2} - 4a_r = r^2 + r - 1.$$
...(*i*)

**Sol.** The homogeneous solution of the difference equation is given by

$$a_{r(h)} = C_1(2)^r + C_2(-2)^r$$

To find the particular solution, let us assume the general form of the solution is

$$= A_1 r^2 + A_2 r + A_3.$$

Putting this solution in L.H.S. of equation (*i*), we get

$$= A_1 (r+2)^2 + A_2 (r+2) + A_3 - 4A_1 r^2 - 4A_2 r - 4A_3$$
$$= -3A_1 r^2 + (4A_1 - 3A_2)r + (4A_1 + 2A_2 - 3A_3)$$
...(*ii*)

Equating equation (*ii*) with R.H.S. of equation (*i*), we get

$$-3A_1 = 1$$
$$4A_1 - 3A_2 = 1$$
$$4A_1 + 2A_2 - 3A_3 = -1$$

After solving these three equations, we get

$$A_1 = -\frac{1}{3},\ A_2 = -\frac{7}{9},\ A_3 = -\frac{17}{27}$$

Therefore, the particular solution is $= -\frac{r^2}{3} - \frac{7}{9}r - \frac{17}{27}$.

$S^2 - 2S + 1 = 0$
$S^2 - S - S + 1 = 0$
$S(S-1) - 1(S-1)$
$(S-1)(S-1)$

**Example 16.** *Find the particular solution of the difference equation*

$$a_{r+2} - 2a_{r+1} + a_r = 3r + 5.$$
...(*i*)

**Sol.** The homogeneous solution of the difference equation is given by

$$a_{r(h)} = C_1 + C_2 r$$
...(*ii*)

(C.P.)

Corresponding to the term $3r + 5$, we assume the general form of the solution as $A_1 r + A_2$ but due to occurrence of these terms in equation (*ii*), we multiply this by suitable power of $r$ so that none of the term will occur in equation (*ii*). Thus multiply by $r^2$.

Hence, the general form of the solution becomes

$$= A_1 r^3 + A_2 r^2 \ldots\ldots$$

Putting this solution in L.H.S. of equation (*i*), we get

$$= A_1(r+2)^3 + A_2(r+2)^2 - 2A_1(r+1)^3 - 2A_2(r+1)^2 + A_1 r^3 + A_2 r^2$$
$$= A_1(r^3 + 8 + 6r^2 + 12r) + A_2(r^2 + 4 + 4r) - 2A_1(r^3 + 1 + 3r^2 + 3r)$$
$$- 2A_2(r^2 + 1 + 2r) + A_1 r^3 + A_2 r^2$$
$$= (12A_1 + 4A_2 - 6A_1 - 4A_2)r + (8A_1 + 4A_2 - 2A_1 - 2A_2)$$
$$= (6A_1)r + (6A_1 + 2A_2) \qquad \ldots(iii)$$

Equating equation (*iii*) with R.H.S. of equation (*i*), we get

$$6A_1 = 3 \quad \therefore \quad A_1 = \frac{1}{2}$$
$$6A_1 + 2A_2 = 5 \quad \therefore \quad A_2 = 1$$

Therefore, the particular solution is $\frac{1}{2}r^3 + r^2$.

**Example 17.** *Find the particular solution of the difference equation*

$$a_{r+2} + a_{r+1} + a_r = r \,.\, 2^r. \qquad \ldots(i)$$

**Sol.** Let us assume the general form of the solution $= (A_0 + A_1 r) \,.\, 2^r$

Now, put this solution in the L.H.S. of equation (*i*), we get

$$= 2^{r+2}[A_0 + A_1(r+2)] + 2^{r+1}[A_0 + A_1(r+1)] + 2^r(A_0 + A_1 r)$$
$$= 4.\, 2^r(A_0 + A_1 r + 2A_1) + 2 \,.\, 2^r(A_0 + A_1 r + A_1) + 2^r(A_0 + A_1 r)$$
$$= r.\, 2^r(7A_1) + 2^r(7A_0 + 10A_1) \qquad \ldots(ii)$$

Equating equation (*ii*) with R.H.S. of equation (*i*), we get

$$7A_1 = 1 \quad \therefore \quad A_1 = \frac{1}{7}$$
$$7A_0 + 10A_1 = 0 \quad \therefore \quad A_0 = \frac{-10}{49}$$

Therefore, the particular solution is $2^r\left(\frac{-10}{49} + \frac{1}{7}r\right)$.

**Example 18.** *Find the particular solution of the difference equation*

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1) \,.\, 2^r. \qquad \ldots(i)$$

**Sol.** The homogeneous solution of the difference equation is given by

$$a_{r(h)} = (C_1 + C_2 r) \,.\, 2^r \qquad \ldots(ii)$$

because it has two real and equal roots *i.e.*, 2 and 2.

To find the particular solution, let us assume the general form of the solution is $= 2^r(A_1 r + A_0)$, but due to occurrence of there terms in equation (*ii*), we multiply this by suitable power of $r$ so that none of the terms will occur in equation (*ii*). Thus multiply by $r^2$.

Hence, the general form of the solution becomes $= 2^r(A_1 r + A_0) \,.\, r^2$

Putting this solution in L.H.S. of equation (*i*), we get

$= 2^r . (A_1 r + A_0) . r^2 - 4 . 2^{r-1} [A_1(r-2) + A_0] . (r-1)^2 + 4 . 2^{r-2} [A_1 (r-2) + A_0] . (r-2)^2$

$= 2^r .(A_1 r + A_0) . r^2 - 2(r^2 + 1 - 2r) . 2^r (A_1 r - A_1 + A_0) + (r^2 + 4 - 4r) . 2^r . (A_1 r - 2A_1 + A_0)$

$= r . 2^r (6A_1) + 2^r (-6A_1 + 2A_0)$ ...(*iii*)

Equating equation (*iii*) with R.H.S. of equation (*i*), we get

$$6A_1 = 1 \quad \therefore \quad A_1 = \frac{1}{6}$$

$$-6A_1 + 2A_0 = 1 \quad \therefore \quad A_0 = 1$$

Therefore, the particular solution is $= r^2 . 2^r \left(\frac{r}{6} + 1\right)$.