DISCRETE MATHEMATICS

CSE-3rd SEM

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DISCRETE MATHEMATICS

Unit-I

Set - unordered collection of distinct objects. These objects are called elements or members.

N = Set of +ve integers
Z = Set of integers
Q = Set of rational numbers
R = Set of real no.
C = Set of complex no.

Universal set (U) - It consists of all points in the plane.

Empty set (ϕ) - It is a set with no elements also called null set.

Subset - If every element in set A is also an element of set B, then A is called a subset of B. ($A \subseteq B$)

Proper subset - If $A \subseteq B$, but $A \neq B$, then it is called proper subset ($A \subset B$)

Theorum's :-

1. For any set A, $\phi \subseteq A \subseteq U$
2. For any set A, $A \subseteq A$
3. If $A \subseteq B$, $B \subseteq C$ then $A \subseteq C$
4. $A = B$ iff $A \subseteq B$ & $B \subseteq A$

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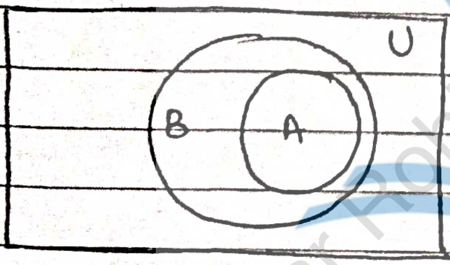
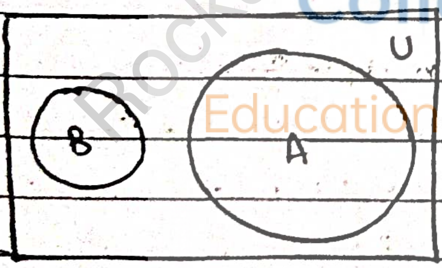
VENN DIAGRAMS :-

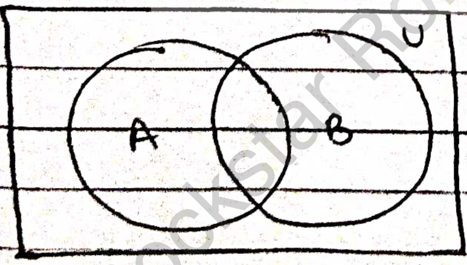
It is a pictorial representation of sets in which sets are represented by enclosed areas in the plane.
The universal set 'U' is represented by the interior of a rectangle. The other sets are represented by discs lying within the rectangle.

a) 

$A \subseteq B$

b) 

A and B are disjoint

c) 

(general form)

SET OPERATIONS :-

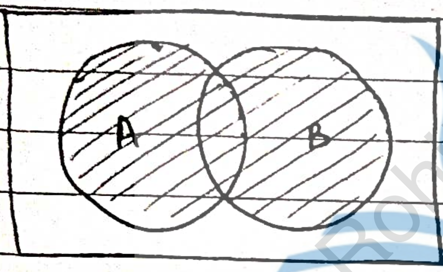
① Union and Intersection :-

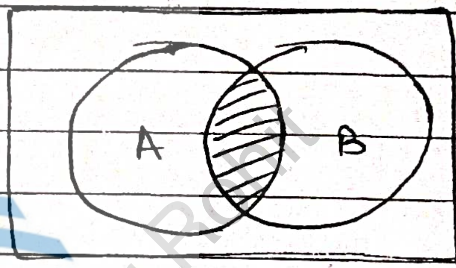
Union: denoted by '$\cup$'

eg: $A \cup B = \{x : x \in A \text{ or } x \in B\}$

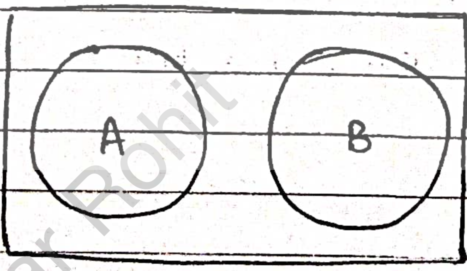
Intersection: denoted by '$\cap$'

eg: $A \cap B = \{x : x \in A \,\&\, x \in B\}$





If $A \cap B = \phi$, that is if A & B don't have any elements in common, then A & B are said to be 'disjoint' or 'non-intersecting'.



$\Leftarrow [A \cap B = \phi]$

Theorum ⇒ The foll. are equivalent :-

① $A \subseteq B$

② $A \cap B = A$

③ $A \cup B = B$

COMPLEMENTS :-

The absolute complement (or simply complement) of a set A, is denoted by A^c. It is the set of elements which belong to U but which do not belong to A.

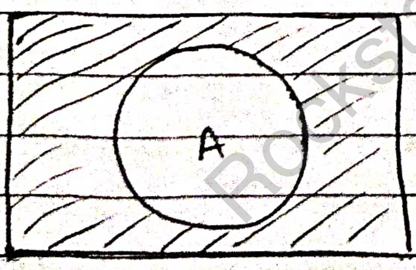
$$A^c = \{x : x \in U, x \in A\}$$

$A^c = A' = \bar{A}$ (Diff. representations)

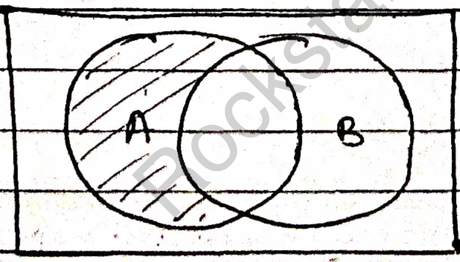
The relative complement of set B w.r.t set A or difference of A & B is denoted by $(A \setminus B)$.
It is the set of elements which belong to A but do not belong to B.

$$A \setminus B = \{x : x \in A, x \in B\}.$$

$A \setminus B$ = A minus B = $A - B$ = $A \sim B$



(A^c)

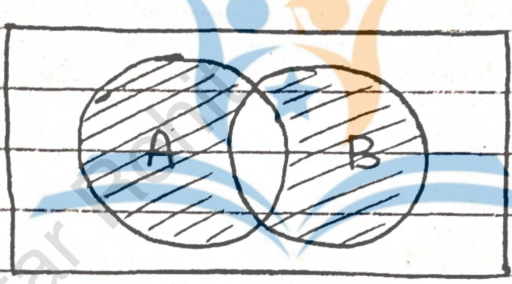


$(A \setminus B)$

SYMMETRIC DIFFERENCE

The symmetric difference of sets A and B is denoted by $A \oplus B$.
It consists of those elements which belong to either A or B, but not to both.

$$A \oplus B = (A \cup B) \setminus (A \cap B)$$



DE-MORGAN'S LAW :-

$$(A \cup B)^c = A^c \cap B^c$$

Proof:- If $x \in (A \cup B)^c$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A \;\&\; x \notin B$

$\therefore x \in A^c$ and $x \in B^c$

Hence, $x \in \cancel{(A \cup B)^c}\; A^c \cap B^c$

Now,

let $x \in A^c \cap B^c$

$\Rightarrow x \notin A \;\&\; x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in (A \cup B)^c$

$$\Rightarrow A^c \cap B^c = (A \cup B)^c$$

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LAWS OF ALGEBRA OF SETS :-

Idempotent laws

1a) $A \cup A = A$ | 1b) $A \cap A = A$

Associative laws

2a) $(A \cup B) \cup C = A \cup (B \cup C)$ | 2b) $(A \cap B) \cap C = A \cap (B \cap C)$

Commutative laws

3a) $A \cup B = B \cup A$ | 3b) $A \cap B = B \cap A$

Distributive laws

4a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ | 4b) $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$

Identity laws

5a) $A \cup \phi = A$ | 5b) $A \cap U = A$

6a) $A \cup U = U$ | 6b) $A \cap \phi = \phi$

Involution laws

7a) — | 7b) $(A^c)^c = A$

Complement laws

8a) $A \cup A^c = U$ | 8b) $A \cap A^c = \phi$

9a) $U^c = \phi$ | 9b) $\phi^c = U$

Demorgan's laws

10a) $(A \cup B)^c = A^c \cap B^c$ | 10b) $(A \cap B)^c = A^c \cup B^c$

DUALITY :-

Suppose E is an eqn of set algebra. The dual E^* of E is the equation obtained by replacing each occurance of $\cup, \cap, U, \phi$ in E by $\cap, \cup, \phi, U$, respectively.

eg: Dual of:
$(U \cap A) \cup (B \cap A) = A$ is $(\phi \cup A) \cap (B \cup A) = A$.

Principle of Duality :-
if any eqⁿ E is an identity, then
it's dual E* is also an identity.

ORDERED SET :-

It is the ordered collection of distinct objects.
eg: {3, 4, 5, 6, 7, 8}
{SUN, MON, TUE, WED, THUR, FRI, SAT}

An ordred pair of objects is a pair of objects arranged in some order.
Thus, in set {a, b} of two objects a & b, a is first and b is 2nd obj of a pair.

ordered triple → {a, b, c}
ordered quadruple → {((a, b), c), d} with first element as ordred triple.

Ordred n-tuple is an ordred pair where first component is an ordred (n-1) tuple.

FINITE SET :-

A set is said to be finite if it contains exactly 'n' distinct elements where 'n' is a non-negative Integer.
Here, 'n' is said to be 'cardinality' of set A. It is denoted by |A|, #A, n(A), Card(A).

A set is called a finite set if there is one-to-one corrospondance between the elements in the set and the elements in some set n, where n is a natural no. & cardinality of the set.
These are also called numerable sets.

INFINITE SET :-

A set, which is not finite, is called infinite set.
A set is said to be 'countably infinite' if there is one-to-one corrospondance b/w elements in the set and elemts in N.
It is also called 'denumerable set'.

A set, which is not countably infinite, is called uncountably infinite set. or uncountable set.

eg: Set R of all the real no. < 1 (i.e decimal).

INCLUSION - EXCLUSION PRINCIPLE :-

let A & B be any finite sets, then:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

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Theorem :- for any finite sets A, B, C :-

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

CLASS, POWER SET :-

Class - collection of sets.

Sets in a given class are called sub-class.

eg: Suppose $S = \{1, 2, 3, 4, [illegible]\}$

Let A be the class of subsets of S which contains 3 elements of S :-

$$A = \{\{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{1,3,4\}\}$$

Power Set :- For a given set S, if we speak of a class containing all the subsets of S, it is called Power Set.

Power set of S = Power(S).

$$\boxed{n(\text{Power}(S)) = 2^{n(S)}}$$

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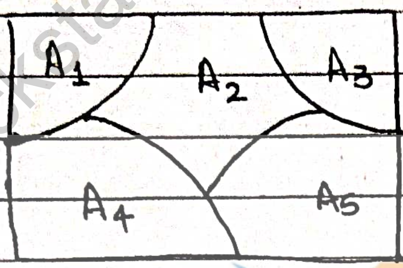
PARTITIONS :-

Let S be a non-empty set. A partition of 'S' is a sub-division of S into non-overlapping, nonempty subsets.



A partition of S is a collection $\{A_i\}$ of non-empty subsets of S such that :-

1. Each 'a' in S belongs to one of the A_i.
2. The sets of $\{A_i\}$ are mutually disjoint i.e. if

$$A_i \neq A_j \text{, then } A_i \cap A_j = \phi.$$

The subsets in partition are called CELLS.

MULTISET :-

It is an unordered collection of elements where an element can occur as a member more than once.
It is a set in which elements are not necessarily distinct.

$$S = \{n_1 \cdot a_1, n_2 \cdot a_2, \ldots, n_i \cdot a_i \ldots\}$$

This denoted that a_1 occurs n_1 times, a_2 occurs n_2 times ... and so on.
n_i = multiplicities of element a_i.

PRODUCT SETS :-

Consider 2 arbitrary set A & B. The set of all ordered pairs (a, b) where $a \in A$ & $b \in B$ is called product, or CARTESIAN PRODUCT, of A & B.

$$A \times B = \{(a,b) : a \in A \text{ \& } b \in B\}$$

INVERSE RELATION :-

Let R be any relation from set A to set B. The Inverse of R, R^{-1}, is the relation from B to A which consist of those ordered pairs which, when reversed, belong to R

$$R^{-1} = \{(b,a) : (a,b) \in R\}$$

The domain and range of R^{-1} are equal to range and domain of R

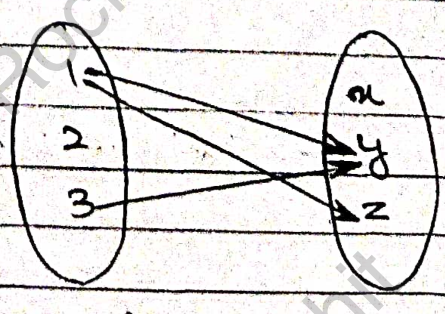
REPRESENTATIONS OF RELATIONS :-

(i)

	x	y	z
1	0	1	1
2	0	0	0
3	0	1	0

(matrix of the relation)

(ii)



(arrow diagram)

TYPES OF RELATIONS :-

① Reflexive Relations :-

A relation R on set 'A' is reflexive if aRa for every $a \in A$, that is,

if $(a,a) \in R \quad \forall a \in A$.

② Irreflexive Relations :-

A relation R on a set is irreflexive if

$(a,a) \notin R \quad \forall a \in A$.

③ Symmetric Relations :-

A relation R on set A is symmetric if whenever $aRb \Rightarrow bRa$ i.e

if $(a,b) \in R$ then $(b,a) \in R$.

④ Antisymmetric Relations :-

A Relation R on set is antisymmetric if there exists $a, b \in A$ such that

$(a,b) \in R$ but $(b,a) \notin R$.

⑤ Asymmetric Relations :-

A relation 'R' on set A is asymmetric if $(a,b) \in R$ then $(b,a) \notin R$. $\forall (a,b) \in A$.

(6) Transitive Relations :-

A relation R on set A is transitive if whenever aRb and bRc then aRc.
That is,
if $(a,b), (b,c) \in R$
then, $(a,c) \in R$.

EQUIVALENCE RELATIONS :-

Consider a non-empty set S.
A relation 'R' on S is an equivalence relation if R is reflexive, symmetric and transitive.
Properties :-
① for every $a \in S$, aRa
② If aRb then bRa
③ If aRb, bRc, then aRc.

EQUIVALENCE RELATIONS AND PARTITIONS :-

Let R be an equivalence relation on set S.
Then the quotient set S/R is a partition of S.
(i) For each a in S, $a \in [a]$
(ii) $[a] = [b]$ iff $(a,b) \in R$
(iii) If $[a] \neq [b]$, then $[a]$ & $[b]$ are disjoint.

$[a] = \{x : (a,x) \in R\}$

$S/R = \{[a] : a \in S\}$

$[a]$ = set of elements of S to which a is related under R

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PARTIAL ORDERING RELATIONS :-

A relation R on a set S is called a partial ordering or a partial order if R is reflexive, antisymmetric & transitive.

eg: Relation ($\subseteq$) is partial ordering as it has 3 desired properties

(i) $A \subseteq A$ for any set A

(ii) $A \subseteq B$ & $B \subseteq A \Rightarrow A = B$

(iii) $A \subseteq B$, $B \subseteq C \Rightarrow A \subseteq C$.

FUNCTIONS :-

Suppose that to each element of set A, we assign ~~a~~ unique element of set B; the collection of such assignments is called a function from A into B.

The set A is called domain of function.

The set B is called co-domain of function.

Let 'f' denote a function from A to B then:

$$f : A \to B.$$

COMPOSITION OF FUNCTIONS :-

Consider function $f : A \to B$ & $g : B \to C$ where codomain of 'f' is domain of 'g'.

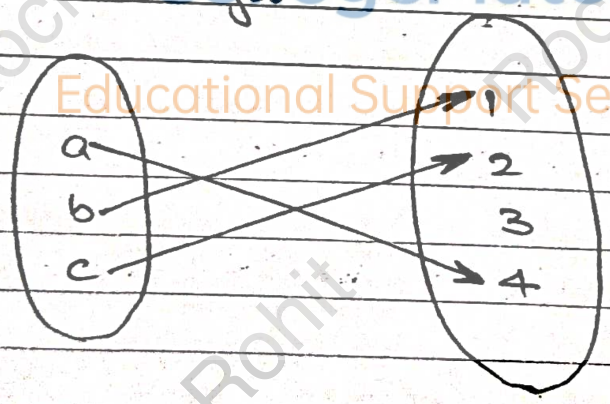
Then we may define a new function from A to C, called 'composition of f and g' written as $g \circ f$:-

$$(g \circ f)(a) = g(f(a))$$

ONE-TO-ONE FUNCTION :- (injective)

If diff. elements in domain A have distinct images.

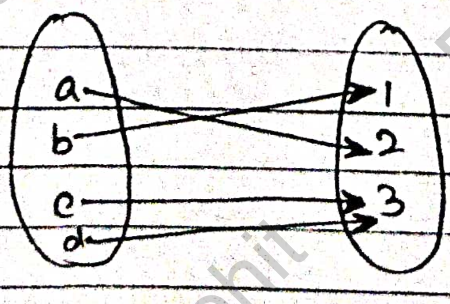
eg:-



ONTO FUNCTIONS :- (Surjective)

If each element of B is image of some element of A.

eg.

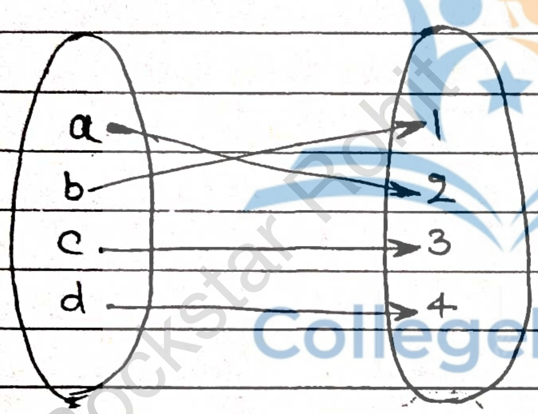


INVERTIBLE FUNCTIONS :- (bijective)

→ If its inverse relation f^{-1} is a function from B to A.

A function $f: A \to B$ is invertible only if
(i) f is one-to-one, as well as,
(ii) f is onto.

eg:-



RECURSIVELY DEFINED FUNCTIONS :-

A function is said to be 'recursively defined' if the function definition refers to itself.

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HOW TO FIND AN INVERSE FUNCTION :-

Q. Let $f: R \to R$ be defined by $f(x) = 2x-3$.

f is one-to-one & onto. Find f^{-1}.

Ans.

Let y be the image of x under the function f:

$$y = f(x) = 2x+3$$

Consequently, x will be the image of y under inverse func^n f^{-1}.

$$x = (y+3)/2$$

Then $f^{-1}(y) = (y+3)/2$

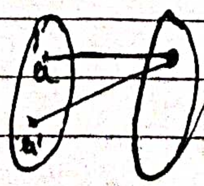
Replace y by x :-

$$f^{-1}(x) = \frac{(x+3)}{2} \quad //$$

x x x

HOW TO PROVE f(x) is ONE-TO-ONE OR ONTO :-

$f(a) = y$
$f(a') = y$
$a = a'$



$2a-3$
$2a'-3 = 2a-3$
$a = a'$

$n^2 = 1$
$a^2 = a'^2$
$\cancel{a = \pm a}$

one-one f^n

y $a \to$ exist.