

Set Theory

* Set →

An unordered collection of distinct objects, called as elements or members of the set.

* Generally, we use capital letters to denote a set and lowercase to denote the elements of sets.

Example:

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$A = \{a, b, c, d\}$ ————— (i)

$A = \{a, d, c, b\}$ ————— (ii)

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* The statement "p is an element of A" or equivalently, "p belongs to A" is written as —

$$p \in A \quad ————— (iii)$$

The statement "p is not an element of A", is written as —

$$p \notin A. \quad ————— (iv)$$

* How to specify a set?

- There are two ways to specify a particular set.

i) Tabular form of a set

If a set is defined by actually listing its members, then it is expressed as —

$$P = \{a, b, c, d\}$$

$$Q = \{1, 2, 3, 4, 5\}$$

This is called as Tabular form
of a Set.

ii) Builder form of a Set

If a set is defined by the properties which its elements must satisfy e.g)

$$A = \{x : x \text{ is a letter in the English alphabet, } x \text{ is a vowel}\}$$

$$A = \{a, e, i, o, u\} \quad \text{--- (vi)}$$

$$P = \{x : x \in N, x \text{ is a multiple of 3}\}$$

↑
set of natural numbers

$$P = \{3, 6, 9, 12, \dots\}$$

3.

Finite Set -

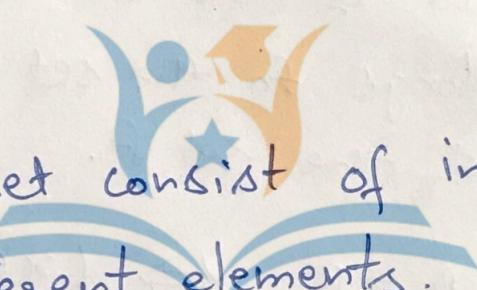
If a set consists of specific number of different elements, then the set is called as finite set.

$$\text{e.g. } \Rightarrow P = \{x : x \in N, 3 < x < 9\} = \{4, 5, 6, 7, 8\}$$

$$Q = \{2, 4, 6, 8\}$$

$$R = \{\text{months of year}\}$$

Infinite Set -



If a set consist of infinite number of different elements.

$$\text{e.g. } \Rightarrow \text{CollegeMates} \{ \text{set of all integers} \}$$

Educational Support Services $\{x \in N : x \text{ is a multiple of } 2\}$

Null Set or Empty Set -

The set that contains no element is called a null set or empty set.

\Rightarrow denoted by $\Rightarrow \phi$.

$$\text{e.g. } \Rightarrow P = \{x : x^2 = 4, x \text{ is odd}\}$$

$$Q = \{x : x^2 = 9, x \text{ is even}\}$$

* The set $A = \{\phi\}$ is not a null set because ϕ is an element belonging to A.

- * The set $\phi = \{0\}$ is not a null set because 0 is the element of the set.

Equality of Sets →

- Two sets A and B are said to be equal and written as $A=B$ if both have the same elements.
- Every element which belongs to A is also an element of the set B and every element which belongs to the set B is also an element of the set A.

$$A = B \Leftrightarrow (x \in A \Leftrightarrow x \in B)$$

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- If there is some element in a set A that does not belong to the set B or vice-versa then $A \neq B$.

- * A set does not change if one or more of its elements are repeated.

- * A set does not change if we change the order in which its elements are tabulated.

• i) $A = \{x : x < 10 \text{ and } x \text{ is even}\}$

$$B = \{2, 4, 6, 8\}$$

$$C = \{x : x > 1 \text{ and } x < 10 \text{ and } x \text{ is even}\}$$

All three sets are equal.

5.

$$\left. \begin{array}{l} \text{i)} \quad P = \{r, s, t\} \\ Q = \{r, r, s, t\} \\ R = \{s, r, t\} \end{array} \right\} \rightarrow \text{All Equal}$$

$$\begin{aligned} \text{ii)} \quad A &= \{a, b, c\} \\ B &= \{b, a, c\} \\ C &= \{b, a\} \end{aligned}$$

$$A = B, \text{ but } C \neq A \\ C \neq B$$

Disjoint Sets 

Two sets A and B are said to be disjoint if no element of A is in B and no element of B is in A .

$$A = \{a, b, c\}, B = \{1, 2, 3\}$$

A and B are disjoint.

~~Universal Sets~~

Subset of a set 

If every element of a set A is also an element of a set B then A is called subset of B and is written as $A \subseteq B$.

B is called superset of A .

$$A \subseteq B = \{x : x \in A \Rightarrow x \in B\}$$

6.

(i) Proper Subset \rightarrow

If A is subset of B and $A \neq B$, then A is said to be proper subset of B .

\Rightarrow If A is a proper subset of B then B is not subset of A i.e., there is atleast one element in B which is not in A .

e.g. \rightarrow

$$\text{(i) Let } A = \{2, 3, 4\}$$

$$B = \{2, 3, 4, 5\}$$

A is proper subset of B that is $A \subset B$.

ii) The null set \emptyset is a ~~proper~~ subset of every set.

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\Rightarrow Let us suppose that -

$$\emptyset \not\subset A$$

Then \exists an element ~~$x \in \emptyset$ s.t. $x \notin A$~~

But \emptyset is null set. Therefore for every x , $x \notin \emptyset$ (By definition)

From above, ~~$x \in \emptyset$ and $x \notin \emptyset$~~ which is a contradiction.

Hence proved.