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Unit-4

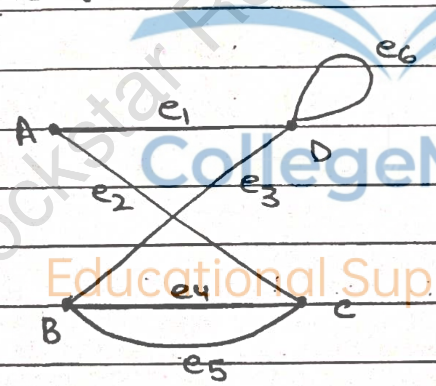
GRAPHS :-

A graph G consists of 2 things :-

(i) A set $V = V(G)$ whose elements are called vertices, points or nodes of G.

(ii) A set $E = E(G)$ of unordered pairs of distinct vertices called edges of G.

MULTI GRAPHS :-



The edges e_4 and e_5 are called multiple edges since they connect the same endpoints. and the edge e_6 is called a loop since it's endpoints are the same vertex.
Such a diagram is called a multigraph

HANDSHAKING LEMMA :-

Sum of degrees of the vertices of the graph are twice the no. of edges in G.

$deg(A) = 2$, $deg(B) = 3$, $deg(C) = 3$, $deg(D) = 3$

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FINITE GRAPHS & TRIVIAL GRAPH :-

A multigraph with finite no. of vertices & edges is called Finite graph.

A finite graph with 1 vertex and no edges i.e. a single point, is called Trivial Graph.

SUBGRAPH :-

Consider a graph $G = G(V, E)$.
A graph $H = H(V', E')$ is called a subgraph of G if the vertices and edges of H are contained in the vertices and edges of G.

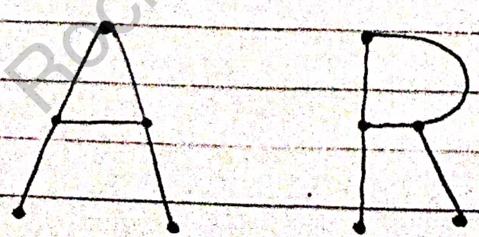
ISOMORPHIC GRAPHS :-

Graphs $G(V, E)$ and $G^*(V^*, E^*)$ are said to be isomorphic if there exists a one-to-one correspondence $f: V \to V^*$ such that:

$\{u, v\}$ is an edge of G iff $\{f(u), f(v)\}$ is an edge of G^*.

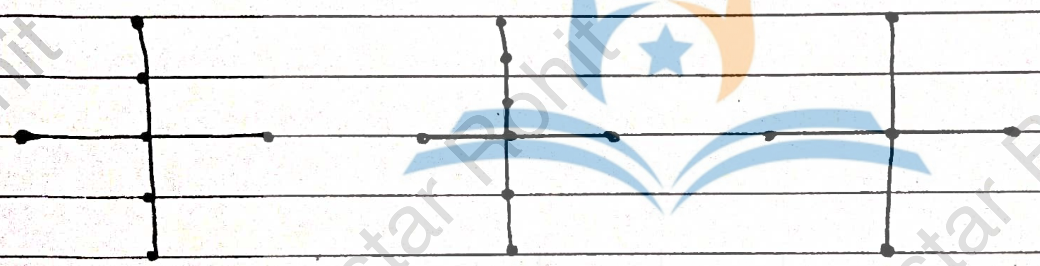
(they may look different)

eg:-



HOMOMORPHIC GRAPHS :-

Given any graph G, we can obtain a new graph by dividing an edge of G with additional vertices.
2 graphs G & G^* are said to be homeomorphic if they can be obtained from the same graph by this method.



(G1) (G2) (GT)

Here, G1 & G2 are isomorphic.

PATHS AND CONNECTIVITY :-

A path in a multigraph G consists of an alternating sequence of vertices and edges of the form:

$$v_0, e_1, v_1, e_2, v_2, \ldots, e_{n-1}, v_{n-1}, e_n, v_n$$

n = length of the path.
If there is no ambiguity, it can also be represented as :-

$$(v_0, v_1, v_2, \ldots, v_n).$$

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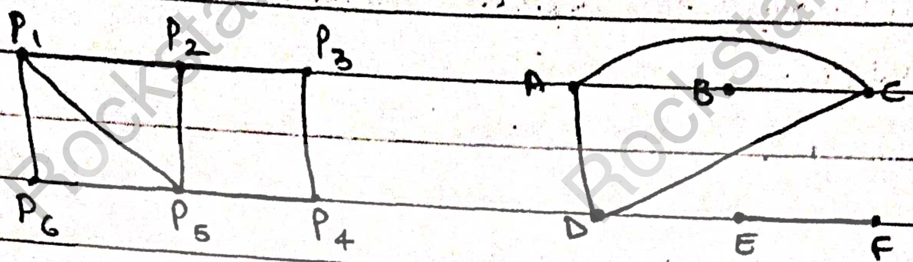
A Graph 'G' is connected if there is a path between any 2 of it's vertices.

eg.

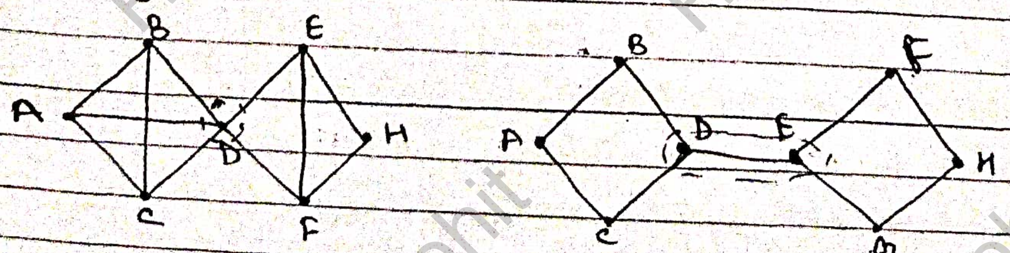


(a) (b).

In (a), it is connected graph, whereas in (b) → it is not connected graph.

CUTPOINTS AND BRIDGES :-

Let G be a connected graph.
A vertex 'v' in G is called 'cutpoint' if G − v is disconnected.
G − v is a graph obtained from G by deleting 'v' and all edges containing v.
Eg → (a)
An edge e of G is called a 'bridge' if G − e is disconnected.
(G − e → graph obtained by deleting edge 'e').
eg → (b)



(a) (b)

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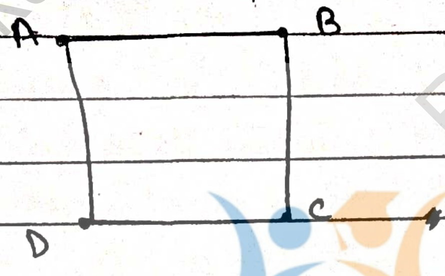
EULER GRAPH :-

A graph G is called an Euler graph if and only if each vertex has even degree.

Eg:-



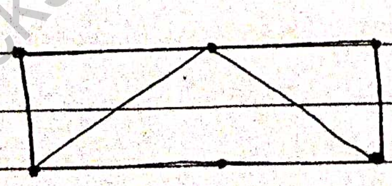
deg (A) = 2
deg (B) = 2
deg (C) = 2
deg (D) = 2

HAMILTONIAN GRAPHS :-

If a Graph G has an hamiltonian circuit, it is a hamiltonian graph.

Hamiltonian circuit is a closed path that visits every vertex in G exactly once.

eg:



Let G be a connected graph with n vertices. Then G is hamiltonian if $n \geq 3$ and $n \leq deg(v)$ for each vertex v in G.

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LABELLED AND WEIGHTED GRAPHS :-

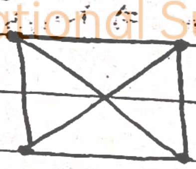
A graph G is called labelled graph if its edges and/or vertices are assigned data of one kind or another.

A graph is weighted if each edge 'e' of G is assigned a non-negative no. w(e), called weight / length of v.

COMPLETE GRAPHS :-

If a graph G, if every vertex is connected to every other vertex.
Denoted by K_n with n vertices.

eg:-



REGULAR GRAPHS :-

A graph is regular of degree k or k-regular graph if every vertex has degree 'k'.
It is regular, if every vertex has same degree. $d(e_1) = d(e_n)$

eg:-

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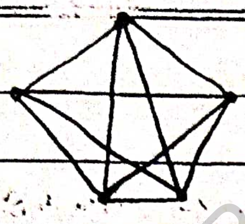


K_5.

BIPARITE GRAPHS :-

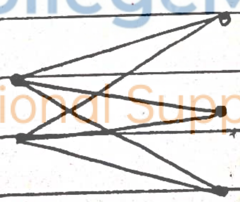
If vertices V can be partitioned into 2 subsets M and N such that each edge of G connects a vertex of M to a vertex of N.

Denoted by $K_{m,n}$

m = vertices in M

n = " in N

eg:

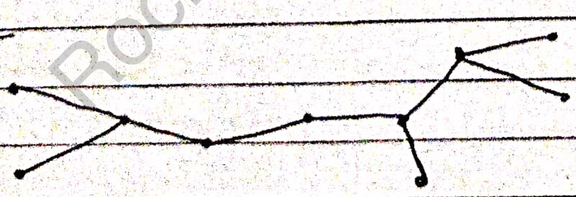


$\Rightarrow K_{2,3}$.

TREE GRAPHS :-

A graph T is called a Tree if T is connected and T has no cycles.

eg:-



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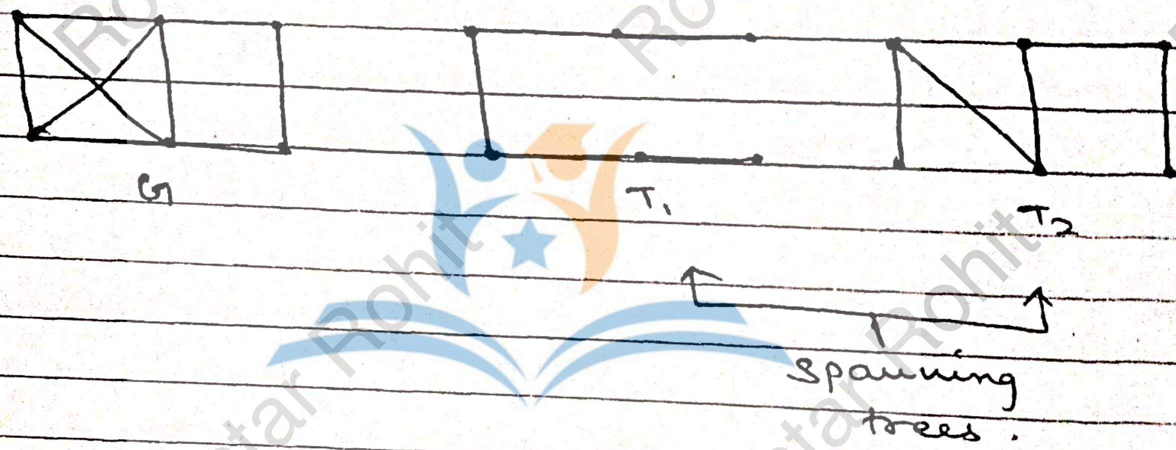
SPANNING TREES :-

A subgraph T of a connected graph G is called a spanning tree of G if T is a tree and T includes all vertices of G.

eg:-



MINIMUM SPANNING TREES :-

A minimal spanning tree of G is a spanning tree whose total weight is as small as possible.

Algorithm 1 :- Input is a connected graph with n vertices.

1) Arrange the edges of 'G' in the order of dec. weights.
2) Proceeding sequentially delete each edge that doesn't disconnect the graph until n-1 edges remain.
3) EXIT.

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KRUSKAL'S ALGORITHM :-

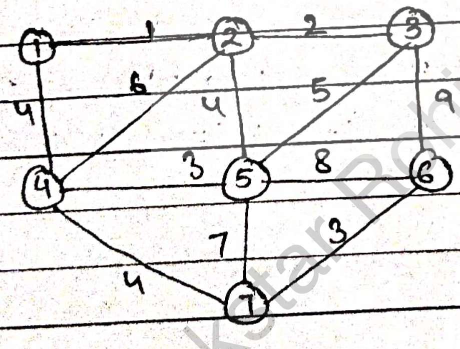
Algorithm :- Input is connected graph G with n vertices

1. Arrange the edges of G in order of inc. wt.
2. Starting only with the vertices of G & proceeding sequentially, add each edge which doesn't result in a cycle until n-1 edges are added.
3. EXIT.

PRIM'S ALGORITHM :-

Prim's algo starts with one vertex and grows the rest of the tree one vertex at a time, by adding associated edge.

Eg:- ~~Tree~~ Graph :-



Step no.	Edge ⟨u,v⟩	Set A
initial	-	{1}
1	⟨1,2⟩	{1,2}
2	{2,3}	{1,2,3}
3	{1,4}	{1,2,3,4}
4	{4,5}	{1,2,3,4,5}
5	⟨4,7⟩	{1,2,3,4,5,7}
6	⟨7,6⟩	{1,2,3,4,5,6,7}

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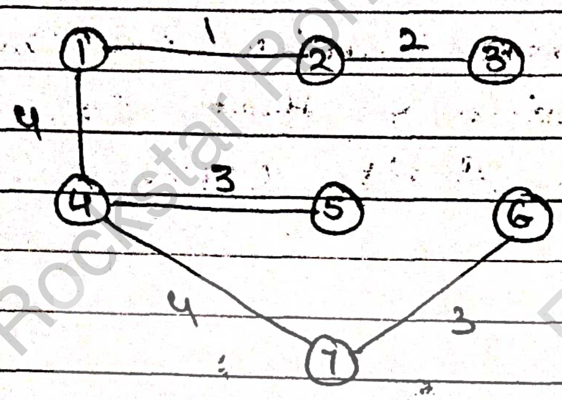
Minimum Spanning tree :-



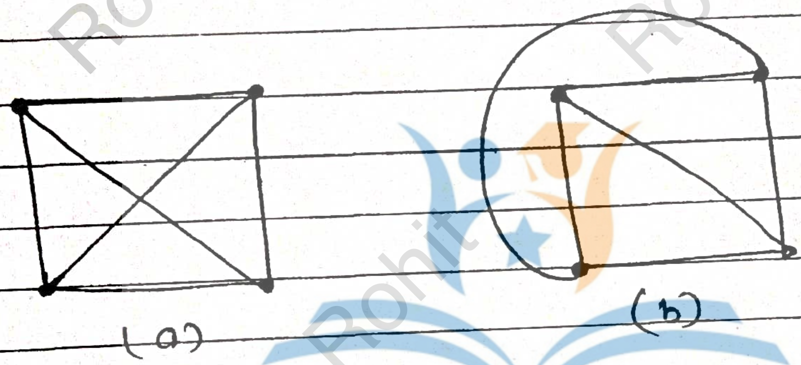
Algo :- Input is connected weighted graph G of n vertices

1. Let $G = \{V, E\}$ & $T = \{A, B\}$
   $A = \phi$, $B = \phi$
2. Let $i \in V$, i is start vertex.
3. $A = A \cup \{i\}$
4. while $(A \neq V)$
   do
   Begin
   Find edge $\{u,v\} \in E$ of minimum length such that
   $u \in A$, $v \in V - A$
   $A = A \cup \{v\}$ &
   $B = B \cup \{u,v\}$
   End
5. EXIT.

PLANAR GRAPHS :-

A graph or multigraph which can be drawn in the plane so that so that it's edges don't cross is called Planar.

eg:



(a) is not planar but (b) is planar

EULER FORMULA :-

V = no. of vertices
E = no. of edges
R = no. of regions of any connected map.

$$\boxed{V - E + R = 2}$$

DIJIKSTRA'S SHORTEST PATH ALGORITHM :-

$G = (A, B)$

A = set of vertices
B = set of edges

U = Set of unvisited vertices
V = Set of visited vertices
$w(i,j)$ = weight of edge (i,j)
$w(i,j) = \infty$ if $(i,j) \notin B$
let $|A| = n$
$L(i)$ = shortest dist. b/w a & vertex i.
$P(i)$ = set of vertices representing shortest path.

Step 1.) ~~for(i)=1 to~~
for(i=1; i≤n ; i++)
$L(i) = \infty$
$L(a) = 0$
$P(i) = \phi$

Step 2.) $V = \phi$ & $U = A$

Step 3.) let 'a' be a source vertex and 'z' be the destination vertex.

Step 4.) While $z \in V$
begin
k = a vertex in U with L(x) minimal
$V = V \cup \{k\}$
for every $j \in U$
if $(L(j) > (L(k) + w(k,j))$ then
begin
$L(j) = Lk + w(k,j)$
$P(j) = P(k) \cup \{j\}$
end
end

Step 5.) STOP.

UNIT- 4 :-

LINEAR HOMOGENOUS RECURRENCE RELATIONS WITH CONSTANT COEFFICIENTS :-

$$a_n = C_1 a_{n-1} + C_2 a_{n-2} + \cdots + C_k a_{n-k} \quad \text{---(1)}$$

where $\underbrace{C_1, C_2, \ldots, C_k \in R}_{\rightarrow \text{constant coefficients}}$ & $C_k \neq 0$.

R= Set of Real numbers.

Order of the recurrance relation is 'k'.
degree of relation = 'k'.

The recurrance relation is linear as R.H.S of eq (1) is sum of multiples of the previous terms of the sequence.
The recurrance relation is homogenous as every term on RHS is multiple of some a_i

General Sol^n => $f_n = A\alpha^n + B\beta^n$

where α & β are the Sol^n for eq: $x^2 - x - 1 = 0$

$$\Downarrow$$

$$\boxed{f_n = f_{n-1} + f_{n-2}}$$

AP, GP and HP :-

AP:-

→ n^{th} term $= a_n = a+(n-1)d$

→ A.M = arithmetic mean = $\frac{\text{sum of all terms in AP}}{\text{no. of terms in AP}}$

→ Sum of 'n' terms of AP $= \frac{n}{2}(a+a_n)$

$$= \frac{n}{2}(2a+(n-1)d)$$

GP:-

→ n^{th} term $= a_n = ar^{n-1}$

→ G.M $= \sqrt[n]{a_1 . a_2 . a_3 \cdots a_n}$

→ Sum of 'n' terms of GP $(r<1) = a\left(\frac{1-r^n}{1-r}\right)$

→ Sum of n terms of GP $(r>1) = a\left(\frac{r^n-1}{r-1}\right)$

→ Sum of infinite terms of GP $= \left(\frac{a}{1-r}\right)$

HP:-

→ a, b, c, d, e are said to be in HP if
$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in AP.

→ HM = $(2ab)/(a+b)$

For 2 nos:-

⇒ AM ≥ GM ≥ HM

⇒ AM . HM = $(GM)^2$

⇒ AM, GM, HM are in GP.