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Relation → Let A and B be two non-empty sets. A relation R is defined to be a subset of $A \times B$ from a set A to B. If $(a,b) \in R$ and $R \subseteq A \times B$ then a is related to b by R i.e. aRb.

If sets A and B are equal, then we say $R \subseteq A \times A$ is a relation on A.

$A = \{a, b, c\}$

$B = \{r, s, t\}$

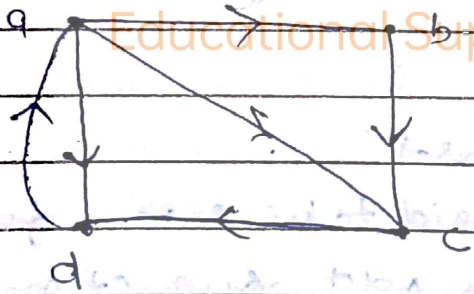
$R = \{(a,r), (b,r), (b,t), (c,s)\}$

Representation of Relations →

(i) Relation as Directed graph →

Let $A = \{a, b, c, d\}$ and $B = A$

Then $R = \{(a,b), (a,c), (a,d), (d,a), (b,c), (c,d)\}$



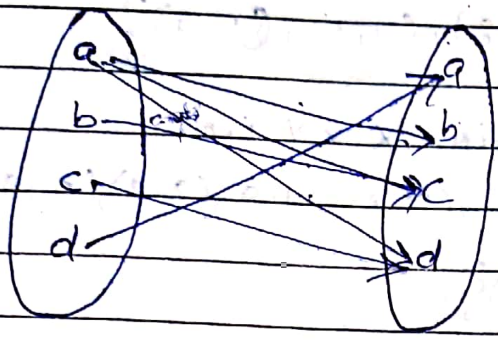
This graph represent the above relation R.

(ii) Relation as Matrix →

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

matrix representation of above Relation R.

(iii) Relation as function or Arrow Diagram :-



Arrow Diagram of above Relation R.

(iv) Relation as a Table :-

	a	b	c	d
a	✗	✓	✓	✓
b	✗	✗	✓	✗
c	✗	✗	✗	✓
d	✓	✗	✗	✗

Table of above Relation R.

4 Domain of Relation :- The domain of relation R from a set A to a set B is the set of all first elements of the ordered pairs which belong to R.

5 Range of Relation :- The range of a relation R is the set of all second elements of the ordered pairs which belong to R.

eg: Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$

and $R = \{(1,a), (1,b), (1,c), (2,b), (2,c), (2,d)\}$

$DOM(R) = \{1, 2\}$

$RAN(R) = \{a, b, c, d\}$

6 Complement of a Relation :- Consider a relation R from a set A to B. The complement of relation R denoted by $\overline{R}$ is a relation from A to B such that

$$\overline{R} = \{(a,b) : (a,b) \notin R\}$$

eg: $X = \{1,2,3\}$, $Y = \{8,9\}$

$R = \{(1,8),(2,8),(1,9),(3,9)\}$

Find $\bar{R}$.

The universal relation $X \times Y$ i.e.

$X \times Y = \{(1,8),(2,8),(3,8),(1,9),(2,9),(3,9)\}$

So $\bar{R} = \{(3,8),(2,9)\}$.

06 **Inverse of a Relation** → Let R be a relation from A to B. The inverse relation of R, denoted by R^{-1}, is a relation from B to A defined by

$$R^{-1} = \{(b,a) : (a,b) \in R\}$$

Thus R^{-1} consists of those ordered pairs which, when reversed, belong to R.

eg.

$R = \{(1,1),(1,2)(1,3),(2,1),(3,1),(3,2),(2,3)\}$

The inverse of Relation R is

$R^{-1} = \{(1,1),(2,1),(3,1),(1,2),(1,3),(2,3),(3,2)\}$

Composition of Relations → Consider a relation R_1 from A to B and R_2 be a relation from B to C. The composition of relation R_1 and R_2 denoted by $R_1 o R_2$, is the relation from A to C and is defined as

$R_1 o R_2 = \{(a,c) : (a,b) \in R_1$ and $(b,c) \in R_2$ for some $b \in B$. [for some $b \in B$ we have aR_1b and bR_2c

eg.

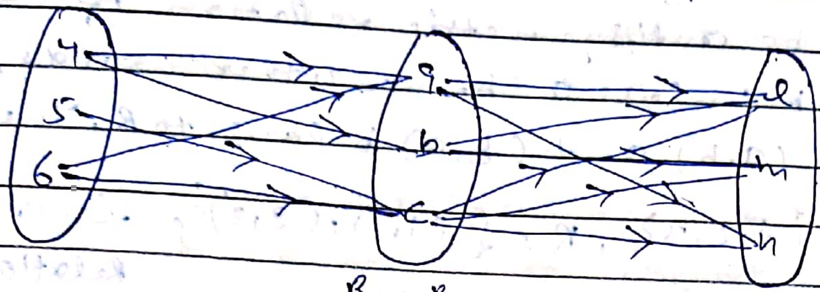
$A = \{4,5,6\}$, $B = \{a,b,c\}$, $C = \{l,m,n\}$

$R_1 = \{(4,a),(4,b),(5,c),(6,a),(6,c)\}$.

$R_2 = \{(a,l),(a,n),(b,l),(b,m),(c,l),(c,m),(c,n)\}$

$\forall \rightarrow$ forall

Composition of R_1 and R_2 is represented as :-



$R_1 \circ R_2$

Types of Relations :-

$A = \{a, b, c, d\}$

(i) **Reflexive Relation** → A relation R on a set A is said to be reflexive if aRa for all $a \in A$. i.e. if $(a,a) \in R$ for all $a \in A$. $R = \{(a,a), (b,b), (c,c), (d,d)\}$

(ii) **Irreflexive Relation** → A relation R on a set A is not reflexive if there exists at least one element $a \in A$ such that $(a,a) \notin R$.

$R = \{(a,b), (b,a), (c,d), (d,c)\}$

(iii) **Symmetric Relation** → A relation R on a set A is said to be symmetric if for every $(a,b) \in R$ implies that (b,a) also belong to R.

$R = \{(a,b), (b,a), (c,d), (d,c)\}$ $\boxed{aRb \text{ then } bRa}$

(iv) **Asymmetric Relation** → A relation R on a set A is called asymmetric relation if $(a,b) \in R$ but $(b,a) \notin R$.

$R = \{(a,b), (b,a), (c,d)\}$

(v) **Transitive Relation** → A relation R in a set 'A' is said to be transitive if aRb and bRc $\Rightarrow$ aRc i.e., if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

$A = \{1, 2, 3\}$ and $R = \{(1,2), (2,1), (1,1), (2,2)\}$ is a transitive Relation.

$\boxed{aRb \text{ and } bRc \text{ then } aRc}$

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(vi) Antisymmetric:- A relation R on a set 'A' is said to be antisymmetric relation if $(a,b) \in R$ but $(b,a) \notin R$ unless $a=b$. In other words, we can say if (a,b) and (b,a) belong to R, then a=b.

e.g. $A = \{1,2,3\}$, $R = \{(1,1),(2,2)\}$ is a Antisymmetric Relation.

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(vii) Equivalance Relation:- A relation R in a set 'A' is said to be equivalance relation if R is reflexive, symmetric and transitive. Thus R is an equivalance.

$A = \{a,b,c,d\}$

$R = \{(a,b),(b,a),(c,c),(d,d),(a,d),(b,d),$
$(a,a),(b,b),(d,a)\}$

It is a Equivalance Relation.

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(viii) Equivalance Class:- An equivalance Relation R on a set A. The Equivalance class of an element of A $a \in A$, is the set of elements of A to which element a is related. It is denoted by [a].

e.g. R be an equivalance relation on the set $A = \{4,5,6,7\}$

$R = \{(4,4),(5,5),(6,6),(7,7),(4,6),(6,4)\}$

The equivalance classes are:-

$[4] = [6] = \{4,6\}$

$[5] = \{5\}$

$[7] = \{7\}$

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(ix) Void Relation:- The relation $R = \phi$ is called the void relation or empty relation in A.

(x) Universal Relation → The universal relation is the largest equivalance relation which can be defined on a given set.

Partition of a set → A Partition ~~of~~ P of S is a ~~collection~~ ~~of~~ subdivision of S into disjoint non-empty sets.

Partial Order Relation → A relation R on a set A is called a partial order relation if it satisfies the following three properties:
(i) Relation R is reflexive.
(ii) Relation R is antisymmetric
(iii) Relation R is transitive.

Partially Order Set (POSET) → Relation R is partially order on a set 'A' then set 'A' is called partially order set.

eg:

$A = \{a, b, c\}$

$R = \{(a,a), (b,b), (c,c), (a,b), (b,c), (a,c)\}$

Relation R is Partial Order Relation and Set A is a partial ordered set.

* eg. xRY if x divides Y.

$A = \{2, 3, 4, 6, 7, 9\}$

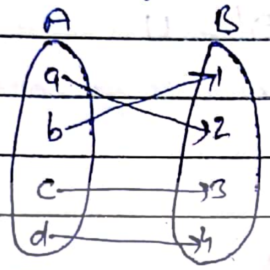
$R = \{(2,2), (2,4), (2,6), (3,3), (3,6), (3,9), (4,4), (6,6), (7,7), (9,9)\}$ set A is Partial Order Set.

* $x \preceq y$ means x precedes Y.

Functions

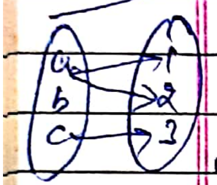
Ran(f)
Im(f) Same thing
f(A)

Function → A function 'f' from a set A to a set B is a relation from A to B such that each element of A is related to exactly one element of the set B. It is denoted as $f: A \to B$ and read as "f is a function from A to B".



Domain Co-Domain

$f(a) = 2$ ← Image
$f(b) = 1$
$f(c) = 3$
$f(d) = 4$ ← Image
Pre-image (range)



Not a function

Note ÷ It is possible that two element have same image but one element has two image is not possible.

Domain of a function → Let f be a function from A to B. The set A is called Domain of the function f.

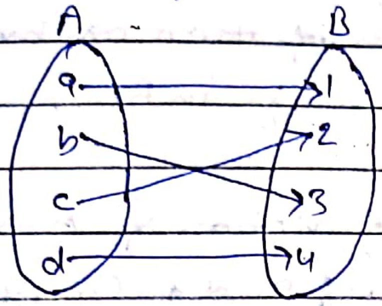
Co-Domain of a Function → Let f be a function from A to B. The set B is called Co-Domain of the function f.

Note ÷ 1. Every element of A must be related with at least one element of B.
2. There may be some element of B which are not related to any element of A.

Range of a function → The range of a function is the set of images of its domain.

Types of Function →

(i) **Injective Function / One-one Function** → If the function $f: A \to B$ is such that different elements of A have different f-images in B. Then f is called injective function or injection.



(Injective as well as Surjective function) called Bijective Function.

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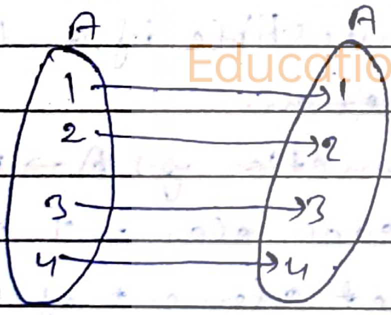
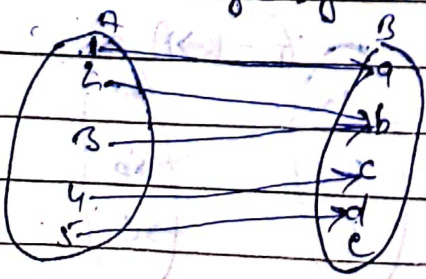
(ii) Surjective Function / OnTo Function → A function $f: A \to B$ is said to be Onto Function if each element of B is the image of some element of A. Or Each element of B have a preimage.

(iii) Bijective Function → A function which is both injective and surjective is called a bijective function or one to one Onto Function.

(iv) Identity Function → Consider any set A. Let the function $f: A \to A$. The function f is called the identity function if each element of set A has image on itself. i.e. $f(x) = x \ \forall \ x \in A$. It is denoted by I.



(v) INTO Function → If the function $f: A \to B$ is such that there is at least one element in B which is not the image of any element in A.



(Many-One Function Also)

(vi) **Many-One Function** → A function $f: A \to B$ is said to be many-one function if two or more elements of set A have the same image in B.

(vii) **Equal Function** → Two function $f: A \to B$ and $g: A \to B$ are said to be equal if and only if

$$f(a) = g(a) \text{ for every } a \in A.$$

eg.

$A = \{1, 2, 3\}$, $B = \{a, b, c\}$

$f: A \to B = \{(1,a), (2,a), (3,c)\}$

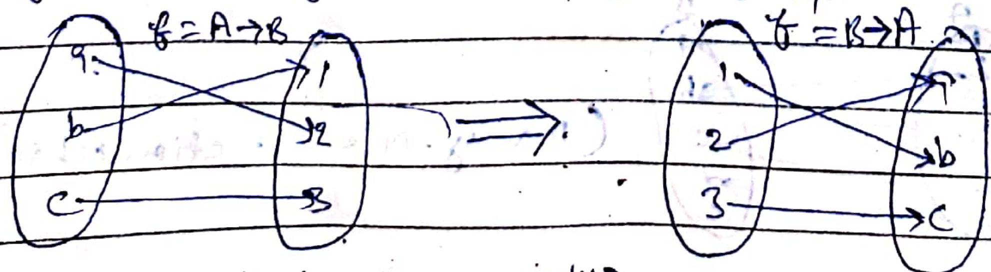
$g: A \to B = \{(1,b), (2,a), (3,c)\}$

$h: A \to B = \{(1,a), (2,a), (3,c)\}$

Function f and h are equal function. and function f, g and g, h are unequal function.

Inverse Function / Invertible Function →

A function $f: A \to B$ is invertible if and only if it is a bijective function.
Consider the bijective function $f: A \to B$. As f is one-one therefore each element of A corresponds to a distinct element of B. As f is Onto function, there is no element of B which is not the image of any element of A. The inverse function for f exists if f^{-1} is a function from B to A.



Note: $f(a) = b \Rightarrow a = f^{-1}(b)$

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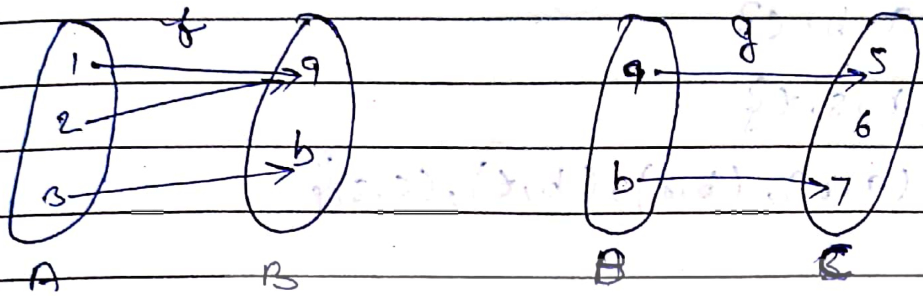
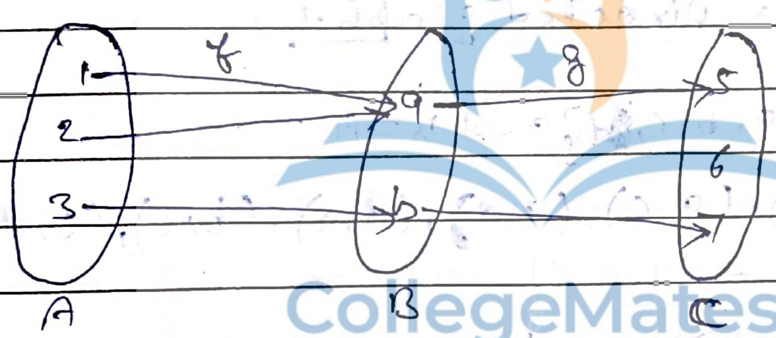
Composition of Functions:-

Let $f: A \to B$ and $g: B \to C$ be two functions. The composition of f with g is a function from A to C defined by $(g \circ f)(x) = g[f(x)]$ $\forall$ $x \in A$

Note:- Domain of $g \circ f$ is the domain of f and co-domain of $g \circ f$ is the co-domain of g.



Composition function $g \circ f$ is



Even and Odd functions:-

A function $f(x)$ is said to be even function of x if $f(-x) = f(x)$ and an odd function if $f(-x) = -f(x)$

Recursively Defined functions:- A function is said to be recursively defined if the function definition refers to itself.