Named Distribution

Named Distribution	Notation	Range	Use Case	PMF/PDF	CDF	Expected Value $E(X)$	Variance $V(X)$	Moment Generating Function $M(t)$	Method of Momenet	Maximum Likelihood	Sufficient Statistic	Note
Bernoulli	$Ber(p)$	$x = 0, 1$	Single trials.	$p^x(1-p)^{1-x}$		p	$p(1-p)$	$(1-p)+pe^t$	$\hat{p} = \bar{X}$	$\hat{p} = \bar{X}$		
Binomial	$Bin(n,p)$	$x = 0, 1, \ldots$ $n \in \mathbb{N}$	x is number of successes out of n trials.	$\binom{n}{x}p^x(1-p)^{n-x}$		np	$np(1-p)$	$(1-p+pe^t)^n$		$\hat{p} = \frac{\bar{X}}{n}$		
Geometric	$Geo(p)$	$x = 1, 2, \ldots$	x is number of trials until first success.	$(1-p)^{x-1}p$	$1-(1-p)^x$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{e^tp}{1-(1-p)e^t}$	$\hat{p} = \frac{1}{\bar{X}}$	$\hat{p} = \frac{1}{\bar{X}}$		Memoryless[1]
Negative Binomial	$NegBin(r,p)$	$x = r, r+1, \ldots$ $r \in \mathbb{N}$	x is number of trials until r successes.	$\binom{x-1}{r-1}p^r(1-p)^{x-r}$		$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{e^tp}{1-(1-p)e^t}\right]^2$				
Hyper Geometric	$HypGeo(m,r,n)$	$x = 1, \ldots, n$ $m, r, n \in \mathbb{N}$	Pick m out of n balls with r blacks. x is the number of black ball picked.	$\frac{\binom{r}{x}\binom{n-r}{m-x}}{\binom{n}{m}}$		$m\left(\frac{r}{n}\right)$	$m\left(\frac{r}{n}\right)\left(1-\frac{r}{n}\right)\left(\frac{n-m}{n-1}\right)$					n is population, m is number of trials, r is number of black ball.
Poisson	$Pois(\lambda)$	$x = 0, 1, \ldots$ $\lambda > 0$	Something happens on average λ times for a period. x is the number of observe happens.	$\frac{\lambda^x e^{-\lambda}}{x!}$		λ	λ	$e^{\lambda(e^t-1)}$		$\hat{\lambda} = \bar{X}$		Poisson can approximate Binomial when $n \to \infty$ and $p \to 0$, that is: $X \sim Bin(n,p)$ then $X \sim Pois(np)$
Uniform	$Uni(a,b)$	$x \in [a,b]$ $a, b \in \mathbb{R}$ $a \leq b$		$\frac{1}{\beta-\alpha}$	$\frac{x-a}{b-a}$	$\frac{b+a}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-eta}{t(b-a)}$		$\hat{\beta} = \max\{X_1, \ldots, X_n\}$	$\beta : \max\{X_1, \ldots, X_n\}$	
Exponential	$Exp(\lambda)$	$x \geq 0$ $\lambda > 0$	Something happens on average λ times for a period. x is the time to wait for one occurance.	$\lambda e^{-\lambda x}$	$1-e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}$	$\hat{\lambda} = \frac{1}{\bar{X}}$	$\hat{\lambda} = \frac{1}{\bar{X}}$		Memoryless[1], Often model time.
Gamma	$Ga(\alpha,\lambda)$	$x > 0$ $\alpha > 0$ $\lambda > 0$	Something happens on average λ times for a period. x is the time to wait for α occurances.	$\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$		$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\left[\frac{\lambda}{\lambda-t}\right]^\alpha$			$\alpha : \sum^n \ln X_i$	$Ga(1,\lambda) = Exp(\lambda)$
Normal (Gaussian)	$N(\mu,\sigma^2)$	$x \in \mathbb{R}$ $\mu \in \mathbb{R}$ $\sigma \in \mathbb{R}$		$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$		μ	σ^2	$e^{\mu t}e^{(\sigma^2t^2)/2}$		$\hat{\mu} = \bar{X}$ $\hat{\sigma^2} = \frac{1}{n}\sum_i(X_i-\bar{X})^2$		Symmetric.
Standard Normal	$Z, N(0,1)$	$x \in \mathbb{R}$		$\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$	$\Phi(x)$	0	1	$e^{t^2/2}$	–	–		Special Case of Normal.
Beta	$Beta(\alpha,\beta)$	$x \in [0,1]$ $\alpha > 0$ $\beta > 0$	Modelling proportion.	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$		$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$					$Beta(1,1) = Uni(0,1)$
Chi-Square	χ^2_n	$x \geq 0$ $n \in \mathbb{N}^+$	A Transformation of Standard Normal. Only takes positive value, model sample variance. Hypothesis tests.	$\frac{x^{n/2-1}}{2^{n/2}\Gamma(n/2)}e^{-x/2}$		n	$2n$	$(1-2t)^{-n/2}$				$\chi^2_n \sim Ga\,(n/2, 0.5)$, $U + V \sim \chi^2_{n+m}$
T	T_n	$t \geq 0$ $n \in \mathbb{N}^+$	Model Standardized quantities. Bell-shaped, looks like Normal but with heavier tails.	$\frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)}\left(1+\frac{t^2}{n}\right)^{-(n+1)/2}$		0						$T = \frac{Z}{\sqrt{U/n}}, Z \sim N(0,1), U \sim \chi^2_n$
F	$F_{m,n}$	$w \geq 0$ $m \in \mathbb{N}^+$ $n \in \mathbb{N}^+$	Model ratio of variance.	$\frac{\Gamma((m+n)/2)}{\Gamma(m/2)\Gamma(n/2)}\left(\frac{m}{n}\right)^{m/2}\left(1+\frac{m}{n}w\right)^{-(m+n)/2}w^{(m/2)-1}$								$W = \frac{U/m}{V/n}, U \sim \chi^2_m, V \sim \chi^2_n$, $X \sim T_n \implies X^2 \implies F_{1,n}$
Sample Mean	$\bar{X}$		$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$	$\frac{1}{n}\sum_{i=1}^n X_i$		μ	$\frac{\sigma^2}{n}$					$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim T_{n-1}$
Sample Variance	S^2			$\frac{1}{n-1}\sum_{i=1}^n(X_i-\bar{X})^2$		σ^2	$\frac{2\sigma^4}{n-1}$					$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

[1] Memoryless : $P(X > n+k | X > n) = P(k)$