# Database Systems, CSCI 4380-01
# Homework # 3 Answers

**Question 1.** You are given relation $R(A, B, C, D, E, F, G, H)$ and set $\mathcal{F}$ of functional dependencies:

$$\mathcal{F} = \{AB \to CD, BD \to EF, DE \to GH, G \to B\}$$

Use the BCNF decomposition to find relations that satisfy the BCNF normal form.

In your decomposition, use the functional dependency $BD \to EF$ first. For each decomposed relation, find the projection of the functional dependencies for that relation.

If the resulting relations are not in BCNF, continue with BCNF decomposition using $G \to B$.

If the resulting relations are not in BCNF, continue with BCNF decomposition using any other functional dependency that you desire.

Finally, after you complete the decomposition, check if the resulting decomposition is dependency preserving. Show your work.

**Answer.**

Check first if it is minimal, it is. Decompose first using $BD \to EF$. $BD^+ = \{B, D, E, F, G, H\}$, we get:

$R1(B, D, E, F, G, H)$ with $F1 = \{BD \to EF, DE \to GH, G \to B\}$ Key: BD, DE, GD. Not in BCNF

$R2(A, B, C, D)$ with $F2 = \{AB \to CD\}$ Key: AB, in BCNF ✓

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Decompose $R1$ using $G \to B$. $G^+ = \{B, G\}$, we get:

$R11(B, G)$ with $F11 = \{G \to B\}$. Key: G, in BCNF ✓

$R12(D, E, F, G, H)$ with $F12 = \{DE \to GH, GD \to EF\}$. Key: GD, DE. In BCNF. ✓

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Final relations:

$(A, B, C, D)$ with $\{AB \to CD\}$

$(B, G)$ with $\{G \to B\}$

$(D, E, F, G, H)$ with $\{DE \to GH, GD \to EF\}$

The union of functional dependency sets is:

$$\mathcal{F}' = \{AB \to CD, G \to B, DE \to GH, GD \to EF\}$$

It is easy to see that this set is not equivalent to the original set.

For example, according to $\mathcal{F}'$, $BD^+ = \{B, D\}$, since it does not include E,F, the original functional dependency $BD \to EF$ is not preserved.

**Question 2.** You are given relation $R(A, B, C, D, E, F, G, H)$ and set $\mathcal{F}$ of functional dependencies $\mathcal{F} = \{AB \to CDE, C \to A, BE \to CG\}$.

Use the 3NF decomposition to find relations that satisfy the 3NF normal form. For each resulting relation, list all functional dependencies that hold for that relation and show if it is also in BCNF or not.

**Answer.** Check if it is minimal.

$\mathcal{F} = \{AB \to CDE, C \to A, BE \to CG\}$.

Put in basis form:

$\mathcal{F} = \{AB \to C, AB \to D, AB \to E, C \to A, BE \to C, BE \to G\}$.

No trivial dependencies. Check if any dependency can be removed. Remove $AB \to C$ to get:

$\mathcal{F} = \{AB \to D, AB \to E, C \to A, BE \to C, BE \to G\}$.

$AB^+ = \{A, B, D, E, C, G\}$ Given $C$ is in the set, we can remove it.

$\mathcal{F} = \{AB \to D, AB \to E, C \to A, BE \to C, BE \to G\}$.

This set is minimal:

$\mathcal{F} = \{AB \to DE, C \to A, BE \to CG\}$.

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Keys: ABFH, BCFH

$R1(A, B, D, E)$, $\{AB \to DE\}$

$R2(A, C)$, $\{C \to A\}$

$R3(B, C, E, G)$, $\{BE \to CG\}$

$R4(A, B, F, H)$, $\{\}$

$R4$ is added because there was no other relation before that contained all the attributes of one of the keys. All relations are in BCNF.

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**Aside note:** What if we did not simplify to get a minimal cover? We would get:

$R1(A, B, C, D, E)$, $\{AB \to CDE, C \to A, BE \to C\}$

$R3(B, C, E, G)$, $\{BE \to CG\}$

$R4(A, B, F, H)$, $\{\}$

The main difference is in R1, in which we combined R1 and R2. For R1, both AB and BE are both keys. It is in 3NF but not BCNF. However, we are able to store (A,C) in a separate relation without causing a problem and this is a more modular solution where we do not have to repeat A values for a given C. Hence, by not using a minimal cover, we ended up with a slightly less optimal solution.