Database Systems, CSCI 4380-01
Homework # 2 Answers
Due Friday February 2, 2018 at 2:00:00 PM

Question 1 (10 points). Given a relation $R(A, B, C, D, E, F)$, are the following two sets of functional dependencies equivalent or not?

$\mathcal{F}_1 = \{AB \to C, CD \to E, BE \to D, D \to F\}$

$\mathcal{F}_2 = \{ABD \to EF, BE \to DEF, AB \to BC, D \to F, CDF \to E\}$

Answer.

($\Rightarrow$) Are all functional dependencies in $\mathcal{F}_2$ implied by $\mathcal{F}_1$?

$ABD^+ = \{A, B, D, C, E, D, F\}$, so $ABD \to EF$ is implied.
$BE^+ = \{B, E, D, F\}$, so $BE \to DEF$ is implied.
$AB^+ = \{A, B, C\}$, so $AB \to BC$ is implied.
$D \to F$ is already in it, so it is implied.
$CDF^+ = \{C, D, F, E\}$, so $CDF \to E$ is implied.

This is true!

($\Leftarrow$) Are all functional dependencies in $\mathcal{F}_1$ implied by $\mathcal{F}_2$?

$AB^+ = \{A, B, C\}$, so $AB \to C$ is implied.
$CD^+ = \{C, D, F, E\}$, so $CD \to E$ is implied.
$BE^+\{B, E, D, F\}$, so $BE \to$ is implied.
$D \to F$ is already in it, so it is implied.

This is true. Hence these two set of functional dependencies are equivalent.

Question 2 (40 points). You are given the following relations and associated set of functional dependencies.

For each part, you must do the following: (1) First, find and list all the keys. (2) Then, assess whether the relation is in Boyce-Codd Normal Form. If it is not, list all functional dependencies that violate it. (3) Finally, assess whether the relation is in Third Normal Form. Discuss why or why not.

(a) $Book(ISBN, Author, Title, Version, Publisher)$, $\mathcal{F} = \{ISBN \rightarrow Title, Publisher\}$

Answer. Key: ISBN, Author, Version. This is not in 3NF or BCNF because ISBN is not a superkey and neither title nor publisher are prime attributes.

(b) $R1(A, B, C, D, E)$, $\mathcal{F}_1 = \{AB \rightarrow C, AB \rightarrow DE\}$

Answer. Key: AB. This relation is both in 3NF and BCNF, because both functional dependencies have a superkey on the left.

(c) $R2(A, B, C, D, E)$, $\mathcal{F}_2 = \{AB \rightarrow CD, B \rightarrow E\}$

Answer. Key: AB. Not in BCNF or 3NF because $B \rightarrow E$ does not have a superkey on the left and E is not a prime attribute.

(d) $R3(A, B, C, D, E, F)$, $\mathcal{F}_3 = \{AB \rightarrow C, C \rightarrow DE, E \rightarrow F, F \rightarrow BC\}$

Answer. Keys: AB, AF, AE, AC. Not in BCNF or 3NF before $C \rightarrow DE$ has no superkey on the left and the right hand side is not all prime keys.

As a side note, the following relation has the keys AB,AF and is in 3NF but not BNCF:

$Rx(A, B, C, D, E, F)$, $\mathcal{F}_x = \{AB \rightarrow CDEF, F \rightarrow B\}$, because $F \rightarrow B$ does not have a super key on the left but B is a prime attribute.

(Note that by accident, some of you ended up solving R3 and some did Rx. We will accept both.)

Question 3 (20 points). Find the minimal basis for the following set of functional dependencies. Show the changes in each step.

$\mathcal{F}_2 = \{ABD \to EF, BE \to DEF, AB \to B, D \to F, CDF \to E\}$

Answer. Step 1: Convert to minimal cover.

$\mathcal{F}_2 = \{ABD \to E, ABD \to F, BE \to D, BE \to E, BE \to F, AB \to B, D \to F, CDF \to E\}$

Step 2: Check which functional depenencies can be removed.

$BE \to E$, $AB \to B$ can be removed, trivial.

$\mathcal{F}_2 = \{ABD \to E, ABD \to F, BE \to D, BE \to F, D \to F, CDF \to E\}$

$BE \to F$ can be removed as it is implied by the rest (in particular $BE \to D$ and $D \to F$)

$\mathcal{F}_2 = \{ABD \to E, ABD \to F, BE \to D, D \to F, CDF \to E\}$

$ABD \to F$ can be removed because ABD^+ includes F according to the set $\mathcal{F}_2'$ below:

$\mathcal{F}_2' = \{ABD \to E, BE \to D, D \to F, CDF \to E\}$

Step 3: Check which functional dependencies can be simplified.

$CDF \to E$ can be simplified to $CD \to E$ giving:

$\mathcal{F}_2'' = \{ABD \to E, BE \to D, D \to F, CDF \to E\}$

because $CD^+ = \{C, D, F, E\}$ with respect to $\mathcal{F}_2''$ and $\mathcal{F}_2'$ both.

$\mathcal{F}_2'' = \{ABD \to E, BE \to D, D \to F, CD \to E\}$

The remaining set is a minimal cover.

Question 4 (30 points). You are given the following:

$\mathcal{F} = \{AB \to CD, AE \to G, GD \to H, HB \to ED\}$ for relation $R(A, B, C, D, E, F, G, H)$

and the decomposition: $R1(A, B, C, D, G)$, $R2(B, E, H)$, $R3(A, B, G, H, F)$

(a) Is the above decomposition lossless? Use chase decomposition to show whether it is lossy or lostless?

Answer. We construct the following table for chase decomposition:

A	B	C	D	E	F	G	H
a	b	c	d	e1	f1	g	h1
a2	b	c2	d2	e	f2	g2	h
a	b	c3	d3	e3	f	g	h

Apply: $AB \to CD$:

A	B	C	D	E	F	G	H
a	b	c	d	e1	f1	g	h1
a2	b	c2	d2	e	f2	g2	h
a	b	c	d	e3	f	g	h

Apply: $GD \to H$:

A	B	C	D	E	F	G	H
a	b	c	d	e1	f1	g	h
a2	b	c2	d2	e	f2	g2	h
a	b	c	d	e3	f	g	h

Apply: $HB \to ED$:

A	B	C	D	E	F	G	H
a	b	c	d	e	f1	g	h
a2	b	c2	d2	e	f2	g2	h
a	b	c	d	e	f	g	h

Since the last row has no subscript, this decomposition is lossless.