ASSIGNMENT 8

Xinhao Luo

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1 Problem 18.20(a)

i) $min(x, y) \leq m \Rightarrow x \leq m \vee y \leq m$

ii)

$$\begin{aligned} P(min(x, y) \leq m) &= P(x \leq m) + P(y \leq m) - P(x \leq m \wedge y \leq m) \\ &= (1 - \frac{1}{2^m}) \times 2 - (1 - \frac{1}{2^m})^2 \\ &= 1 - \frac{1}{4^m} \end{aligned} \tag{1}$$

2 Problem 18.33

(l) Not Binomial: each draw is not independent, and the probability is not fixed

(m) Binomial

(o) i) Binomial

ii) Not binomial, as the number of trails is not fixed

(p) Binomial

(q) Not Binomial, each pick is not independent

3 Problem 19.11

1. In total, there are 16 possible outcomes: 11 lose and 5 wins
2. $E = \frac{10\times5-11x}{16} = \frac{25}{8} - \frac{11}{16}x$

4 Problem 19.35

a) i) Calculate the possibility of Double-head coins after 10 flip

$$
\begin{aligned}
P(\text{Double-head}\|\text{10 Head}) &= \frac{P(\text{10 Head}\|\text{ Double-side}) \times P(\text{ Double-side})}{P(\text{10 Head})} \\
&= \frac{P(\text{10 Head}\|D)P(\text{Double-Head})}{P(\text{10 Head}\|\text{Double-Head}))P(\text{Double-Head}) + P(10Head\|\text{Fair Coins})P(\text{Fair Coins})} \\
&= \frac{1 \times \frac{1}{1025}}{1 \times \frac{1}{1025} + (\frac{1}{2})^{10} \times \frac{1024}{1025}} \\
&= \frac{1}{2}
\end{aligned} \tag{2}
$$

ii) $E = \frac{1}{2} \times 100 + \frac{1}{2} \times \frac{1}{2} \times 100 = 75$

b) (a) From Part a, we know that $P(\text{Double-head}) = \frac{1}{2}$

(b)

$$
\begin{aligned}
E[\text{Head}] &= E[\text{Head}\|\text{Double-Head}] \times P(\text{Double-Head}) + E[\text{Head}\|\text{Fair}] \times P(\text{Fair}) \\
&= \frac{1}{P(\text{Head}\|\text{Double-Head})} \times P(\text{Double-Head}) + \frac{1}{P(\text{Head}\|\text{Fair})} \times P(\text{Fair}) \\
&= \frac{1}{1} \times \frac{1}{2} + \frac{1}{\frac{1}{2}} \times \frac{1}{2} \\
&= 1.5
\end{aligned} \tag{3}
$$

5 Problem 19.54

a) i) $P(\text{One male}) = \frac{1}{3}$

ii) $E(\text{One male}) = \frac{1}{\frac{1}{P}} = 3$

iii) $E(\text{Two male}) = 3 \times 2 = 6$

b) From Part a, we only need to change the $P(\text{One Male}) = \frac{1}{2}$, and we can get $E(\text{Two Male}) = 4$

c) From Part a, we only need to change the $P(\text{One Male}) = \frac{2}{3}$, and we can get $E(\text{Two Male}) = 3$

6 Problem 20.11

a) $\frac{1}{10!}$

b) 0

c) $\frac{\binom{10}{2}}{10!}$

d) $\frac{1}{10} + \frac{9}{10} \times \frac{1}{9} + \frac{8}{10} \times \frac{1}{8} + ... + \frac{1}{10} \times 1 = 1$

7 Problem 21.37

1. i) $E[X] = 1$

   ii) i) Let $X = X_1 + X_2 ... + X_{100}$

      ii) From Prob. 20.11, We know $P[X_1] = P[X_2]... = P[X_{100}]$

      iii) $E[X] = E[X_1 + X_2 ... + X_{100}] = 1 \times \frac{1}{100} 100 = 1$

      iv) $var[X] = E[X^2] - E[X]^2$, and we know $E[X] = 1$

      v)

$$\begin{aligned} E[X]^2 &= E[(X_1 + X_2 ... + X_{100})^2] \\ &= \sum_{i=1}^{100} E[(X_i)^2] + 2 \sum_{i,j \in (1,100)} E[X_i, X_j] \end{aligned} \tag{4}$$

      vi) Since X_i are Bernoulli, $E[{X_i}^2] = p = \frac{1}{100}$

      vii) $E[X_i, X_j] = E[X_i = 1 and X_j = 1] = \frac{1}{100} \times \frac{1}{99}$

      viii) The total number of terms like $E[X_i, X_j] = \binom{100}{2}$

      ix)

$$\frac{1}{100} \times 100 + 2 \times \frac{100 \times 99}{2} \times \frac{1}{100 \times 99} - 1 = 1 \tag{5}$$

      x) The answer is 1

2. i) Since $E[X] = 1$

   ii) $P[X = 50] = P[X \geq 50] - P[X \geq 51] = \frac{1}{50} - \frac{1}{51}$

   iii) $P[X > 50] \leq P[X \geq 50] - P[X = 50] = \frac{1}{50} - \frac{1}{50}\frac{1}{51} = \frac{1}{51}$