ASSIGNMENT 7

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Tuesday 12^{th} May, 2020

1 Problem 15.39(q)

Assume that friend is mutual: only both side agree then they become friends, so (A, B) is equivalent to (B, A)

1. There are in total 6 pairs among 4 people: (A, B), (A, C), (A, D), (B, C), (B, D), (C, D)
2. Each pair may have two different status: they are not friends, or they are friends. We use 1 to represent friendship, while 0 is the friendship does not exist
3. The binary sequence with length 6 has in total $2^6 = 64$ possible result
4. The possibility when randomly decide friendship is $\frac{1}{64}$

2 Problem 16.4

a) There are two world, and there is only one world with 100 black raven, so the chance is 1/2

b) (a)

$$\begin{aligned} P(\text{all black ravens}\|\text{one black raven exist}) &= \frac{P(\text{all black ravens} \cap 1 \text{ Black Raven exists})}{P(1 \text{ Black Raven exists})} \\ &= \frac{\frac{1}{2} \times \frac{100}{10^6}}{\frac{1}{2} \times \frac{100}{10^6} + \frac{1}{2} \times \frac{1000}{10^6}} \\ &= \frac{1}{11} \end{aligned} \tag{1}$$

3 Problem 16.37

a) $\frac{5}{100} \times 1 \times 1 + \frac{95}{100} \times \frac{1}{2} \times \frac{1}{2} = \frac{23}{80}$

b) $\frac{5}{100} \times 0 + \frac{95}{100} \times \frac{1}{2} \times \frac{1}{2} = \frac{19}{80}$

c) $\frac{5}{100} \times 1 \times 1 + \frac{95}{100} \times \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{21}{40}$

4 Problem 16.40

a) $P(\text{two girls}\|\text{one is a girl}) = \frac{P(\text{two girls}\cap\text{one is a girl})}{P(\text{one is a girl})} = \frac{1}{3}$

b) Set $Q = P(\text{a girl named Leilitoon})$

$$
\begin{aligned}
P(\text{Two girls}\|\text{A girl named Leilitoon}) &= \frac{P(\text{two girls} \cap \text{a girl named Leilitoon})}{P(\text{a girl named Leilitoon})} \\
&= \frac{\frac{1}{4}Q^2 + 2 \times \frac{1}{4}Q(1-Q)}{\frac{1}{4}Q^2 + 2 \times \frac{1}{4}Q(1-Q) + 2 \times \frac{1}{4}Q} \\
&= \frac{2-Q}{4-Q}
\end{aligned} \tag{2}
$$

Since Leiliton is a rare name, Q will be close to 0, so the final possibility would tend to be $\frac{2}{4} = \frac{1}{2}$

c) Set $Q = P(\text{Sunday}) = \frac{1}{7}$
From part c), we may have $P(\text{two girls}\|\text{Sunday}) = \frac{2-Q}{4-Q}$, where $Q = \frac{1}{7}$ with similar reasoning
$\frac{2-\frac{1}{7}}{4-\frac{1}{7}} = \frac{13}{27}$

5 Problem 17.9

The 8×8 chessboard has 64 squares in total (32 whites and 32 blacks)

a) $P(A \| B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{\frac{1}{2}} = 0 \neq P(A) = \frac{1}{2}$, so A and B are dependent event

b) $P(A \| B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} = P(A) = \frac{1}{2}$, so A and B are independent event

c) $P(A \| B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2}} = \frac{1}{2} = P(A) = \frac{1}{2}$, so A and B are independent event

6 Problem 17.28

$P(100 \text{ sided die and five time die with same number}) = 1 - \frac{100}{100} \times \frac{99}{100} \times \frac{98}{100} \times \frac{97}{100} \times 96100 = \frac{150859}{1562500}$