ASSIGNMENT 2

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1 Problem 3.53

(a) $x \in \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

(b) $x \in \mathbb{R}$

(c)
- $x \in \mathbb{N}, y \in \mathbb{N}$
- $x \in \mathbb{Z}, y \in \mathbb{Z}$
- $x \in \mathbb{Q}, y \in \mathbb{Q}$
- $x \in \mathbb{R}, y \in \mathbb{R}$

(d) $x \in \mathbb{R}$; $y \in \mathbb{N}$

2 Problem 4.9

(a) set "$n^3 + 5$ is odd" as p, "n is even" as q

- Direct Prove
  i) set p as true
  ii) $n^3 + 5 = 2k + 1$, while k is integer
  iii) n^3 is even
  iv) $n^3 = n \times n \times n$, since an even result can only come from even number times an even number, and a number can only either be even or odd, so n is even
  v) $p \rightarrow q$ is true ■
- Contraposition
  i) set q as false
  ii) n is odd
  iii) $n^3 = n \times n \times n$. Since odd number times an odd number will end in odd result, n^3 is odd
  iv) an odd number add an odd number will end in even result, so $n^3 + 5$ is even
  v) p is false
  vi) $p \rightarrow q$ is true ■

(b) set "3 does not divide n" as p, "3 divides $n^2 + 2$" as q

- Direct Prove
  i) set p as true
  ii) $\begin{cases} \text{n = 3k + 1} \\ \text{n = 3k + 2} \end{cases}$, k is integer
  iii) $\begin{cases} 9k^2 + 6k + 1 + 2 \\ 9k^2 + 6k + 4 + 2 \end{cases} \Rightarrow \begin{cases} 3(3k^2 + 2k + 1) \\ 3(3k^2 + 2k + 2) \end{cases}$, under both conditions, $n^2 + 2$ is a multiple of 3
  iv) q is true
  v) $p \rightarrow q$ is true ■
- Contraposition
  i) set q as false
  ii) for n, k as integer, there are three condition: $\begin{cases} n = 3k \\ n = 3k + 1 \\ n = 3k + 2 \end{cases}$
  iii) $n^2 + 2 \Rightarrow \begin{cases} 9k^2 + 2 \\ 9k^2 + 6k + 3 = 3(3k^2 + 2k + 1) \\ 9k^2 + 6k + 6 = 3(3k^2 + 2k + 2) \end{cases}$
  iv) Only $9k^2 + 2$ match $\neg q$
  v) $n = 3k$, n divides 3, p is false
  vi) $p \rightarrow q$ is true ■

3 Problem 4.15

(e) Direct Prove; set $k \in \mathbb{Z}$, p as "$n^2 + 3n + 4$ is even"

1. $n = \begin{cases} 2k \\ 2k+1 \end{cases}$
2. $n^2 + 3n + 4 = \begin{cases} 4k^2 + 6k + 4 \\ 4k^2 + 10k + 8 \end{cases} \Rightarrow \begin{cases} 2(2k^2 + 3k + 2) \\ 2(2k^2 + 5k + 4) \end{cases}$
3. under all conditions, the result of $n^2 + 3n + 4$ can be divided by 2
4. p is true ■

(w) Contraposition; set p as "a and b are positive real numbers with $ab < 10000$, q as "$min(a, b) < 100$"

1. set q as false
2. $min(a, b) \geq 100$
3. the minimum value of a, b is 100
4. $ab = 100 \times 100 = 10000$, $10000 \nless 10000$
5. p is false
6. $p \rightarrow q$ must be true ■

4 Problem 4.26

Truth table

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

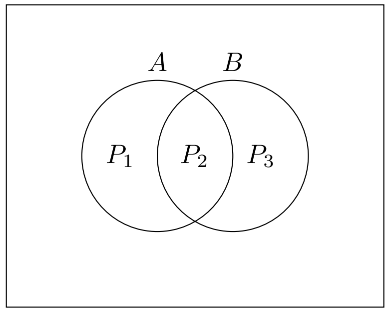
(b)
- **Prove.** (Direct Prove) From the truth table above, in order to prove $\neg(p \to q)$ true, we need to prove p as true and q as false
- **Disprove.** (show a counterexample) From the truth table above, in order to prove $p \to q$, for example, we can make p as true, and q as true, so $\neg(p \to q)$ will always be false

(d) $\forall x : ((\forall n : P(n)) \to Q(x))$
- **Prove.** (Show for general object) Showing that $P(n)$ is false for all n, then based on the truth table, the claim will always be true.
- **Disprove.** (show a counterexample) Given an pair of n, x that makes $P(n)$ as true while $Q(x)$ is false.

(f) $\exists x : ((\exists n : P(n)) \to Q(x))$
- **Prove.** (Show an example) Given an pair of x, n that makes $P(n)$ as true while $Q(x)$ is true.
- **Disprove.** (show an counterexample) Given an n that makes $P(n)$ as true while there is no corresponded x which makes $Q(x)$ is false.

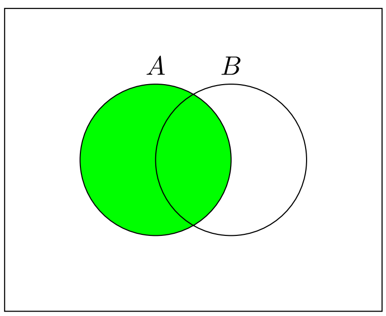
5 Problem 4.36(j)

Set P_1, P_2, P_3 to the corresponded areas shown below.

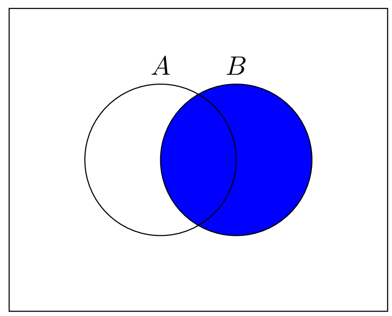


(1)

$|A| \Rightarrow P_1 + P_2$

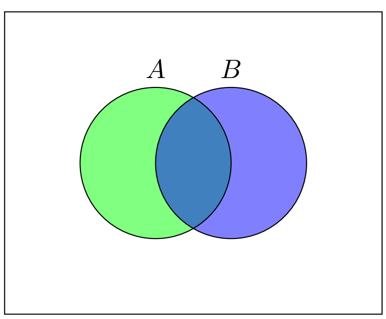


$|B| \Rightarrow P_2 + P_3$

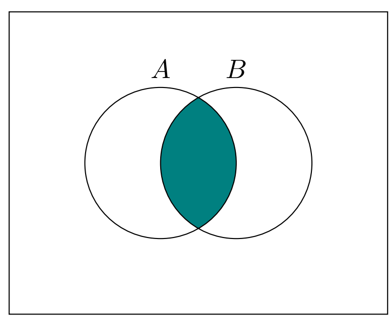


(2)

$A \cup B \Rightarrow P_1 + P_2 + P_3$



$A \cap B \Rightarrow P_2$



(3) From the graph above, $(P_1 + P_2) + (P_2 + P_3) = (P_1 + P_2 + P_3) + (P_2) = P_1 + 2P_2 + P_3$.

The equation is true. $\blacksquare$

6 Problem 4.45(b)

i) Set $a = 1$; set Definition (2) as p

(1) a function can either approach ∞ or a

(2) from (ii), $f(n) \to 1$

(3) $f(n)$ will not approach ∞

(4) p is false ■

ii) Set $a = 1$; set Definition (2) as p

(1) Assume $\neg p$ (contradiction)

(2) $f(n) \to a$ if for any $\epsilon > 0$, there is n_ϵ such that for all $n \geq n_\epsilon$, $|f(n) - a| > \epsilon$.

(3) $\frac{n+3}{n+1} > \epsilon \Rightarrow \frac{2}{\epsilon} - 1 > n$

(4) set $\epsilon = 2$, $2/2 - 1 = 0$

(5) $0 \leq 1$, which is a **fishy** conclusion

(6) p has to be true when $a = 1$ ■

iii) Set $a = 2$, $\epsilon = 0.1$, $n = 2$; set Definition (2) as p

(1) when $n = 2$, $f(n) = \frac{5}{3}$

(2) $|f(n) - 2| = \frac{1}{3}$

(3) $|f(n) - a| > \epsilon$

(4) p is false when $a = 2$ ■

7 Problem 5.7(f)

Define the claim P(n): $(1-\frac{1}{2})(1-\frac{1}{3})...(1-\frac{1}{n})=\frac{1}{n}$

i) **Base case** $P(1) = 1-\frac{1}{2}=\frac{1}{2}$

ii) **Induction Step** Showing $P(n) \to P(n+1)$ for all $n \geq 2$, using a direct proof

(1) Assume $P(n)$ is T: $(1-\frac{1}{2})(1-\frac{1}{3})...(1-\frac{1}{n})=\frac{1}{n}$

(2) Show $P(n+1)$ is T: $(1-\frac{1}{2})(1-\frac{1}{3})...(1-\frac{1}{n})(1-\frac{1}{n+1})=\frac{1}{n+1}$

(3)

$$\begin{aligned}(1-\frac{1}{2})(1-\frac{1}{3})...(1-\frac{1}{n})(1-\frac{1}{n+1}) &= [(1-\frac{1}{2})(1-\frac{1}{3})...(1-\frac{1}{n})](1-\frac{1}{n+1})\\ &= (\frac{1}{n})(1-\frac{1}{n+1})\\ &= \frac{n+1-1}{n(n+1)}\\ &= \frac{1}{n+1}\end{aligned} \tag{1}$$

(4) This is exactly what was to be shown. So, $P(n+1)$ is T

■