

Pop quiz Lec 6

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1 Pop Quiz 1

Write down the claim $P(n+1)$ where $P(n) = 1^2 + 2^2 + 3^2 \dots + n^2 = (1/6)(n)(n+1)(2n+1)$
 $P(n+1) = 1^2 + 2^2 + 3^2 \dots + n^2 + (n+1)^2 = (1/6)(n+1)(n+1+1)(2n+2+1)$

2 Pop Quiz 2

Show $(1/6)n(n+1)(2n+1) + (n+1)^2 = (1/6)(n+1)(n+2)(2n+3)$ using algebra

$$\begin{aligned}(1/6)n(n+1)(2n+1) + (n+1)^2 &= (1/6)(n+1)(n+2)(2n+3) \\ (1/6)n(2n+1) + (n+1) &= (1/6)(n+2)(2n+3) \\ (n^2 + 7n + 6)/6 &= (n^2 + 7n + 6)/6\end{aligned}\tag{1}$$

The equation is true.

3 Pop Quiz 3

Prove by contradiction, $2\sqrt{n} + (1/\sqrt{n+1}) \leq 2\sqrt{n+1}$

Prove.

i) Assume the claim is false

ii)

$$\begin{aligned}2 * \sqrt{n} + (1/\sqrt{n+1}) &> 2\sqrt{n+1} \\ 2 * \sqrt{n(n+1)} + 1 &> 2(n+1) \\ 2 * \sqrt{n^2 + n} &> 2n + 1 \\ 4(n^2 + n) &> 4n^2 + 4n + 1 \\ 0 &> 1\end{aligned}\tag{2}$$

iii) $0 > 1$ is a fishy conclusion

iv) then $2 * \sqrt{n} + (1/\sqrt{n+1}) \leq 2\sqrt{n+1}$