ASSIGNMENT 6

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1 Problem 13.8

a) 26^5

b) $\frac{26!}{21!}$

c) 26^2

d) $26^2 * 2$

e) $26^2 * 2 - 1$

2 Problem 13.44

a) $\frac{10!}{4!6!} = 210$

b) $\frac{10!}{(10-4)!} = 5040$

c) $\binom{14-1}{10-1} = \frac{13!}{9!4!} = 715$

d) 10^4

3 Problem 13.50

a) $\binom{10-1}{4-1} = 84$

b) $\binom{10+4-1}{4-1} = 286$

c) $\binom{12+4-1}{4-1} = 455$

4 Problem 13.51(a)

1. The number of all possible combination of 20-bit binary string is 2^{20}
2. Set $Q(n)$ as the number of 20-bit binary string does not contain "00"
3. Base case:
   (a) $Q(0) = 0$
   (b) $Q(1) = 2$
   (c) $Q(2) = 3$
   (d) $Q(3) = 5$
4. For $n \geq 4$, we have two cases:
   (a) String start from "1"
   So we have "1"+$Q(n-1)$
   (b) String start from "0"
   The second digit must be "1"
   So we have "01"+$Q(n-2)$
5. In conclusion,
$$Q(n) = Q(n-1) + Q(n-2) \tag{1}$$
6. The result will then be $2^{20} - Q(20)$, $Q(20) = 17711$
7.

5 Problem 13.61

$a = 2x$, $b = \sqrt{x}$, $n = 10$

$$\begin{aligned}\binom{10}{k}(2x)^k(\sqrt{x})^{10-k} &= \binom{10}{k}2^k x^k x^{5+\frac{k}{2}} \\ &= \binom{10}{k}2^k x^{5+\frac{k}{2}}\end{aligned} \tag{2}$$

x^3 $5 + \frac{k}{2} = 3$, $k = -4$, coefficient is 0 (section not exist)

x^4 $5 + \frac{k}{2} = 4$, $k = -2$, coefficient is 0 (section not exist)

x^5 $5 + \frac{k}{2} = 5$, $k = 0$, $\binom{10}{0}2^0x^5 = x^5$, the coefficient is 1

x^6 $5 + \frac{k}{2} = 6$, $k = 2$, $\binom{10}{2}2^2x^6 = 180x^6$, the coefficient is 180

x^7 $5 + \frac{k}{2} = 7$, $k = 4$, $\binom{10}{4}2^4x^7 = 3360x^5$, the coefficient is 3360

6 Problem 14.5

a) $\binom{10+4-1}{4-1} = \frac{13!}{10!3!} = 286$

b) $\frac{15!}{5!5!5!}$

c) $\binom{10+10-1}{10-1}$

d) $\frac{9!}{3!3!3!}$

7 Problem 14.14

a) $\binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}$

b) $2^5 + 2^4 * 5$

c) $2^5 + 2^4 * 5$

d) $(2^5 + 2^4 * 5) * 2 - 2$