Pop quiz 2

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1 prove if r is a irrational (contraposition), then $\sqrt{r} is irrational$

1. set p as r is irrational, q as $\sqrt{r}$ is irrational
2. from $p \to q$, we can get $\neg q \to \neg p$, then the statement become: if $\sqrt{r}$ is rational, then r is rational
3. since for any rational number x, x^2 is also a rational number. For $\sqrt{r}^2 = r$, since $\sqrt{r}$ is a rational number, r is also a rational number. $\neg q \to \neg p$ is true.
4. $p \to q$ is true

2 Prove (contradition) that for all integer $a, b, a^2 - 4b^2 \neq 2$

1. set a, b are all possible integers as p, $a^2 - 4b^2 \neq 2$ as q
2. to prove $p \to q$ true, we may prove the following statement is false: $p \to \neg q$
3. For all integers a, b, $a + b$ and $a - b$ will also be integers. In equation $(a - 2b)(a + 2b) = 2$ from $\neg q$, $a - 2b$ and $a + 2b$ have to be integers as well
4. $(1, 2), (-1, -2), (2, 1), (-2, -1)$ are possible combinations, while solution of a, b in all combinations are not integers.
5. Since a, b can either be an integer or an non-integer. Statement $p \to \neg q$ is false.
6. $p \to q$ is true