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*specjalność: Analiza danych*

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Locally-informed proposals in Metropolis-Hastings  
algorithm with applications

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## Abstract

The Markov Chain Monte Carlo methods (abbrev. MCMC) are a family of algorithms used for sampling from a given probability distribution. They prove very effective when the state space is large. This fact can be used to solve many hard deterministic problems – one of them being *traveling salesmen problem*. It will be used in this thesis to test a new approach of *locally-informed propolsals* as a modification of well known *Metropolis-Hastings* algorithm. In this thesis we will present the implementation of modified algorithm, experiments based on it, results and a comparison of to previous MCMC methods.

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Metody próbkowania Monte Carlo łańcuchami Markowa są rodziną algorytmów używanych do próbkowania z danego rozkładu prawdopodobieństwa. Okazują się efektywne zwłaszcza gdy przestrzeń stanów jest wielka. Ten fakt może być wykorzystany przy rozwiązywaniu wielu deterministycznych problemów – jednym z nich jest *problem komiwojażera*. Zostanie on użyty w tej pracy do przetestowania nowego podejścia *lokalnie poinformowanego*, jako modyfikacji dobrze znanego algorytmu *Metropolis-Hastingsa*. W tej pracy zaprezentujemy implementację zmodyfikowanego algorytmu, eksperymentów bazujących na nim, wyników oraz porównania z poprzednimi metodami próbkowania Monte Carlo.

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# 1 Introduction

The Markov Chain Monte Carlo methods (abbrv. MCMC) are a family of algorithms used for sampling from a given probability distribution. At first they do not seem useful for solving practical deterministic problems, but with some tweaks they can become a powerful tool. It happens especially when space of possible solutions is enormous and computing becomes infeasible for machines. These offer a shortcut for obtaining “close enough” answers.

At their core, MCMC methods generate a Markov Chain (abbrv. MC) with a defined distribution and sample using it. The convergence of the chain is assured by ergodic theorems. The most known of them is *Metropolis-Hastings* algorithm, which constructs a MC using another set of distributions, maybe simpler ones.

In this thesis we work on *locally-informed proposals*, which involve determining *local* distribution – which comes down to finding transition probabilities of the state. They are a bit more complex and computationally heavy, but offer better results with less iterations.

To test this method we will need a deterministic problem which quickly becomes infeasible for machines to compute – one of them is a well-known traveling salesman problem. The testing is carried out using its benchmark training set *tsplib95* and implementation is provided in *Python3*.

## 2 Markov Chains

## 3 Markov Chain Monte Carlo methods

## 4 Traveling salesman problem

## 5 Decoding encrypted text

## 6 Code description?

## 7 Conclusions

## References

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- [2] C. Karpiński. On use of monte carlo markov chains to decode encrypted text and to solve travelling salesman problem. 2020.
- [3] C. J. Maddison, D. Duvenaud, K. J. Swersky, M. Hashemi, and W. Grathwohl. Oops i took a gradient: Scalable sampling for discrete distributions. 2021.

## A Source code