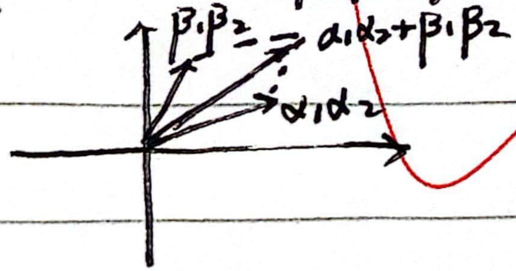
1. 证明：

由Cauchy不等式

$$(|\alpha_1|^2+|\beta_1|^2)(|\alpha_2|^2+|\beta_2|^2)$$

$$\geqslant (|\alpha_1||\alpha_2|+|\beta_1||\beta_2|)^2$$

$$= (|\alpha_1\alpha_2|+|\beta_1\beta_2|)^2$$



三角不等式

由图 $(|\alpha_1\alpha_2|+|\beta_1\beta_2|)^2$

$$\geqslant |\alpha_1\alpha_2+\beta_1\beta_2|^2$$

故 $|\alpha_1\alpha_2+\beta_1\beta_2|^2 \leqslant (|\alpha_1|^2+|\beta_1|^2)(|\alpha_2^2|+|\beta_2|^2)$.

2.证明：

$f(z)=u+iv \qquad z=x+iy \qquad \bar{z}=x-iy \quad \therefore \begin{cases} x=\dfrac{z+\bar{z}}{2} \\ y=\dfrac{z-\bar{z}}{2i} \end{cases}$

若$f(z)$是全纯的

则 $\begin{cases} \dfrac{\partial u}{\partial x}=\dfrac{\partial v}{\partial y} \\ \dfrac{\partial v}{\partial x}=-\dfrac{\partial u}{\partial y} \end{cases}$

$$\frac{\partial u}{\partial z}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial z}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial z}$$

$$=\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{2i}\frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial \bar{z}}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \bar{z}}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \bar{z}} \qquad \frac{\partial v}{\partial \bar{z}}=\frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial \bar{z}}+\frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial \bar{z}}$$

$$=\frac{1}{2}\frac{\partial u}{\partial x}-\frac{1}{2i}\frac{\partial u}{\partial y} \qquad =\frac{1}{2}\frac{\partial v}{\partial x}-\frac{1}{2i}\frac{\partial v}{\partial y}$$

故 $\dfrac{\partial f}{\partial \bar{z}}=\dfrac{\partial u}{\partial \bar{z}}+i\dfrac{\partial v}{\partial \bar{z}}$

$$=\frac{1}{2}\frac{\partial u}{\partial x}-\frac{1}{2i}\frac{\partial u}{\partial y}+i\left(\frac{1}{2}\frac{\partial v}{\partial x}-\frac{1}{2i}\frac{\partial v}{\partial y}\right)$$

$$=\frac{1}{2}\frac{\partial u}{\partial x}+\frac{i}{2}\frac{\partial u}{\partial y}-\frac{i}{2}\frac{\partial u}{\partial y}-\frac{1}{2}\frac{\partial u}{\partial x}$$

$$=0.$$

证毕

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3.解: $\cos(2+3i)$

$$=\cos 2\cos 3i-\sin 2\sin 3i$$

$$\cos 3i=\frac{e^{3}+e^{-3}}{2}\qquad \sin 3i=\frac{e^{-3}-e^{3}}{2i}$$

故 $\cos(2+3i)$

$$=\frac{e^{3}+e^{-3}}{2}\cos 2+i\,\frac{e^{-3}-e^{3}}{2}\sin 2$$

$\sin(2+3i)$

$= \sin 2 \cos 3i + \cos 2 \sin 3i$

$$= \frac{e^{3}+e^{-3}}{2}\sin 2 + i\,\frac{e^{3}-e^{-3}}{2}\cos 2$$

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4. 解：记 $z=x+iy$

$$\int_{|z|=1} Re(z)\,dz$$

$$=\int_{x^2+y^2=1} x\,d(x+iy)$$

$$=\int_{x^2+y^2=1} x\,dx+i\,x\,dy$$

令 $x=\cos\theta \quad y=\sin\theta$

则 $\int_{x^2+y^2=1} x\,dx+i\,x\,dy$

$$=\int_0^{2\pi}\cos\theta\,d\cos\theta+i\cos\theta\,d\sin\theta$$

$$=\cancel{\int_0^{2\pi}}\frac{1}{2}\cos^2\theta\Big|_0^{2\pi}+i\int_0^{2\pi}\cos^2\theta\,d\theta$$

$$=\frac{i}{4}\int_0^{2\pi}(1+\cos2\theta)\,d(2\theta)$$

$$=\frac{i}{4}(2\theta+\sin2\theta)\Big|_0^{2\pi}$$

$$=\pi i$$

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5. 解： $x^3=3x+1$

令 $x=2\cos\alpha$ 得 $8\cos^3\alpha-6\cos\alpha=1$

即 $2(4\cos^3\alpha-3\cos\alpha)=2\cos 3\alpha=1$

$\cos 3\alpha=\frac{1}{2}$

故 $3\alpha=2k\pi\pm\frac{\pi}{3}$

$\alpha=\frac{2k\pi}{3}\pm\frac{\pi}{9}$

故 $x=2\cos\left(\frac{2k\pi}{3}\pm\frac{\pi}{9}\right)$

解为 $x=2\cos\frac{\pi}{9}$ ，$2\cos\frac{5\pi}{9}$ ，$2\cos\frac{7\pi}{9}$