
DATA130026.01 Optimization

Assignment 5

Due Time: at the beginning of the class, April. 6, 2023

1. Consider the maximization problem

$$\begin{aligned} \max \quad & x_1^2 + 2x_1x_2 + 2x_2^2 - 3x_1 + x_2 \\ \text{s.t.} \quad & x_1 + x_2 = 1 \\ & x_1, x_2 \geq 0. \end{aligned}$$

- (i) Is the problem convex?
- (ii) Find all the KKT points of the problem.
- (iii) Find the optimal solution of the problem.

2. Consider the optimization problem

$$\begin{aligned} \text{(P)} \quad \min \quad & x_1 - 4x_2 + x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + 2x_3 = -2 \\ & x_1^2 + x_2^2 + x_3^2 \leq 1. \end{aligned}$$

- (i) Given a KKT point of problem (P), must it be an optimal solution?
- (ii) Find the optimal solution of the problem using the KKT conditions.

3. Consider the optimization problem

$$\text{(P)} \quad \min \{ \mathbf{a}^T \mathbf{x} : \mathbf{x}^T \mathbf{Q} \mathbf{x} + 2\mathbf{b}^T \mathbf{x} + c \leq 0 \},$$

where $\mathbf{Q} \in \mathbb{R}^{n \times n}$ is positive definite, $\mathbf{a} (\neq \mathbf{0})$, $\mathbf{b} \in \mathbb{R}^n$, and $c \in \mathbb{R}$.

- (i) For which values of $\mathbf{Q}, \mathbf{b}, c$ is the problem feasible?
- (ii) For which values of $\mathbf{Q}, \mathbf{b}, c$ are the KKT conditions necessary?
- (iii) For which values of $\mathbf{Q}, \mathbf{b}, c$ are the KKT conditions sufficient?
- (iv) Under the condition of part (ii), find the optimal solution of (P) using the KKT conditions.

4. Consider the optimization problem

$$\begin{aligned} \min \quad & x_1^2 - x_2^2 - x_3^2 \\ \text{s.t.} \quad & x_1^4 + x_2^4 + x_3^4 \leq 1. \end{aligned}$$

- (i) Is the problem convex?
- (ii) Find all the KKT points of the problem.
- (iii) Find the optimal solution of the problem.

5. **[Only required for DATA130026h.01.]** Questions 5.27 and 5.31 in CVX book (https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf).