DATA130026 Optimization
Assignment 14
Due Time: at the beginning of the class, Jun. 22, 2023

1. Consider the nonsmooth function

$$f(x) = \max_{1 \le i \le K} x_i + \frac{1}{2}\|x\|^2,$$

where $x \in \mathbb{R}^n$, $K \in [1, n]$ is a given positive integer.

(a) Calculate the minimizer x^* and the associated objective value f^*.

(b) Show that $f(x)$ is G-Lipschitz continuous for all $\|x\| \le 1/\sqrt{K}$, where $G = 1 + \frac{1}{\sqrt{K}}$.

(c) Suppose the initial point $x_0 = 0$ and you use subgradient method (let the subgradient of x be $x + e_j,\ j = \min\{n | x_n = \max_{1 \le i \le K} x_i\}$) to solve $\min f(x)$, where the stepsize can be arbitrary chosen. Show that after k ($k < K$) iterations, we have

$$f_{best}^k - f^* \ge \frac{G\|x_1 - x^*\|}{2(1+\sqrt{K})},$$

where $f_{best}^k = \min_{1 \le i \le k} f(x_i)$.

2. Write a MATLAB code for solving the Lasso problem using subgradent method:

$$\min_{x \in \mathbb{R}^n} \frac{1}{2}\|Ax - b\|_2^2 + \tau\|x\|_1$$

where $\tau > 0$ is a weighting parameter, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^n$ are given data. Choose $x = 0$ as the starting point. Terminate your code after 10000 iterations. Use the following Matlab code to generate the data:

m = 100; n = 500; s = 50;
A = randn(m,n);
xs = zeros(n,1); picks = randperm(n); xs(picks(1:s)) = randn(s,1);
b = A*xs;tau=0.001;

Use constant step size, constant step length, dimnishing step size and Polyak's step size. Try three different constants or parameter for constant step size, constant step length and dimnishing step size. Plot fours figures to show the evolutions for $f(x_k) - f^*$ (the optimal value can be computed by CVX) for the four step size rules.