DATA130026.01 Optimization
Assignment 15
Due Time: N/A

1. Compute the projection of a given point x to the second order cone $Q = \{(x,t) \in \mathbb{R}^{n+1} \mid \|x\| \leq t\}$.

2. Show that the projection onto the set $C = [l, u] = \{x \in \mathbb{R}^n | l \leq x \leq u\}$ can be written as

$$P_C(x)_i = \begin{cases} l_i & \text{if } x_i \leq l_i, \\ x_i & \text{if } l_i \leq x_i \leq u_i, \\ u_i & \text{if } x_i \geq u_i. \end{cases}$$

3. Consider the Lasso problem

$$\min_{x \in \mathbb{R}^n} \frac{1}{2} \|Ax - b\|_2^2 + \tau \|x\|_1$$

where $\tau > 0$ is a weighting parameter, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ are given data. Use the following Matlab code to generate the data:

```
m = 100; n = 500; s = 50;
A = randn(m,n);
xs = zeros(n,1); picks = randperm(n); xs(picks(1:s)) = randn(s,1);
b = A*xs;
```

Choose $x = 0$ as the starting point. Implement the proximal gradient method to solve the problem. Terminate your code after 1000 iterations or the norm of gradient mapping is less than `1e-6`. Implement your algorithm with fixed step size $1/L$, where L is the Lipschitz constant for the smooth part, and backtracking line search. (You need to show how to compute L.)

Did you get a solution exactly equal to the given xs? How about try a different τ, or different m? Give a summary how the parameters τ and m affect the solution. Plot the results.