DATA130026.01 Optimization
Assignment 7
Due Time: at the beginning of the class, Apr. 20, 2023

1. Questions 5.6 and 5.11 in CVX book (https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf).

2. *Strong duality in linear programming.* We prove that strong duality holds for the LP

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax \preceq b \end{array}$$

and its dual

$$\begin{array}{ll} \text{maximize} & -b^T z \\ \text{subject to} & A^T z + c = 0, \quad z \succeq 0, \end{array}$$

provided at least one of the problems is feasible. In other words, the only possible exception to strong duality occurs when $p^\star = \infty$ and $d^\star = -\infty$.

   (a) Suppose $p^\star = \infty$ and the dual problem is feasible. Show that $d^\star = \infty$. *Hint.* Show that there exists a nonzero $v \in \mathbf{R}^m$ such that $A^T v = 0, v \succeq 0, b^T v < 0$. If the dual is feasible, it is unbounded in the direction v.

   (b) Consider the example

$$\begin{array}{ll} \text{minimize} & x \\ \text{subject to} & \begin{bmatrix} 0 \\ 1 \end{bmatrix} x \preceq \begin{bmatrix} -1 \\ 1 \end{bmatrix}. \end{array}$$

   Formulate the dual LP, and solve the primal and dual problems. Show that $p^\star = \infty$ and $d^\star = -\infty$.

3. **[Only required for DATA130026h.01.]** Question 5.10 in CVX book (https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf).