DATA130026 Optimization
Assignment 11
Due Time: at the beginning of the class, May 25, 2023

1. Let $f(x) := \|x\|_2^\beta$, where $\beta > 0$ is given. Suppose you use Pure Newton's method (with stepsize 1) to minimize f, and initial point $x_0 \neq 0$.

   (a) If $\beta > 1$ and $\beta \neq 2$, then x_k converges to 0 linearly. Explain why we do not have local quadratical convergence shown in the class.

   (b) If $0 < \beta < 1$, then the method diverges.

2. An engineer has decided to verify numerically that the exponential function $x \to \exp(x) = e^x$ grows faster than any polynomial. In order to do so, he/she studies the optimization problem to

$$\min f(x) = x^\alpha - e^x,$$

   where α is the highest power of the polynomial (we assume it is an even, positive integer number). The engineer uses a Newton method (with unit steps!) to solve the problem. He/she argues that if the exponential function grows faster than any polynomial, then the sequence $\{x_k\}$ generated by the method should diverge to infinity, because the objective function f can be decreased indefinitely by increasing the value of x.

   (a) State the Newton iteration explicitly for the given problem (1).

   (b) Construct a numerical example (that is, choose a value of $\alpha \in \{2, 4, \dots\}$ and a starting point of the Newton algorithm) illustrating the engineers error in reasoning.

   (c) Find the error in the engineer's reasoning and formally explain it.