DATA130026 Optimization
Assignment 13
Due Time: at the beginning of the class, Jun. 8, 2023

1. For each of the following functions on $\mathbb{R}^n$, explain how to calculate a subgradient at a given x.

   (a) $f(x) = \sup_{0 \le t \le 1} p(t)$, where $p(t) = x_1 + x_2 t + \ldots + x_n t^{n-1}$.

   (b) $f(x) = x_{[1]} + x_{[2]} + \ldots + x_{[k]}$, where $x_{[i]}$ denotes the ith largest elements of x.

   (c) $f(x) = \|Ax - b\|_2 + \|x\|_2$ where $A \in \mathbb{R}^{m \times n}$.

2. (subgradient of the maximum eigenvalue function). Consider the function $f : S^n \to \mathbb{R}^n$ given by $f(X) = \lambda_{\max}(X)$ (recall that S^n is the set of all $n \times n$ symmetric matrices). Let $X \in S^n$ and let v be a normalized eigenvector of X ($\|v\|_2 = 1$) associated with the maximum eigenvalue of X. Show that $vv^T \in \partial f(X)$.