DATA130026 Optimization
Assignment 3
Due Time: at the beginning of the class, Mar. 23, 2023

Give an explicit solution of each of the following LPs.

1. Minimizing a linear function over an affine set.

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & Ax = b. \end{array}$$

2. Minimizing a linear function over a halfspace.

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & a^T x \leq b. \end{array}$$

3. Minimizing a linear function over a rectangle.

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & l \leq x \leq u. \end{array}$$

   where l and u satisfy $l \leq u$.

4. Minimizing a linear function over the probability simplex.

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & e^T x = 1,\ x \geq 0, \end{array}$$

   where e is the all one vector.

5. Minimizing a linear function over a unit box with a total budget constraint.

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & e^T x = \alpha,\ 0 \leq x \leq 1, \end{array}$$

   where α is an integer between 0 and n. What happens if α is not an integer (but satisfies $0 \leq \alpha \leq n$)? What if we change the equality to an inequality $e^T x \leq \alpha$.

6. Minimizing a linear function over a unit box with a weighted budget constraint.

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & d^T x = \alpha,\ 0 \leq x \leq 1, \end{array}$$

   where $d > 0$, and $0 \leq \alpha \leq e^T d$.