# RF device modeling in SCUFF-EM

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## 1 Overview

The SCUFF-EM package includes a module for RF device modeling within the framework of integral-equation electromagnetism. The core functionality of the module is implemented within the SCUFF-SOLVER high-level interface to SCUFF-EM and may be accessed either **(a)** in API form from C++ or python programs, or **(b)** from the command line via the SCUFF-RF binary application code.

The SCUFF-EM RF module extends the existing functionality of the SCUFF-EM core library in two main ways:

1. It introduces the notion of a *port.* This is a region of your structure (be it an antenna, a coaxial cable, etc.) that interfaces with RF circuitry; more specifically, a port consists of a positive terminal, into which a current of arbitrary complex amplitude is injected, and a negative terminal from which the same current is extracted. The fields radiated by these currents define the incident field in the SIE scattering problem solved by SCUFF-EM.

   Ports are defined by geometric entities (points, lines, or polygons) identifying regions of meshed structures. Information on ports is stored internally in the SCUFF-EM RF module via data structures named `RWGPort` and `RWGPortEdge`.

2. It introduces a new type of post-processing operation to compute the *impedance parameters* of a multiport geometry. (All of the varous other

types of post-processing calculations offered by core SCUFF-EM—including computation and visualization of fields, induced moments, power, etc.—are available as well.)

In this memo we will discuss the implementation of these features.

## 2 The concept of an `RWGPort`

The key extension of the LIBSCUFF core library provided by SCUFF-RF is the idea of an `RWGPort`. This is a physical region of a material body that interfaces with RF circuitry. More specifically, a port consists of a positive terminal and an (optional) negative terminal, where each terminal is a small region of a surface into (or from) which an external current is forced; if the port current is $I$, then a total current $I$ is forced into the positive terminal and an equal current is extracted from the negative terminal. The fields radiated by the resulting spatially localized current distributions then constitute the incident field in a scattering problem. (A port may have no negative terminal, in which case the current forced into the positive terminal may be thought of as flowing out through the ground plane of the substrate (if present) or from a negative terminal at spatial infinity.

Figure 1 shows some examples of `RWGPort` terminals. As this figure makes clear, each port is fully defined by one or more vertices; ports are specified to SCUFF-EM simply by giving the coordinates of these vertices.

To each `RWGPort` terminal correspond one or more triangles with distinguished edges (indicated by arrows in Figure 1), to which SCUFF-EM assigns *half-RWG* basis functions $\mathbf{h}(\mathbf{x})$. A half-RWG basis function is just what it sounds like—it is defined by a single triangle, with a distinguished choice of vertex, and it describes a surface-current density supported only on that triangle, emanating from the vertex and flowing normally outward through the opposite edge. In a *full*-RWG function $\mathbf{b}(\mathbf{x})$ this outflow of current is sunk into the adjacent panel (which, in SCUFF-EM parlance, would be the 'negative panel' of the full RWG function), and thus full RWG functions carry no net current; half-RWG functions, on the other hand, have no negative panel and describe currents that appear from out of nowhere, as is appropriate for representing ports driven by a given current injected by fixed external sources.

The *perimeter* of a port terminal is the sum of the lengths of all triangle edges it contains. This number is relevant for determining the weight with which each half-RWG basis function in a port terminal is populated to describe a port current $I$. If $L_p^{\pm}$ are the perimeters of the positive and negative ports of port $p$, then a port current $I_p$ is described by weighting each basis function in the positive terminal with weight $+\frac{I_p}{L_p^+}$ and each basis function in the negative terminal with weight $-\frac{I_p}{L_p^-}$. Thus, defining

$$\mathbf{K}_p(I_p;\mathbf{x}) \equiv \begin{pmatrix}\text{spatial distribution of electric surface-current} \\ \text{density due to port } p \text{ driven by current } I_p\end{pmatrix} \tag{1}$$

we have

$$\mathbf{K}_p(I_p;\mathbf{x}) = \frac{I_p}{L_p^+}\sum_{a\in\mathcal{P}_p^+}\mathbf{h}_a(\mathbf{x}) - \frac{I_p}{L_p^-}\sum_{a\in\mathcal{P}_p^-}\mathbf{h}_a(\mathbf{x}). \tag{2}$$

where the notation $a\in\mathcal{P}_p^{\pm}$ indicates that $a$ runs over the indices of all half-RWG basis functions on the $\pm$ terminal of port $p$.

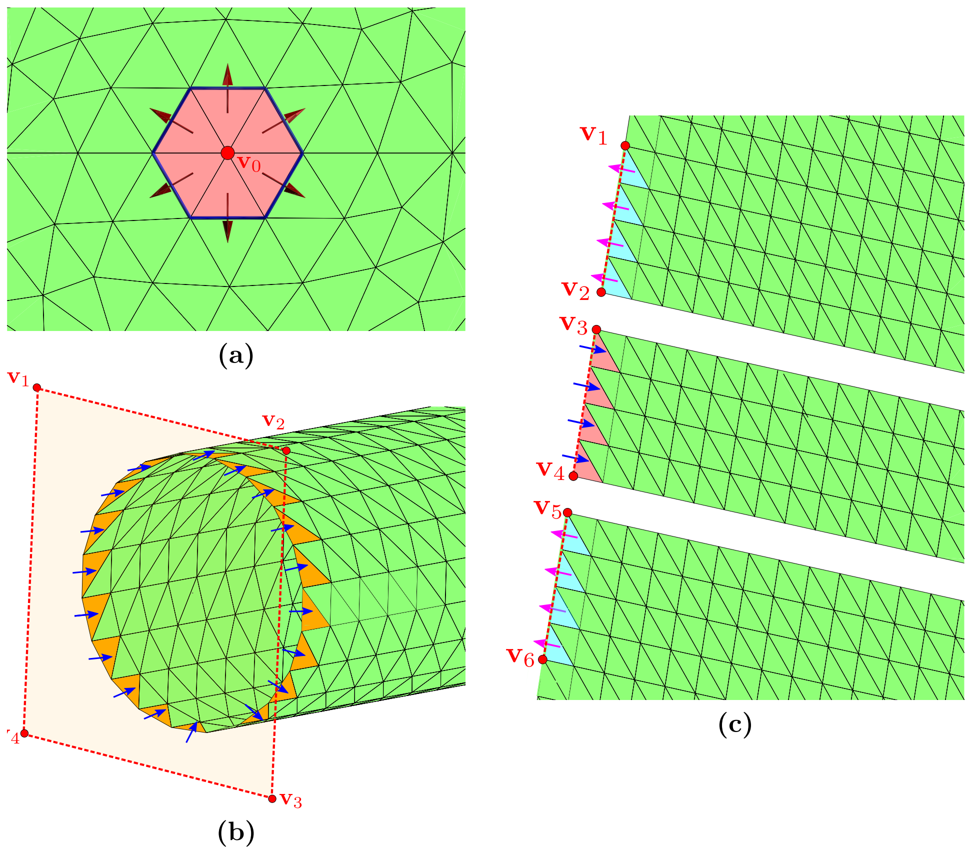


Figure 1: Examples of `RWGPort` terminals. **(a)** Port terminal defined by a single point $\mathbf{v}_0$ in the interior of a meshed surface. **(b)** Port terminal defined by a planar polygon; the port terminal is the union of all triangle edges lying on the boundary of the meshed structure and inside the polygon. **(c)** Port terminals defined by line segments: the positive port terminal (blue arrows) consists of all triangle edges on the boundary of the meshed structure lying within the line segment $\overline{\mathbf{v}_3\mathbf{v}_4}$, while the negative terminal (red arrows) consists of all triangle edges contained in line segments $\overline{\mathbf{v}_1\mathbf{v}_2}$ and $\overline{\mathbf{v}_5\mathbf{v}_6}$.

In this memo I use indices $a, b$ for half-RWG functions and $\alpha, \beta$ for full-RWG functions.

**Some technical implementation details** Any meshed representation of an open surface in a SCUFF-EM geometry is automatically assigned a set of half-RWG basis functions, one for each exterior panel edge; these are stored within the `RWGSurface` structure for the surface in an array called `HalfRWGEdges`. (Full RWG edges are stored separately, in the `Edges` array.) Ordinarily, the half-RWG functions assigned to exterior edges are inert in SCUFF-EM scattering problems; they are identified when the geometry is read and their properties are stored in the `HalfRWGEdges` array, but they are never populated with surface-current weights and do not contribute to the SIE system or to post-processing quantities. However, when a half-RWG function for an exterior edge is identified as belonging to a port terminal, it is promoted to an active participant in scattering calculations, contributing to the RHS vector and to post-processing quantities like scattered fields.

In addition to ports defined on boundaries of open surfaces, it is also possible to define ports based at individual vertices in the *interior* of surfaces. In this case, although the panels that share that vertex already belong to full-RWG functions, we create separate new half-RWG functions for them to describe their their role in supporting the port-current distribution. Thus, each instantiation of a point-based port results in the creation of new `HalfRWGEdge` structures that are tacked on to the end of the `HalfRWGEdge` array for the `RWGSurface` in question.

# 3 SIE scattering problems with port excitation

In RF device modeling, structures are excited by user-specified input currents forced into one or more ports of a structure, and the objective is to compute impedance parameters (or admittance parameters or $S$-parameters) for the multiport network (possibly accompanied by other quantities such as radiated-field profiles).

On the other hand, in the usual surface-integral-equation (SIE) approach to electromagnetic scattering implemented by the SCUFF-EM core library, the excitation is provided by a user-specified incident electromagnetic field configuration (such as a plane wave or the field of a point dipole), and the objective is to compute quantities such as absorbed or scattered power.

How do we squeeze the former problem into the latter framework? In essence, we take the incident field in the SIE scattering problem to be the field radiated by the currents forced into the ports of a structure, and we obtain impedance-matrix elements by calculating complex power dissipation in the structure. In this section I discuss how these steps are implemented in the SCUFF-EM RF module. (The problem of computing fields radiated by port-driven structures requires no new implementation beyond the existing SCUFF-EM algorithms for computing fields radiated by excited structures.)

## 3.1 Port currents: A new type of incident field

Consider an $N$-port RF device geometry driven by a set of $N$ complex-valued currents $\{I_p\}, p = 1, \cdots, N$. The full surface-current distribution of all driven ports is given by superposing (2) for all ports:

$$\mathbf{K}^{\text{port}}(\mathbf{x}) = \sum_{a\in\mathcal{P}_p^{\pm}} \underbrace{\pm\frac{I_p}{L_p^{\pm}}}_{\xi_a} \mathbf{h}_a(\mathbf{x})$$

The electric field radiated by this current distribution, which defines the incident field in the scattering problem solved by SCUFF-EM, is then a sum of contributions from HRWG basis functions:

$$\mathbf{E}^{\text{inc}}(\mathbf{x}) = \underbrace{\int \mathbf{\Gamma}(\mathbf{x},\mathbf{x}')\cdot\mathbf{K}(\mathbf{x}')\,d\mathbf{x}'}_{\mathbf{\Gamma}\star\mathbf{K}} \tag{3}$$

$$= \sum_a \xi_a \mathbf{\Gamma}\star\mathbf{h}_a \tag{4}$$

with $\mathbf{\Gamma} \equiv \mathbf{\Gamma}^{\text{EE}}$ the $3\times 3$ dyadic Green's function giving the electric field of an electric current in the medium of the scattering geometry.[1]

[1]In a homogeneous medium with wavevector $k$ and relative wave impedance $Z_r$ we have $\mathbf{\Gamma}^{\text{EE}}(\mathbf{x},\mathbf{x}') = ikZ_0Z_r\mathbb{G}(k;\mathbf{x}-\mathbf{x}')$ with $\mathbb{G}_{ij}(\mathbf{r}) \equiv \left(\delta_{ij}+\frac{1}{k^2}\partial_i\partial_j\right)\frac{e^{ik|\mathbf{r}|}}{4\pi|\mathbf{r}|}$ the usual Helmholtz dyadic, but this discussion is more general; in particular, $\mathbf{\Gamma}^{\text{EE}}$ may include the effect of a layered dielectric substrate.

Figure 2: The incident field in the BEM scattering problem is the field radiated by half-RWG basis functions associated with the exterior edges comprising the `RWGPort`. Each half-RWG basis function is populated with a strength proportional to the port current.

The usual EFIE formulation of scattering from PEC bodies then yields an integral equation for the unknown electric surface-current density $\mathbf{K}^{\text{ind}}$ induced on the scatterers by the incident field:

$$\int \mathbf{\Gamma}_{\|}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}^{\text{ind}}(\mathbf{x}')d\mathbf{x}' = -\mathbf{E}_{\|}^{\text{inc}}(\mathbf{x}) \tag{5}$$

Inserting (4), expanding $\mathbf{K}^{\text{ind}}$ as a linear combination of RWG basis functions $\mathbf{K}^{\text{ind}} = \sum k_\alpha \mathbf{b}_\alpha$, and Galerkin testing yields a discrete linear system for the expansion coefficients:

$$\mathbf{M} \cdot \mathbf{k} = \mathbf{r} \tag{6}$$

where the elements of $\mathbf{M}$ are the $\mathbf{\Gamma}$ interactions of full RWG basis functions,

$$\mathbf{M}_{\alpha\beta} = \Big\langle \mathbf{b}_\alpha \Big| \mathbf{\Gamma} \Big| \mathbf{b}_\beta \Big\rangle$$

and the elements of the RHS vector are $\mathbf{\Gamma}$ interactions of full RWG functions with half RWG functions:

$$r_\alpha = -\sum_a \xi_a \Big\langle \mathbf{b}_\alpha \Big| \mathbf{\Gamma} \Big| \mathbf{h}_a \Big\rangle. \tag{7}$$

where the sum runs over all edges on all port terminals and $\xi_a \equiv \pm \frac{I_p}{L_p^{\pm}}$ is the common weight of all half-RWG functions $\mathbf{h}_a$ on the $\pm$ terminal of port $p$. Formally solving (6), the induced surface-current density reads

$$\begin{aligned} \mathbf{K}^{\text{ind}}(\mathbf{x}) &= \sum_\alpha k_\alpha \mathbf{b}_\alpha(\mathbf{x}) \\ &= \sum_{\alpha\beta} \mathbf{b}_\alpha(\mathbf{x}) W_{\alpha\beta} r_\beta \end{aligned} \tag{8}$$

with $\mathbf{W} = \mathbf{M}^{-1}$ the inverse EFIE matrix.

# 4 Calculation of impedance matrix

For an $N_P$-port system driven by a set of port currents $\{I_p\}$, the complex power is a quadratic function of the $\{I_p\}$, whose coefficients define the *impedance parameters* $Z_{pq}$ (entries of the *impedance matrix* $\mathbf{Z}$) of the system:

$$\frac{1}{2}\int \mathbf{K}^* \cdot \mathbf{E}\, dA \equiv \frac{1}{2}\sum I_p^* Z_{pq} I_q = \frac{1}{2}\mathbf{I}^\dagger \mathbf{Z}\mathbf{I} \tag{9}$$

where the integral extends over all surfaces in the SIE geometry. (Note that we consider the full *complex* power, i.e. both real and imaginary parts of the integral of $\mathbf{J}^* \cdot \mathbf{E}$.)

In this section I derive an expression relating the impedance parameters $Z_{pq}$ to quantities computed by SCUFF-EM. The derivation proceeds in multiple steps.

1. **Total surface current.** For a $N_P$-port system driven by port currents $\{I_p\}$, the total surface current at a point $\mathbf{x}$ on a meshed surface is a linear function of the $\{I_p\}$:

$$\mathbf{K}(\mathbf{x}) = \sum I_p \mathbf{K}_p(\mathbf{x})$$

where $\mathbf{K}_p$ receives both a *direct* contribution from the half-RWG basis functions $\{\mathbf{h}(\mathbf{x})\}$ carrying the port currents and an *induced* contribution from the full-RWG basis functions populated by the coefficients obtained by solving the scattering problem, equation (8):

$$\mathbf{K}_p(\mathbf{x}) = \sum_{a\in\mathcal{P}_p^\pm} \pm\frac{1}{L_p^\pm}\left\{\mathbf{b}_a(\mathbf{x}) - \sum_{\alpha\beta}\mathbf{b}_\alpha(\mathbf{x})W_{\alpha\beta}r_{\beta a}\right\} \tag{10}$$

2. **Total electric field.** The total $\mathbf{E}$-field is similarly linear the $\{I_p\}$:

$$\mathbf{E}(\mathbf{x}) = \sum_p I_p \mathbf{E}_p(\mathbf{x}), \quad \mathbf{E}_p(\mathbf{x}) = \sum_{b\in\mathcal{P}_p^\pm} \pm\frac{1}{L_p^\pm}\left\{\mathbb{E}_a(\mathbf{x}) - \sum_{\alpha\beta}\mathbb{E}_\alpha(\mathbf{x})W_{\alpha\beta}r_{\beta a}\right\} \tag{11}$$

where $\{\mathbb{E}_\alpha, \mathbb{E}_a\} \equiv \mathbf{\Gamma}\star\{\mathbf{b}_\alpha, \mathbf{b}_a\}$ are the electric fields due to individual full- or half-RWG functions weighted with unit strength.

3. **Complex power.** Inserting (10) and (11) into (9), the impedance parameter $Z_{pq}$ becomes a sum of 4 terms:

$$\begin{aligned} Z_{pq} &\equiv \frac{1}{2}\int \mathbf{J}_p(\mathbf{x})\mathbf{E}_q(\mathbf{x})\, d\mathbf{x} \\ &= T_1 + T_2 + T_3 + T_4 \end{aligned}$$

The first term describes interactions between half-RWG basis functions:

$$T_1 \equiv \sum_{ab} \frac{1}{2L_p^\pm L_q^\pm} \Big\langle \mathbf{h}_a \Big| \mathbf{E}_b \Big\rangle$$

The second two terms describe interactions between half-RWG and full-RWG functions:

$$\begin{aligned} T_2 &= -\sum_{a\alpha\beta b} \frac{1}{2L_p^\pm L_q^\pm} \underbrace{\Big\langle \mathbf{b}_a \Big| \mathbf{E}_\alpha \Big\rangle}_{r_{p\alpha}} W_{\alpha\beta} r_{q\beta} \\ &= -\frac{1}{2}\mathbf{r}_p^T \mathbf{W} \mathbf{r}_q \\ T_3 &= -\frac{1}{2}\mathbf{r}_q^T \mathbf{W} \mathbf{r}_p \end{aligned}$$

The final term describes the interaction of full-RWG and full-RWG functions:

$$\begin{aligned} T_4 &\equiv \sum_{a\alpha\beta\gamma\delta b} \frac{1}{2L_p^\pm L_q^\pm} r_{p\beta} W_{\alpha\beta} \underbrace{\Big\langle \mathbf{b}_\alpha \Big| \mathbf{E}_\gamma \Big\rangle}_{W^{-1}_{\alpha\gamma}} W_{\gamma\delta} r_{q\delta} \\ &= +\frac{1}{2L_p^\pm L_q^\pm} \mathbf{r}_p^T \mathbf{W} \mathbf{r}_q \end{aligned}$$

Thus, when the dust settles we find that $|T_2| = |T_3| = |T_4|$ and the impedance parameter reads

$$Z_{pq} = \sum_{\pm} \frac{1}{2L_p^\pm L_q^\pm} \Big[ -\mathbf{r}_p^T \mathbf{W} \mathbf{r}_q + \sum_{ab} \Big\langle \mathbf{h}_a \Big| \mathbf{E}_b \Big\rangle \Big]$$

Figure 3: The port voltage is the line integral of the total **E** field along the straight line connecting the port's positive reference point to its negative reference point.

### .1 Port voltages: Alternative computation of impedance matrix

The port voltage is defined to be the directed line integral of the total **E** field over the straight line connecting the port's positive reference point to its negative reference point (Figure 3).

$$\begin{aligned} V^{\text{port}} &= \int_{P^+}^{P^-} \mathbf{E}(\mathbf{x}) \cdot d\mathbf{l} \\ &= \int_{P^+}^{P^-} \Big[ i\omega \mathbf{A}(\mathbf{x}) - \nabla\Phi(\mathbf{x}) \Big] \cdot d\mathbf{l} \\ &= \Big[ \Phi\big(P^+\big) - \Phi\big(P^-\big) \Big] + i\omega \int_{P^+}^{P^-} \mathbf{A}(\mathbf{x}) \cdot d\mathbf{l} \end{aligned}$$

## Averaging the potential due to $\mathcal{P}$ over an edge L

The electrostatic potential due to a unit charge density on panel $\mathcal{P}$, averaged over the length of some panel edge **L**, is

$$\Big\langle \phi^{\mathcal{P}} \Big\rangle_{\mathbf{L}} = \frac{1}{4\pi\epsilon_0 |\mathbf{L}|} \int_{\mathbf{L}} d\mathbf{x} \int_{\mathcal{P}} d\mathbf{x}' \frac{1}{|\mathbf{x} - \mathbf{x}'|}$$