Surface Impedance Boundary Conditions in SCUFF-EM

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1 The Surface-Impedance Boundary Condition

The usual boundary condition imposed at the surface of a perfectly electrically conducting (PEC) scatterer is that the total tangential electric field vanish:

$$\mathbf{E}_{\parallel}^{\text{tot}}(\mathbf{x}) = 0. \tag{1}$$

At the surface of an *imperfectly* electrically conducting (IPEC) scatterer with dimensionless relative surface impedance ζ, the boundary condition (1) is modified to read

$$\mathbf{E}_{\parallel}^{\text{tot}}(\mathbf{x}) = \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{H}^{\text{tot}}(\mathbf{x}) \tag{2}$$

where $Z_0 \approx 377\,\Omega$ is the impedance of vacuum.

I will refer to (2) as the "impedance boundary condition" (IBC).

2 Two SIE formulations for IPEC bodies

2.1 Review: SIE formulation for PEC bodies

I will consider two distinct SIE formulations for IPEC bodies. These are both variants of the usual SIE procedure for PEC bodies, which—by way of review—I summarize thusly:

1. We introduce an electric surface current $\mathbf{K}(\mathbf{x})$ on the surface of a PEC scatterer. This current is related to the total tangential $\mathbf{H}$-field according to
$$\mathbf{K}(\mathbf{x}) = \hat{\mathbf{n}} \times \mathbf{H}^{\text{tot}}(\mathbf{x}). \tag{3}$$
2. We do *not* need to introduce a magnetic surface current; such a current would be proportional to the total tangential $\mathbf{E}$ field, but this vanishes in view of the boundary condition (1):
$$\mathbf{N}(\mathbf{x}) = -\hat{\mathbf{n}} \times \mathbf{E}^{\text{tot}}(\mathbf{x}) \equiv 0. \tag{4}$$
3. $\mathbf{K}$ gives rise to scattered $\mathbf{E}$ and $\mathbf{H}$ fields according to
$$\mathbf{E}^{\text{scat}} = \int \mathbf{\Gamma}^{\text{EE}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}')d\mathbf{x}' \tag{5}$$
$$\mathbf{H}^{\text{scat}} = \int \mathbf{\Gamma}^{\text{ME}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}')d\mathbf{x}'. \tag{6}$$
4. We solve for $\mathbf{K}$ by demanding that the scattered field to which it gives rise satisfy the boundary condition (1):
$$\mathbf{E}^{\text{scat}}_{\parallel}(\mathbf{x}) = -\mathbf{E}^{\text{inc}}_{\parallel}(\mathbf{x}) \tag{7}$$
or
$$\underbrace{\int \mathbf{\Gamma}^{\text{EE}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}')d\mathbf{x}'}_{\mathbf{\Gamma}^{\text{EE}}_{\parallel}\star\mathbf{K}} = -\mathbf{E}^{\text{inc}}(\mathbf{x}). \tag{8}$$

2.2 First SIE formulation for IPEC bodies

My first SIE formulation for IPEC bodies

1. As in the PEC case, to each IPEC I continue to assign an electric surface current $\mathbf{K}$ related to the total $\mathbf{H}$ field by equation (3).
2. As in the PEC case, I continue to assign *no* magnetic surface current $\mathbf{N}$ to IPEC surfaces.
3. To determine $\mathbf{K}$ from $\mathbf{E}^{\text{inc}}$, I use (2) in place of (1) and (7). This has the effect of replacing (8) with
$$\mathbf{\Gamma}^{\text{EE}} \star \mathbf{K} - \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{\Gamma}^{\text{ME}} \star \mathbf{K} = -\mathbf{E}^{\text{inc}} + \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{H}^{\text{inc}} \tag{9}$$

2.3 Second SIE formulation for IPEC bodies

My alternative SIE formulation for IPEC bodies goes like this:

1. As before, to each IPEC surface I continue to assign an electric surface current $\mathbf{K}$ related to the total $\mathbf{H}$ field by equation (3).
2. Unlike before, I now assign to each IPEC surface a nonvanishing magnetic current, which is not an independent unknown but is instead determined by $\mathbf{K}$ via the (2):
$$\mathbf{N}(\mathbf{x}) = -\hat{\mathbf{n}} \times \mathbf{E}^{\text{tot}}(\mathbf{x}) = -\zeta Z_0 \hat{\mathbf{n}} \times \mathbf{K}(\mathbf{x}).$$
3. $\mathbf{K}$ and $\mathbf{N} \equiv \mathbf{N}[\mathbf{K}]$ give rise to scattered $\mathbf{E}$ and $\mathbf{H}$ fields according to
$$\mathbf{E}^{\text{scat}} = \mathbf{\Gamma}^{\text{EE}} \star \mathbf{K} + \mathbf{\Gamma}^{\text{EM}} \star \mathbf{N}, \qquad \mathbf{H}^{\text{scat}} = \mathbf{\Gamma}^{\text{ME}} \star \mathbf{K} + \mathbf{\Gamma}^{\text{MM}} \star \mathbf{N} \tag{10}$$
4. I solve for $\mathbf{K}$ by demanding that the scattered fields (10) satisfy the boundary condition (2):
$$\mathbf{E}^{\text{scat}}_{\parallel}(\mathbf{x}) - \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{H}^{\text{scat}}_{\parallel}(\mathbf{x}) = -\mathbf{E}^{\text{inc}}_{\parallel}(\mathbf{x}) + \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{H}^{\text{inc}}_{\parallel}(\mathbf{x}).$$
or
$$\begin{aligned} \int \Big\{ & \mathbf{\Gamma}^{\text{EE}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}') \\ & - \zeta Z_0 \mathbf{\Gamma}^{\text{ME}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \Big[\hat{\mathbf{n}} \times \mathbf{K}(\mathbf{x}')\Big] \\ & - \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{\Gamma}^{\text{EM}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}') \\ & + \zeta^2 Z_0^2 \hat{\mathbf{n}} \times \mathbf{\Gamma}^{\text{MM}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \Big[\hat{\mathbf{n}} \times \mathbf{K}(\mathbf{x}')\Big] \Big\} = -\mathbf{E}^{\text{inc}}_{\parallel}(\mathbf{x}) + \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{H}^{\text{inc}}_{\parallel}(\mathbf{x}). \end{aligned}$$