Surface Impedance Boundary Conditions in SCUFF-EM

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1 The Surface-Impedance Boundary Condition

The usual boundary condition imposed at the surface of a perfectly electrically conducting (PEC) scatterer is that the total tangential electric field vanish:

$$\mathbf{E}_{\parallel}^{\text{tot}}(\mathbf{x}) = 0. \tag{1}$$

At the surface of an *imperfectly* electrically conducting (IPEC) scatterer with dimensionless relative surface impedance ζ, the boundary condition (1) is modified to read

$$\mathbf{E}_{\parallel}^{\text{tot}}(\mathbf{x}) = \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{H}^{\text{tot}}(\mathbf{x}) \tag{2}$$

where $Z_0 \approx 377\,\Omega$ is the impedance of vacuum.

I will refer to (2) as the "impedance boundary condition" (IBC).

2 Two SIE formulations for IPEC bodies

2.1 Review: SIE formulation for PEC bodies

I will consider two distinct SIE formulations for IPEC bodies. These are both variants of the usual SIE procedure for PEC bodies, which—by way of review—I summarize thusly:

1. We introduce an electric surface current $\mathbf{K}(\mathbf{x})$ on the surface of a PEC scatterer. This current is related to the total tangential $\mathbf{H}$-field according to
$$\mathbf{K}(\mathbf{x}) = \hat{\mathbf{n}} \times \mathbf{H}^{\text{tot}}(\mathbf{x}). \tag{3}$$

2. We do *not* need to introduce a magnetic surface current; such a current would be proportional to the total tangential $\mathbf{E}$ field, but this vanishes in view of the boundary condition (1):
$$\mathbf{N}(\mathbf{x}) = -\hat{\mathbf{n}} \times \mathbf{E}^{\text{tot}}(\mathbf{x}) \equiv 0. \tag{4}$$

3. $\mathbf{K}$ gives rise to scattered $\mathbf{E}$ and $\mathbf{H}$ fields according to
$$\mathbf{E}^{\text{scat}} = \int \mathbf{\Gamma}^{\text{EE}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}') d\mathbf{x}' \tag{5}$$
$$\mathbf{H}^{\text{scat}} = \int \mathbf{\Gamma}^{\text{ME}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}') d\mathbf{x}'. \tag{6}$$

4. We solve for $\mathbf{K}$ by demanding that the scattered field to which it gives rise satisfy the boundary condition (1):
$$\mathbf{E}^{\text{scat}}_{\parallel}(\mathbf{x}) = -\mathbf{E}^{\text{inc}}_{\parallel}(\mathbf{x}) \tag{7}$$
or
$$\underbrace{\int \mathbf{\Gamma}^{\text{EE}}_{\parallel}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}') d\mathbf{x}'}\, \mathbf{\Gamma}^{\text{EE}}_{\parallel} \star \mathbf{K} = -\mathbf{E}^{\text{inc}}(\mathbf{x}). \tag{8}$$

2.2 SIE formulation for IPEC bodies

My first SIE formulation for IPEC bodies goes like this:

1. As in the PEC case, to each IPEC I continue to assign an electric surface current $\mathbf{K}$ related to the total $\mathbf{H}$ field by equation (3).

2. In contrast to the PEC case, I now assign to each IPEC surface a non-vanishing magnetic current, which is not an independent unknown but is instead determined by $\mathbf{K}$ via the (2):
$$\mathbf{N}(\mathbf{x}) = -\hat{\mathbf{n}} \times \mathbf{E}^{\text{tot}}(\mathbf{x}) = -\zeta Z_0 \hat{\mathbf{n}} \times \mathbf{K}(\mathbf{x}).$$

3. $\mathbf{K}$ and $\mathbf{N} \equiv \mathbf{N}[\mathbf{K}]$ give rise to scattered $\mathbf{E}$ and $\mathbf{H}$ fields according to

$$\mathbf{E}^{\text{scat}} = \mathbf{\Gamma}^{\text{EE}} \star \mathbf{K} + \mathbf{\Gamma}^{\text{EM}} \star \mathbf{N}, \qquad \mathbf{H}^{\text{scat}} = \mathbf{\Gamma}^{\text{ME}} \star \mathbf{K} + \mathbf{\Gamma}^{\text{MM}} \star \mathbf{N} \tag{9}$$

4. I solve for $\mathbf{K}$ by demanding that the scattered fields (9) satisfy the boundary condition (2):

$$\mathbf{E}_{\parallel}^{\text{scat}}(\mathbf{x}) - \zeta Z_0 \hat{\mathbf{n}} \times \mathbf{H}_{\parallel}^{\text{scat}}(\mathbf{x}) = -\mathbf{E}_{\parallel}^{\text{inc}}(\mathbf{x}) + \zeta Z_0 s \hat{\mathbf{n}} \times \mathbf{H}_{\parallel}^{\text{inc}}(\mathbf{x}).$$

or

$$\begin{aligned}\int \Big\{ &\mathbf{\Gamma}_{\parallel}^{\text{EE}}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}') \\ &- Z_s \mathbf{\Gamma}_{\parallel}^{\text{ME}}(\mathbf{x},\mathbf{x}') \cdot \Big[\hat{\mathbf{n}} \times \mathbf{K}(\mathbf{x}')\Big] \\ &- Z_s \hat{\mathbf{n}} \times \mathbf{\Gamma}_{\parallel}^{\text{EM}}(\mathbf{x},\mathbf{x}') \cdot \mathbf{K}(\mathbf{x}') \\ &+ Z_s^2 \hat{\mathbf{n}} \times \mathbf{\Gamma}_{\parallel}^{\text{MM}}(\mathbf{x},\mathbf{x}') \cdot \Big[\hat{\mathbf{n}} \times \mathbf{K}(\mathbf{x}')\Big] \Big\} = -\mathbf{E}_{\parallel}^{\text{inc}}(\mathbf{x}) + Z_s \hat{\mathbf{n}} \times \mathbf{H}_{\parallel}^{\text{inc}}(\mathbf{x}).\end{aligned}$$

3 Alternative formulation