

The master problem attempts to minimize the cost “ $c^t$ ” of building new lines “ $x$ ” plus the hourly operation costs estimated by  $\eta$ . This  $\eta$  is the value of the worst-case of hourly costs for **a certain network**, assuming producers are still making optimal decisions to mitigate the scenarios.  $\eta$  develops as a closer and closer upper bound to the true worst-case hourly costs for **any network** as the main problem iterates. The algorithm takes in values of generation capacity “ $Gen_g^{max}$ ” and demand “ $Dem_d^{max}$ ” from the subproblem and outputs “ $x^*$ ”, the choice of lines that minimize the costs, as well as a new lower bound to the final optimization

The formulation of the master problem, assuming the same model conditions as in [1], follows. See [2] for further details. Every “ $p$ th” iteration of the mater problem will take in  $Gen_{g,p}$  and  $Dem_{d,p}$  as parameters and will add a new set of the variables to consider in the solve:

flow ( $Flow_{l,p}$ )

load shedding ( $Shed_{d,p}$ )

generation ( $Gen_{g,p}$ )

Then, a copy of the feasibility constraints, listed below, are added appended to problem. Keep in mind, the minimization is not just against the last iteration of the subproblem, “ $k$ ”, but rather against every iteration of the subproblem  $p = 1 \dots k$ , adding new variables and constraints for ever  $p$ . Thus, at iteration “ $k$ ”:

$$\begin{aligned}
 & \min_{x,y_p} c^t x + \eta \\
 & \sum_g Gen_{g,p} - \sum_{l|s(l)=n} Flow_{l,p} + \sum_{l|r(l)=n} Flow_{l,p} = \sum_d (Dem_{d,p} - Shed_{d,p}) \quad \forall n, \forall p \leq k \\
 & 0 \leq Gen_{g,p} \leq Gen_g^{max} \quad \forall g, \forall p \leq k \\
 & 0 \leq Shed_{d,p} \leq Dem_d^{max} \quad \forall d, \forall p \leq k \\
 & Flow_l = x(Sus_l)(\theta_{s(l),p} - \theta_{r(l),p}) \quad \forall l, \forall p \leq k \\
 & -Cap_l \leq Flow_{l,p} \leq Cap_l \quad \forall l, \forall p \leq k \\
 & -\pi \leq \theta_{n,p} \leq \pi \quad \forall n, \forall p \leq k \\
 & \theta_{n,p} = 0 \quad n:ref \\
 & \eta \geq \sigma * \left[ \sum_g C_g^E * Gen_{g,p} + \sum_d C_d^D * Shed_{d,p} \right] \quad \forall p \leq k
 \end{aligned}$$

“ $Cap$ ” is the max capacity of any line, and “ $Sus$ ” is the electrical susceptance of a line. As can be seen, the number of variables in the linear solve at each iteration grows by the sum of the sizes vectors of  $Flow$ ,  $Shed$ , and  $Gen$ . Constraints also grow by a similar factor.

## References

- [1] Antonio J. Conejo, Luis Baringo Morales, S. Jalal Kazempour, and Afzal S. Siddiqui. 2016. Investment in Electricity Generation and Transmission: Decision Making Under Uncertainty (1st ed.). Springer Publishing Company, Incorporated.
- [2] Ruiz, C, and A.J Conejo. “Robust Transmission Expansion Planning.” *European Journal of Operational Research*, vol. 242, no. 2, 2015, pp. 390–401.