In the robust method of [1], the model split into two parts, a master problem and a sub problem. The subproblem receives, from a solve of a master problem, values of "x^*" (the network of lines built) as a fixed parameter. Then, it combines the two-stage inner problem into a single linear maximization problem by using a Fortuny-Amat transformation and the KKT optimality conditions to represent the minimization of hourly decision costs. This is reasonable since KKT conditions are sufficient for optimality for any linear problem. This reformation into a mixed integer linear problem (MILP) allows for efficient algorithms for solving linear systems to be applied, such as a branch and bound. Then a master problem takes in the scenario realization for the subproblem and uses a column-and-constraint method of adding new variables and constraints to solve another MILP. This process is repeated in a loop until convergence. See [2] for more numerical examples, and step by step instructions for constructing such a model.

Algorithm

In the algorithm, the subproblem and master problem are repeated back and forth, finding upper and lower bounds until those bound converge as follows:

- **Setup**) Find the cost of just the preexisting lines, if any, at the start of simulation and set this as an absolute lower bound. Set $k = 0$. Then, go to **Step 2.**

- **Step 1**) Input d^* and y^* from all previous *k* iterations of the subproblem into the master problem to solve the for maximization $\min_{x,y_p} c^t x + \eta$ to find a new set of lines, x^*. Update the lower bound with the value of the objective function.

- **Step 2**) Input x^* into the subproblem and solve the minimization $\max_{d,y} c^t x + b^t y$ to find a new set of d^* and y^*. Update the upper bound only if it is lower than the previous upper bound.

- **Step 3**) If the upper and lower bound are within a predefined epsilon, then **stop**. Otherwise, increment to iteration $k + 1$. Return to **Step 1**.

Upon completion there will be a power grid plan "x^*" that is within ($\epsilon * x^{optimal}$) of $x^{optimal}$. Of course, epsilon can be adjusted depending on computational time limitations. As can be noticed in **Step 2**, each iteration does not necessarily lead to a better (lower) upper bound, and thus the solution may not be closer to optimality after each iteration. On the other hand, even on the largest data sets used in the literature this method only takes less than 10 iterations to converge within a reasonable epsilon [1][2]. However, a single MILP solve could take an hour or more for larger data sets.

References

[1] Ruiz, C, and A.J Conejo. "Robust Transmission Expansion Planning." *European Journal of Operational Research*, vol. 242, no. 2, 2015, pp. 390–401.

[2] Antonio J. Conejo, Luis Baringo Morales, S. Jalal Kazempour, and Afzal S. Siddiqui. 2016. Investment in Electricity Generation and Transmission: Decision Making Under Uncertainty (1st ed.). Springer Publishing Company, Incorporated.