

For the first series of unit tests on constraint-and-column, individual portions of the master and subproblems were tested as they were developed. As they were simple enough, one could just compare to transportation optimization toy problem with known results, or by comparing to a program typed up in GAMS. Afterward, several tests of a completed master and subproblem are provided in folder **BASIC**. Note, since I continued to make changes to these algorithms by including more parameters, these do not match the current program setup.

After bringing components of constraint-and-column together so that a “main” algorithm could iterate through a “master” and “sub” to find a final answer, I compared to the GAMS program provided by [1]. See folder **GAMS**. As such, I was able to quickly test many different data sets to establish not just that my code found the correct final answer, but that each step in the process had the same intermediate results. Unfortunately, since the provided GAMS program was hard coded with a certain set of parameters, once I made significant changes to the constraint-and-column, I would have to almost start from scratch to rework the GAMS code. Furthermore, GAMS software also requires a license to solve linear problems needed in a 24-bus or higher problem, since the trial license is limited in size. Another option is CPLEX, which has an unlimited academic license but requires making a third version of the code.

However, comparing to problem with known solutions also provides further validation. [1] also provides two sample 6-bus solves in detail that I could compare to, see **TEP**. After that I could compare to [2], which had solutions for a 6-bus solve, see **MINGUEZ**. As of right now, [2] uses a slightly different uncertainty budget, so the full set of results can only be compared once the new budget constraints are added. To see a use of a 24-bus solve from [3] see **24Bus**, however this is still in progress.

Unfortunately, I did not find a source of incremental results for the Bender’s Decomposition scheme besides the 6-bus and 46-bus examples provided in accompanying paper [4]. Of course, the final answer of a solve can be compared to answers found using other methods, especially in any case where the optimal is found to within an error of machine epsilon (there should be no variation in optimal based on the algorithm used). But still some sort of better test cases needs to be found. Luckily, a significant amount of code can be reused from the constraint-and-column, so the new algorithms could call from a same basic set of tested functions to cut down on new tests and needing to test every component.

Finally, [2] and [4] have 118-bus problem that took 9 + hours to solve so I’ll reserve that test for later. As I implement the hybrid method from [2] and the method splitting short-term from long-term in [6] I will include comparison to the results from those papers. The final test is to compare my code to a commercial, not just to check answers but the answers but more for computational complexity and is something I’d like to repeat for several hundred semi-random datasets, but I have to commit to the comparison program.

## References

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