Finite Difference Method applied to DML simulation. Part 2a. Rev 3.

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03 Avr. 2025

Abstract

Simulation of Distributed Mode Loudspeaker (DML) thanks to the Finite Difference Method (FDM)

Contents

Disclaimer : this paper is written in the context of DIY DML building with the target to identify some design rules to help in the panel construction. This document is not written in the context of any academic or scientific work. Its content is reviewed only by the feedback it can get while posting it in audio DIY forum like diyAudio.

The pdf format of the paper is directly extract from a python script See Github py2pdf for more information about this method.

Revision 1 : The way to fill the system matrix is completely reviewed to be more readable, compact, efficient (pythonic?). For that a function that fills the system matrix according an explicit stencil is created. The code is rearranged for a better readability.

Revision 2 : The code is even more compacted now thanks to a new external module of functions to fill the system matrix.

Revision 3 : Introduction of the free edge boundary conditions and of a new way to manage the simply supported and clamped conditions. Check of the stencils according to Barton's paper added. 03 avr 25: Stencil correction. The implementation is fully following Barton's paper.

This document was exported from a python script. Done : April 3, 2025.

1 Introduction

This part 2a is the application of part 1 and part 1b in a Python script up to the calculation of the system matrix.

The novelty from rev 3 is the matrix is filled according to the stencils related to the free edge boundary conditions. The specific stencils used for the simply supported and clamped conditions are no more used but replaced by a constraint on the displacement :

- w=0 at the edge points for a simply supported plate
- w=0 at the edge points and also at the adjacent points for a clamped plate.

This approach is a trial. It is based on the idea those conditions could be described as it is done in practice by adding some fixations.

Beyond the simply supported and clamped conditions (from the equation point of view) which have mainly an interest in a theoritical approach, the idea is to open to "more real" conditions like a limited number of fixed points or damper or spring at the edges. The impact on the precision will be check in the next parts.

It opens also to future steps to add local mass, damper or spring like for example an exciter.

This paper is a step in the use of the finite difference method by checking the script fills correctly the matrix.

For the ones having no or very low interest in this python implementation, skip it and go to part 2b to see the results of the method.

Note : In 2D, the matrix library numpy from python works in [row, column], so [y, x] coordinates or [k, j] here after indexes. Then while printing, x axis (j index) is horizontal, y axis (k index) is vertical; [0,0] is top left.

2 The testing plate

See figure 1.

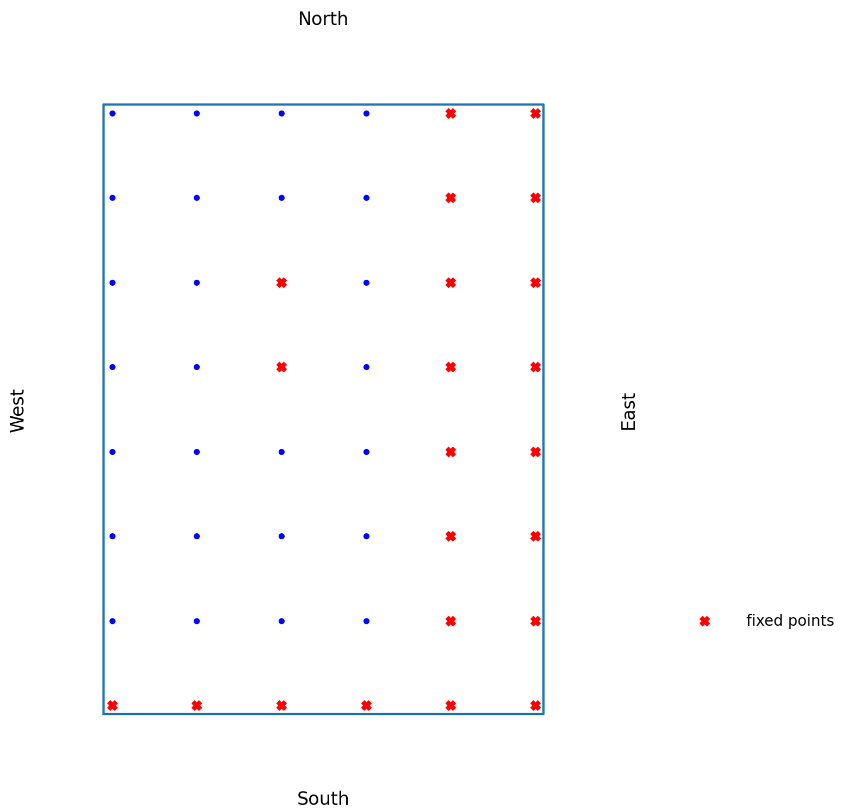

Figure 1: Testing plate

3 The script outputs

```
2025-04-03 18:13
The plate is Lx= 0.25 m by Ly= 0.35 m
dxmm= dymm= 50 mm
The mesh is Nj= 6 points by Nk= 8 points

Point index used in the system matrix
    0  1  2  3  4  5
 0  0  8 16 24 32 40
 1  1  9 17 25 33 41
 2  2 10 18 26 34 42
 3  3 11 19 27 35 43
 4  4 12 20 28 36 44
 5  5 13 21 29 37 45
 6  6 14 22 30 38 46
 7  7 15 23 31 39 47

Stencils with nux=0.01and nuy=0.1 for debug
Stencil d4w/dx4 for inner points (Sx area)
 POI:0/2
 [[ 1 -4  6 -4  1]]

Stencil d4w/dy4 for inner points (Sy area)
 POI:2/0
 [[ 1]
 [-4]
 [ 6]
 [-4]
 [ 1]]

Stencil d4w/dx2dy2 for inner points (Sxy area)
 POI:1/1
 [[ 1 -2  1]
 [-2  4 -2]
 [ 1 -2  1]]

Stencil d4w/dx4 for West Edge points (EW area)
 POI:2/0
 [[  0.19   0.     0.  ]
 [ -4.36   3.8    0.  ]
 [ 10.34 -11.6    2.  ]
 [ -4.36   3.8    0.  ]
 [  0.19   0.     0.  ]]

Stencil d4w/dx4 for the neighbor points of the West Edge (IW area)
 POI:1/1
 [[-0.1  0.   0.   0. ]
 [-1.8  5.  -4.   1. ]
 [-0.1  0.   0.   0. ]]

Stencil d4w/dx4 for North West corner (CNW point)
 POI:0/0
 [[  9.98 -11.6    2.  ]
 [ -8.36   7.6    0.  ]
 [  0.38   0.     0.  ]]

Stencil d4w/dx4 for point next to the North West Corner on the North Edge (XNW point)
 POI:0/1
 [[-2  5 -4  1]]
```

```
Stencil d4w/dx4 for the point next to the North West Corner on the West Edge (XWN point)
 POI:1/0
 [[ -3.98   3.8    0.  ]
 [ 10.15 -11.6    2.  ]
 [ -4.36   3.8    0.  ]
 [  0.19   0.     0.  ]]

Stencil d4w/dx2dy2 for West Edge points (EW area)
 POI:2/0
 [[-0.1]
 [ 0.4]
 [-0.6]
 [ 0.4]
 [-0.1]]

Stencil d4w/dx2dy2 for North West corner (CNW point)
 POI:0/0
 [[-4.22  4.04 -0.02]
 [ 4.4  -4.    0.  ]
 [-0.2   0.    0.  ]]

Stencil d4w/dx2dy2 for the point next to the North West Corner on the West Edge (XWN point)
 POI:1/0
 [[ 0.2]
 [-0.5]
 [ 0.4]
 [-0.1]]


Check the stencils as shown in Barton's paper, nu= 0
----------
Points on free edge ES
[[  0.   0.   0.   0.   0.]
 [  0.   0.   0.   0.   0.]
 [  0.   0.   2.   0.   0.]
 [  0.   4. -12.   4.   0.]
 [  1.  -8.  16.  -8.   1.]]

Point adjacent to free edge IS
[[ 0.  0.  0.  0.  0.]
 [ 0.  0.  1.  0.  0.]
 [ 0.  2. -8.  2.  0.]
 [ 1. -8. 19. -8.  1.]
 [ 0.  2. -6.  2.  0.]]

Point on free corner CSE
[[  0.   0.   0.   0.   0.]
 [  0.   0.   0.   0.   0.]
 [  0.   0.   0.   0.   2.]
 [  0.   0.   0.   8. -12.]
 [  0.   0.   2. -12.  12.]]

Point inside free corner IES
[[ 0.  0.  0.  0.  0.]
 [ 0.  0.  0.  1.  0.]
 [ 0.  0.  2. -8.  2.]
 [ 0.  1. -8. 18. -6.]
 [ 0.  0.  2. -6.  2.]]

Point adjacent to corner XES
[[  0.   0.   0.   0.   0.]
```

```
 [  0.   0.   0.   0.   1.]
 [  0.   0.   0.   4.  -8.]
 [  0.   0.   2. -12.  15.]
 [  0.   0.   0.   4.  -6.]]

Inside point
[[ 0.  0.  1.  0.  0.]
 [ 0.  2. -8.  2.  0.]
 [ 1. -8. 20. -8.  1.]
 [ 0.  2. -8.  2.  0.]
 [ 0.  0.  1.  0.  0.]]


Check the stencils as shown in Barton's paper, nu= 0.3
----------
Points on free edge ES
[[  0.     0.     0.     0.     0.  ]
 [  0.     0.     0.     0.     0.  ]
 [  0.     0.     2.     0.     0.  ]
 [  0.     3.4  -10.8    3.4    0.  ]
 [  0.91  -6.44  13.06  -6.44   0.91]]

Point adjacent to free edge IS
[[ 0.   0.   0.   0.   0. ]
 [ 0.   0.   1.   0.   0. ]
 [ 0.   2.  -8.   2.   0. ]
 [ 1.  -8.  19.  -8.   1. ]
 [ 0.   1.7 -5.4  1.7  0. ]]

Point on free corner CSE
[[ 0.    0.    0.    0.    0.  ]
 [ 0.    0.    0.    0.    0.  ]
 [ 0.    0.    0.    0.    1.82]
 [ 0.    0.    0.    5.6  -9.24]
 [ 0.    0.    1.82 -9.24  9.24]]

Point inside free corner IES
[[ 0.   0.   0.   0.   0. ]
 [ 0.   0.   0.   1.   0. ]
 [ 0.   0.   2.  -8.   1.7]
 [ 0.   1.  -8.  18.  -5.4]
 [ 0.   0.   1.7 -5.4  1.4]]

Point adjacent to corner XES
[[  0.     0.     0.     0.     0.  ]
 [  0.     0.     0.     0.     0.91]
 [  0.     0.     0.     3.4   -6.44]
 [  0.     0.     2.   -10.8   12.15]
 [  0.     0.     0.     3.4   -4.62]]

Inside point
[[ 0.  0.  1.  0.  0.]
 [ 0.  2. -8.  2.  0.]
 [ 1. -8. 20. -8.  1.]
 [ 0.  2. -8.  2.  0.]
 [ 0.  0.  1.  0.  0.]]


Sxy : All but edges
    0  1  2  3  4  5
 0  .  .  .  .  .  .
```

```
 1  .  8  8  8  8  .
 2  .  8  8  8  8  .
 3  .  8  8  8  8  .
 4  .  8  8  8  8  .
 5  .  8  8  8  8  .
 6  .  8  8  8  8  .
 7  .  .  .  .  .  .
```

```
Sx : Inner points + Inner boundary S and N + edge S and N
    0  1  2  3  4  5
 0  .  .  8  8  .  .
 1  .  .  8  8  .  .
 2  .  .  8  8  .  .
 3  .  .  8  8  .  .
 4  .  .  8  8  .  .
 5  .  .  8  8  .  .
 6  .  .  8  8  .  .
 7  .  .  8  8  .  .
```

```
Sy : Inner points + Inner boundary E and W + edge E and W
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  8  8  8  8  8  8
 3  8  8  8  8  8  8
 4  8  8  8  8  8  8
 5  8  8  8  8  8  8
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .
```

```
EN : North edge points but corner and next to corner
    0  1  2  3  4  5
 0  .  .  8  8  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .
```

```
ES : South edge points but corner and next to corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  8  8  .  .
```

```
EW : West edge points but corner and next to corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  8  .  .  .  .  .
 3  8  .  .  .  .  .
 4  8  .  .  .  .  .
 5  8  .  .  .  .  .
 6  .  .  .  .  .  .
```

```
 7  .  .  .  .  .  .

EE : East edge points but corner and next to corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  8
 3  .  .  .  .  .  8
 4  .  .  .  .  .  8
 5  .  .  .  .  .  8
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .

CNW : North West Corner
    0  1  2  3  4  5
 0  8  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .

CNE : North East Corner
    0  1  2  3  4  5
 0  .  .  .  .  .  8
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .

CSW : South West Corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  8  .  .  .  .  .

CSE : South East Corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  8

IN : Inner points next to North edge
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  8  8  8  8  .
```

```
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .

IS : Inner points next to South edge
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  8  8  8  8  .
 7  .  .  .  .  .  .

IW : Inner points next to West edge
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  8  .  .  .  .
 2  .  8  .  .  .  .
 3  .  8  .  .  .  .
 4  .  8  .  .  .  .
 5  .  8  .  .  .  .
 6  .  8  .  .  .  .
 7  .  .  .  .  .  .

IE : Inner points next to East edge
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  8  .
 2  .  .  .  .  8  .
 3  .  .  .  .  8  .
 4  .  .  .  .  8  .
 5  .  .  .  .  8  .
 6  .  .  .  .  8  .
 7  .  .  .  .  .  .

XNW : Point of North edge, next to West corner
    0  1  2  3  4  5
 0  .  8  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .

XNE : Point of North edge, next to East corner
    0  1  2  3  4  5
 0  .  .  .  .  8  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .
```

```
XEN : Point of East edge, next to North corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  8
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .

XES : Point of East edge, next to South corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  8
 7  .  .  .  .  .  .

XSE : Point of South edge, next to East corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  8  .

XSW : Point of South edge, next to West corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  8  .  .  .  .

XWN : Point of West edge, next to North corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  8  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .

XWS : Point of West edge, next to South corner
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
```

```
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  8  .  .  .  .  .
 7  .  .  .  .  .  .

BN : North edge all points
    0  1  2  3  4  5
 0  8  8  8  8  8  8
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .

BS : South edge all points
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  8  8  8  8  8  8

BW : West edge all points
    0  1  2  3  4  5
 0  8  .  .  .  .  .
 1  8  .  .  .  .  .
 2  8  .  .  .  .  .
 3  8  .  .  .  .  .
 4  8  .  .  .  .  .
 5  8  .  .  .  .  .
 6  8  .  .  .  .  .
 7  8  .  .  .  .  .

BE : East edge all points
    0  1  2  3  4  5
 0  .  .  .  .  .  8
 1  .  .  .  .  .  8
 2  .  .  .  .  .  8
 3  .  .  .  .  .  8
 4  .  .  .  .  .  8
 5  .  .  .  .  .  8
 6  .  .  .  .  .  8
 7  .  .  .  .  .  8

BNI : Inner points next to North edge (all points)
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  8  8  8  8  8  8
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  .  .  .  .  .  .
 7  .  .  .  .  .  .
```

```
BSI : Inner points next to South edge (all points)
    0  1  2  3  4  5
 0  .  .  .  .  .  .
 1  .  .  .  .  .  .
 2  .  .  .  .  .  .
 3  .  .  .  .  .  .
 4  .  .  .  .  .  .
 5  .  .  .  .  .  .
 6  8  8  8  8  8  8
 7  .  .  .  .  .  .

BWI : Inner points next to West edge (all points)
    0  1  2  3  4  5
 0  .  8  .  .  .  .
 1  .  8  .  .  .  .
 2  .  8  .  .  .  .
 3  .  8  .  .  .  .
 4  .  8  .  .  .  .
 5  .  8  .  .  .  .
 6  .  8  .  .  .  .
 7  .  8  .  .  .  .

BEI : Inner points next to East edge (all points)
    0  1  2  3  4  5
 0  .  .  .  .  8  .
 1  .  .  .  .  8  .
 2  .  .  .  .  8  .
 3  .  .  .  .  8  .
 4  .  .  .  .  8  .
 5  .  .  .  .  8  .
 6  .  .  .  .  8  .
 7  .  .  .  .  8  .
```

```
Ax matrix (extract) 32 points
        0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
 0  0  8 -8  2  .  .  .  .  . -8  4  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 1  1 -2  7 -4  1  .  .  .  .  2 -8  2  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 2  2  1 -4  8 -4  1  .  .  .  .  2 -8  2  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .  .  .  .  .  .
 3  3  .  1 -4  8 -4  1  .  .  .  .  2 -8  2  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .  .  .  .  .
 4  4  .  .  1 -4  8 -4  1  .  .  .  .  2 -8  2  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .  .  .  .
 5  5  .  .  .  1 -4  8 -4  1  .  .  .  .  2 -8  2  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .  .  .
 6  6  .  .  .  .  1 -4  7 -2  .  .  .  .  .  2 -8  2  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .  .
 7  7  .  .  .  .  .  2 -8  8  .  .  .  .  .  .  4 -8  .  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .
 8  8 -2  .  .  .  .  .  .  .  5  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  1  .  .  .  .  .  .  .
 9  9 -1  . -1  .  .  .  .  .  .  5  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  1  .  .  .  .  .  .
10 10  . -1  . -1  .  .  .  .  .  .  5  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  1  .  .  .  .  .
11 11  .  . -1  . -1  .  .  .  .  .  .  5  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  1  .  .  .  .
12 12  .  .  . -1  . -1  .  .  .  .  .  .  5  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  1  .  .  .
13 13  .  .  .  . -1  . -1  .  .  .  .  .  .  5  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  1  .  .
14 14  .  .  .  .  . -1  . -1  .  .  .  .  .  .  5  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  1  .
15 15  .  .  .  .  .  .  . -2  .  .  .  .  .  .  .  5  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  1
16 16  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .
17 17  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  . -4  .  .  .  .  .  .
18 18  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  . -4  .  .  .  .  .
19 19  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  . -4  .  .  .  .
20 20  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  . -4  .  .  .
21 21  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  . -4  .  .
22 22  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  . -4  .
23 23  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  . -4
```

```
24 24  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .  .
25 25  .  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .  .
26 26  .  .  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .  .
27 27  .  .  .  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .  .
28 28  .  .  .  .  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .  .
29 29  .  .  .  .  .  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .  .
30 30  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6  .
31 31  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  .  .  .  .  .  .  .  6
```

```
Ay matrix (extract) 32 points
       0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
 0  0  8 -8  2  .  .  .  .  . -8  4  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 1  1 -2  5 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 2  2  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 3  3  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 4  4  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 5  5  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 6  6  .  .  .  .  1 -4  5 -2  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 7  7  .  .  .  .  .  2 -8  8  .  .  .  .  .  .  4 -8  .  .  .  .  .  .  .  2  .  .  .  .  .  .  .  .
 8  8 -2  2  .  .  .  .  .  .  7 -8  2  .  .  .  .  . -4  2  .  .  .  .  .  .  1  .  .  .  .  .  .  .
 9  9 -1  .  .  .  .  .  .  .  .  5 -4  1  .  .  .  . -1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
10 10  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
11 11  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
12 12  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
13 13  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
14 14  .  .  .  .  .  .  . -1  .  .  .  .  1 -4  5  .  .  .  .  .  .  .  . -1  .  .  .  .  .  .  .  .
15 15  .  .  .  .  .  .  2 -2  .  .  .  .  .  2 -8  7  .  .  .  .  .  .  2 -4  .  .  .  .  .  .  .  1
16 16  1  .  .  .  .  .  .  . -4  2  .  .  .  .  .  .  8 -8  2  .  .  .  .  . -4  2  .  .  .  .  .  .
17 17  .  .  .  .  .  .  .  . -1  .  .  .  .  .  .  .  .  5 -4  1  .  .  .  . -1  .  .  .  .  .  .  .
18 18  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .  .
19 19  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .  .
20 20  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .  .
21 21  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .  .  .  .  .  .
22 22  .  .  .  .  .  .  .  .  .  .  .  .  .  .  . -1  .  .  .  .  1 -4  5  .  .  .  .  .  .  .  . -1
23 23  .  .  .  .  .  .  .  1  .  .  .  .  .  .  2 -4  .  .  .  .  .  2 -8  8  .  .  .  .  .  .  2 -4
24 24  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  . -4  2  .  .  .  .  .  .  8 -8  2  .  .  .  .  .
25 25  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  . -1  .  .  .  .  .  .  .  .  5 -4  1  .  .  .  .
26 26  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .  .
27 27  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .  .
28 28  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1  .
29 29  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1 -4  6 -4  1
30 30  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  . -1  .  .  .  .  1 -4  5  .
31 31  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1  .  .  .  .  .  .  2 -4  .  .  .  .  .  2 -8  8
```

```
Axy matrix (extract) 32 points
       0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
 0  0 -8  8 -2  .  .  .  .  .  8 -4  .  .  .  .  .  . -2  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 1  1  2 -5  4 -1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 2  2 -1  4 -6  4 -1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 3  3  . -1  4 -6  4 -1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 4  4  .  . -1  4 -6  4 -1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 5  5  .  .  . -1  4 -6  4 -1  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 6  6  .  .  .  . -1  4 -5  2  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 7  7  .  .  .  .  . -2  8 -8  .  .  .  .  .  . -4  8  .  .  .  .  .  .  . -2  .  .  .  .  .  .  .  .
 8  8  2  .  .  .  .  .  .  . -5  .  .  .  .  .  .  .  4  .  .  .  .  .  .  . -1  .  .  .  .  .  .  .
 9  9  1 -2  1  .  .  .  .  . -2  4 -2  .  .  .  .  .  1 -2  1  .  .  .  .  .  .  .  .  .  .  .  .  .
10 10  .  1 -2  1  .  .  .  .  . -2  4 -2  .  .  .  .  .  1 -2  1  .  .  .  .  .  .  .  .  .  .  .  .
```

11	11	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.	.	.	.	.	.	.	.	.	.	.
12	12	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.	.	.	.	.	.	.	.	.	.
13	13	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.	.	.	.	.	.	.	.	.
14	14	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.	.	.	.	.	.	.	.
15	15	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	-5	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.	-1
16	16	-1	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.	-6	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.
17	17	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.	.	.	.	.
18	18	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.	.	.	.
19	19	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.	.	.
20	20	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.	.
21	21	.	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1	.
22	22	.	.	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.	1	-2	1
23	23	.	.	.	.	.	.	.	-1	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.	-6	.	.	.	.	.	.	.	4
24	24	.	.	.	.	.	.	.	.	-1	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.	-6	.	.	.	.	.	.	.
25	25	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.	.
26	26	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.	.
27	27	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.	.
28	28	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.	.
29	29	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2	.
30	30	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	-2	1	.	.	.	.	.	-2	4	-2
31	31	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-1	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.	-6

SysMat matrix (extract) 32 points

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
0	0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
1	1	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2	2	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.
3	3	.	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.
4	4	.	.	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.
5	5	.	.	.	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.
6	6	.	.	.	.	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.
7	7	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
8	8	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
9	9	.	-4	1	.	.	.	.	.	-4	18	-8	1	.	.	.	.	1	-8	2	.	.	.	.	.	.	1	.	.	.	.	.	.
10	10	.	1	-4	1	.	.	.	.	1	-8	19	-8	1	.	.	.	.	2	-8	2	.	.	.	.	.	.	1	.	.	.	.	.
11	11	.	.	1	-4	1	.	.	.	.	1	-8	19	-8	1	.	.	.	.	2	-8	2	.	.	.	.	.	.	1	.	.	.	.
12	12	.	.	.	1	-4	1	.	.	.	.	1	-8	19	-8	1	.	.	.	.	2	-8	2	.	.	.	.	.	.	1	.	.	.
13	13	.	.	.	.	1	-4	1	.	.	.	.	1	-8	19	-8	1	.	.	.	.	2	-8	2	.	.	.	.	.	.	1	.	.
14	14	.	.	.	.	.	1	-4	.	.	.	.	.	1	-8	18	-4	.	.	.	.	.	2	-8	1	.	.	.	.	.	.	1	.
15	15	.	.	.	.	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.
16	16	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.	.	.	.	.	.
17	17	.	1	.	.	.	.	.	.	1	-8	2	.	.	.	.	.	-4	19	-8	1	.	.	.	.	1	-8	2	.	.	.	.	.
18	18	.	.	1	.	.	.	.	.	.	2	-8	2	.	.	.	.	1	-8	20	-8	1	.	.	.	.	2	-8	2	.	.	.	.
19	19	.	.	.	1	.	.	.	.	.	.	2	-8	2	.	.	.	.	1	-8	20	-8	1	.	.	.	.	2	-8	2	.	.	.
20	20	.	.	.	.	1	.	.	.	.	.	.	2	-8	2	.	.	.	.	1	-8	20	-8	1	.	.	.	.	2	-8	2	.	.
21	21	.	.	.	.	.	1	.	.	.	.	.	.	2	-8	2	.	.	.	.	1	-8	20	-8	1	.	.	.	.	2	-8	2	.
22	22	.	.	.	.	.	.	1	.	.	.	.	.	.	2	-8	1	.	.	.	.	1	-8	19	-4	.	.	.	.	.	2	-8	1
23	23	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.	.	2	.
24	24	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	2	-8	2	.	.	.	.	.
25	25	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	1	-8	2	.	.	.	.	.	-4	19	-8	1	.	.	.	.
26	26	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	2	-8	2	.	.	.	.	1	-8	20	-8	1	.	.	.
27	27	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	2	-8	2	.	.	.	.	1	-8	20	-8	1	.	.
28	28	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	2	-8	2	.	.	.	.	1	-8	20	-8	1	.
29	29	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	2	-8	2	.	.	.	.	1	-8	20	-8	1
30	30	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	2	-8	1	.	.	.	.	1	-8	19	-4
31	31	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	2	-8	2

Fixed points

```
[ 7 15 23 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 18 19]
```

4 Python script code

4.1 Import modules

```
import os  # for py2pdf
import shutil  # for py2pdf backup
import christianpylib.py2pdf as p2p  # import py2pdf library
import numpy as np
import mod_fdm_print_FFFF as fdmp  # import the module of specific functions to print and plot
# from fdm_applied_to_dml_part4 import markdownfile
# from fdm_applied_to_dml_part4 import pdffile
import mod_fdm_FFFF as fdmbm  # import the module of specific functions to build the matrices of a free
```

4.2 Functions

```
Empty section : all the functions are in external modules
```

4.3 Prepare py2pdf

```
script_name = os.path.basename(__file__).split(".")[0]  # get this script file name without extension
py2pdfdir = "../data/part2a3"  # where the script logs are recorded
logfile = py2pdfdir + "/DML_FDM3.txt"  # define the logfile
p2p.clearlog(logfile)  # clear the logfile (in case script is ran several times)
```

4.4 Parameters

```
# Dimensions (in m otherwise mentioned in the variable name : ie Lmm in mm)
Lxmm = 250
Lymm = 350  # use locally mm to avoid rounding problems
h = 0.02  # plate thickness

# Boundary conditions C = clamped, SS = simply supported, F = free, North, West, South, East
BoundDict = {
    "North": "F",
    "West": "F",
    "South": "SS",
    "East": "C"
}

# Material
Ex = 60e6
Ey = 60e6  # Young modulus x and y direction in Pa
rho = 25.0  # density in kg/m^3
nux = 1 # unrealistic but useful for test (tests where done for 0 and 2 to check the stencils)
nuy = nux  # Poisson ration x and y direction

# Mesh
dxmm = 50  # distance between points of the mesh in x direction in mm
dymm = dxmm  # same in y

# Some parameters
Nj = Lxmm // dxmm + 1  # number of points in the mesh, j index = x direction = horizontal sides
Nk = Lymm // dymm + 1  # same in k index = y direction = vertical sides
# some conversion to meter
Lx = Lxmm / 1000
```

```
Ly = Lymm / 1000
dx = dxmm / 1000
dy = dymm / 1000
# 2D matrix containing the index i of each points
plate2d = np.transpose(np.arange(Nk * Nj).reshape(Nj, Nk))
# Prepare the sub-areas
# AreasLim, AreasName, AreasMatrix are dictionaries
# key is the area short name (ie Sx, EW, IE...)
AreasLim, AreasName = fdmbm.AreasDict()
# AreasMatrix = fdmbm.AreasMat(AreasLim)
AreasSubMat = fdmbm.AreasMat(AreasLim, plate2d)

# Fixed points ------------------------------------
# Apply Boundary Conditions
FixedPoints = fdmbm.Fixedpoints(BoundDict, AreasSubMat)
# Add specific fixed points
AddFixedPoints = ((2,2), (3,2))
for row, col in AddFixedPoints:
    i = row + Nk * col
    FixedPoints = np.append(FixedPoints, [i])
# --------------------------------------------------
# Summary
p2p.print_twice(logfile, "The plate is Lx=", Lx, "m by Ly=", Ly, "m")
p2p.print_twice(logfile, "dxmm= dymm=", dxmm, "mm")
p2p.print_twice(logfile, "The mesh is Nj=", Nj, "points by Nk=", Nk, "points")

# Show the plate
fdmp.plot_plate2(Nj, Nk, FixedPoints, py2pdfdir + "/plate.png")  # show the mesh
```

4.5 System matrix filling process

```
# Create the system matrix
SysMat = np.zeros([Nk * Nj, Nk * Nj])  # System Matrix

# Let's fill the system submatrices
Ax, Ay, Axy = fdmbm.fill_main_matrices(Nj, Nk, nux, nuy, AreasSubMat)

# Magic : here is the System Matrix
# ---------------------------------
SysMat = (Ax + Ay + 2*Axy)  # FFFF version
# ------------
# Apply fixed points
Sysmat = fdmbm.applyfixedpoints(SysMat, FixedPoints)
# ---------------------------------
```

4.6 Script output printing

```
p2p.print_twice(logfile, "")
p2p.print_twice(logfile, "Point index used in the system matrix")
no0 = False
fdmp.printmat(logfile, plate2d, Nj, Nk, no0)
p2p.print_twice(logfile, "")

# Print the primary stencils
fdmp.report_stencils(logfile)
p2p.print_twice(logfile, "")

# Print the stencils according to Barton's paper
```

```python
fdmp.check_Barton(logfile, 0) # case nu=0
p2p.print_twice(logfile, "")
fdmp.check_Barton(logfile, 0.3) # case nu=0
p2p.print_twice(logfile, "")

# Print the point areas (inner, boundaries, inner boundaries...)
fdmp.report_areas(logfile, AreasSubMat, AreasName, plate2d)
p2p.print_twice(logfile, "")

# Print the sub matrices
N = 32  # number of elements to print
fdmp.report_mat(logfile, Ax, plate2d, "Ax", N)
p2p.print_twice(logfile, "")
fdmp.report_mat(logfile, Ay, plate2d, "Ay", N)
p2p.print_twice(logfile, "")
fdmp.report_mat(logfile, Axy, plate2d, "Axy", N)
p2p.print_twice(logfile, "")
fdmp.report_mat(logfile, SysMat, plate2d, "SysMat", N)
p2p.print_twice(logfile, "")

# Print fixed points
p2p.print_twice(logfile, "Fixed points")
p2p.print_twice(logfile, FixedPoints)
p2p.print_twice(logfile, "")
```

4.7 py2pdf

```python
# WARNING : no output after this line can be recorded automatically (reason is the markdown is created
# export the markdown from the python file
msg, date_ext = p2p.py2md(script_name, "# ~~")
# prepare the command to launch the pdf report creation
markdownfile = "../docs/fdm_applied_to_dml_part2a3.md"
pdffile = "../docs/FDM_applied_to_DML_part2a3.pdf"
pdfbackup = "../p2pbackup/FDM_applied_to_DML_part2a3" + date_ext + ".pdf.bak"

# cmd = ('pandoc --pdf-engine=xelatex --variable mainfont="Ubuntu" --filter pandoc-citeproc --number-se
cmd = ('pandoc --pdf-engine=xelatex --variable mainfont="Arial" '+ markdownfile + ' -s --citeproc --num
# Run the command and capture the return code
return_code = os.system(cmd)
# Check the return code to determine success or failure
if return_code == 0:
    print("Pandoc command executed successfully. Hurrah the pdf was created!")
    shutil.copy(pdffile, pdfbackup)
else:
    print("Error: Pandoc command failed with exit code.", return_code)
```

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