Finite Difference Method applied to DML simulation. Part 1b.

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Abstract

Simulation of Distributed Mode Loudspeaker (DML) thanks to the Finite Difference Method (FDM)

Contents

Disclaimer : this paper is written in the context of DIY DML building with the target to identify some design rules to help in the panel construction. This document is not written in the context of any academic or scientific work. Its content is reviewed only by the feedback it can get while posting it in audio DIY forum like diyAudio.

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1 Introduction

In part 1, the application of the finite difference method to simply supported or clamped thin rectangular plate was shown. This document extends to the case of the free edge plate to prepare its implementation in a python script.

The main source for this document is MV Barton 1948 : Finite Difference Equations for the Analysis of Thin Rectangular Plates with Combinations of Fixed and Free Edges [1]

A second source [2] helped to extend to the case of an orthotropic material.

2 Free edge isotropic thin plate boundary conditions

The boundary conditions of a free edge plate are (ref : [1]) as follow.

For an edge parallel to the y axis :

$$\frac{\partial^2 w}{\partial x^2} + \nu * \frac{\partial^2 w}{\partial y^2} = 0$$

$$\frac{\partial^3 w}{\partial x^3} + (2 - \nu) * \frac{\partial^3 w}{\partial x \partial y^2} = 0$$

For an edge parallel to the x axis :

$$\frac{\partial^2 w}{\partial y^2} + \nu * \frac{\partial^2 w}{\partial x^2} = 0$$

$$\frac{\partial^3 w}{\partial y^3} + (2 - \nu) * \frac{\partial^3 w}{\partial y \partial x^2} = 0$$

where ν is the the Poisson's coefficient.

In addition at a corner :

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = 0$$

$$\frac{\partial^2 w}{\partial x \partial y} = 0$$

3 Isotropic thin plate stencils

From [1], the stencils for the isotropic plate in free edge conditions are in **figure 1**.

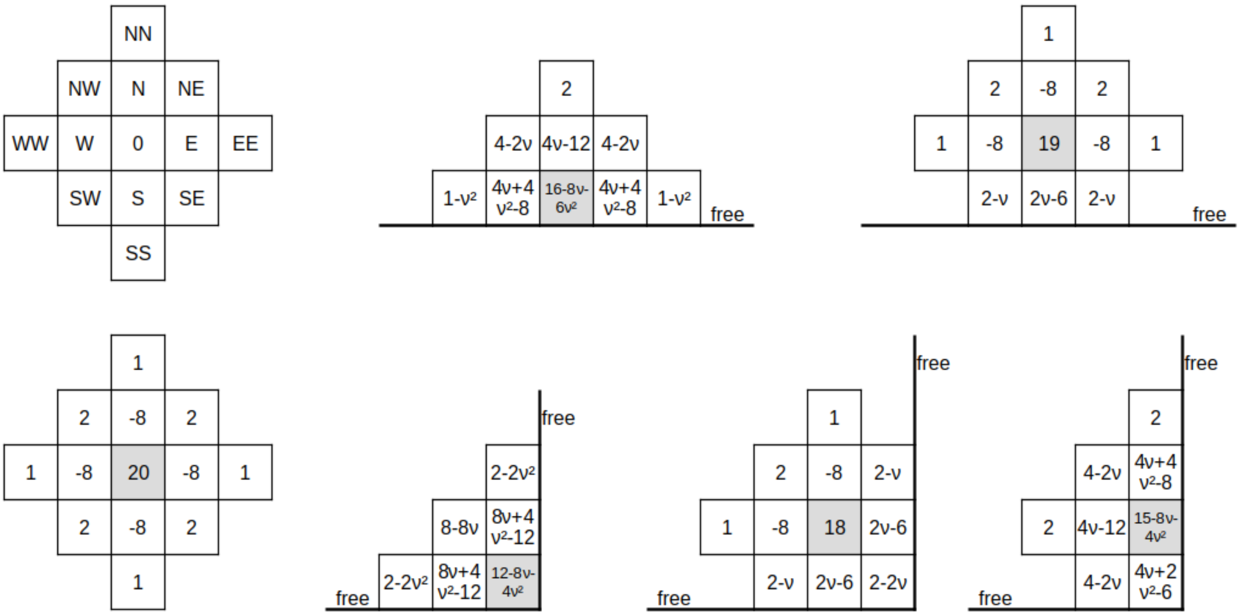

Figure 1: Free edge stencils

4 Free edge orthotropic thin plate boundary conditions

Using the equations for a thin orthotropic plate shown in [2] the conditions from [1] are now :

For an edge parallel to the y axis :

$$\frac{\partial^2 w}{\partial x^2} + \nu_{21} * \frac{\partial^2 w}{\partial y^2} = 0$$

$$\frac{\partial^3 w}{\partial x^3} + (2 - \nu_{21}) * \frac{\partial^3 w}{\partial x \partial y^2} = 0$$

For an edge parallel to the x axis :

$$\frac{\partial^2 w}{\partial y^2} + \nu_{12} * \frac{\partial^2 w}{\partial x^2} = 0$$

$$\frac{\partial^3 w}{\partial y^3} + (2 - \nu_{12}) * \frac{\partial^3 w}{\partial y \partial x^2} = 0$$

With (from wikipedia Poisson's ratio, orthotropic material:

- E_i the Young modulus along axis i.
- $G_{ij} is the shear modulus in direction j on the plane whose normal is in direction i.
- ν_{ij} is the Poisson ratio that corresponds to a contraction in direction j when an extension is applied in direction i.

In addition : $E_i.\nu_{ji} = E_j.\nu_{ij}$

Other notation : $E_x, E_y, \nu_{xy}, \nu_{yx}$

In the following figures, the Poisson's coefficients according to the directions are denoted ν and σ

As in part 1, the partial derivatives are replaced by approximations based on the values of the points surrounding the point of interest. See for example in the wiki page "Finite difference coefficient".

With the free edges, the complexity of the boundary conditions is higher than in the clamped or simply supported conditions. The use of "substitution schematics" makes the transformation of the main stencil in the boundary conditions easier.

See figure 2

The points denoted u, v , x, y, c can be subsituted in the original stencils by the shaded cells when the stencil goes out of the plate.

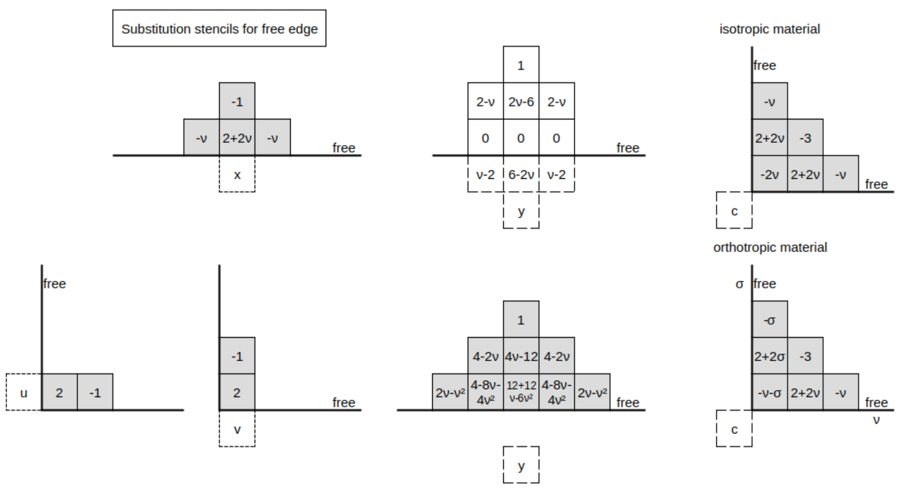

Figure 2: Substitution schematics

5 $\frac{\partial^4 w}{\partial x^4}$, $\frac{\partial^4 w}{\partial y^4}$ stencils

The points out of the plate are replaced thanks to the substitution schematics.

See figure 3

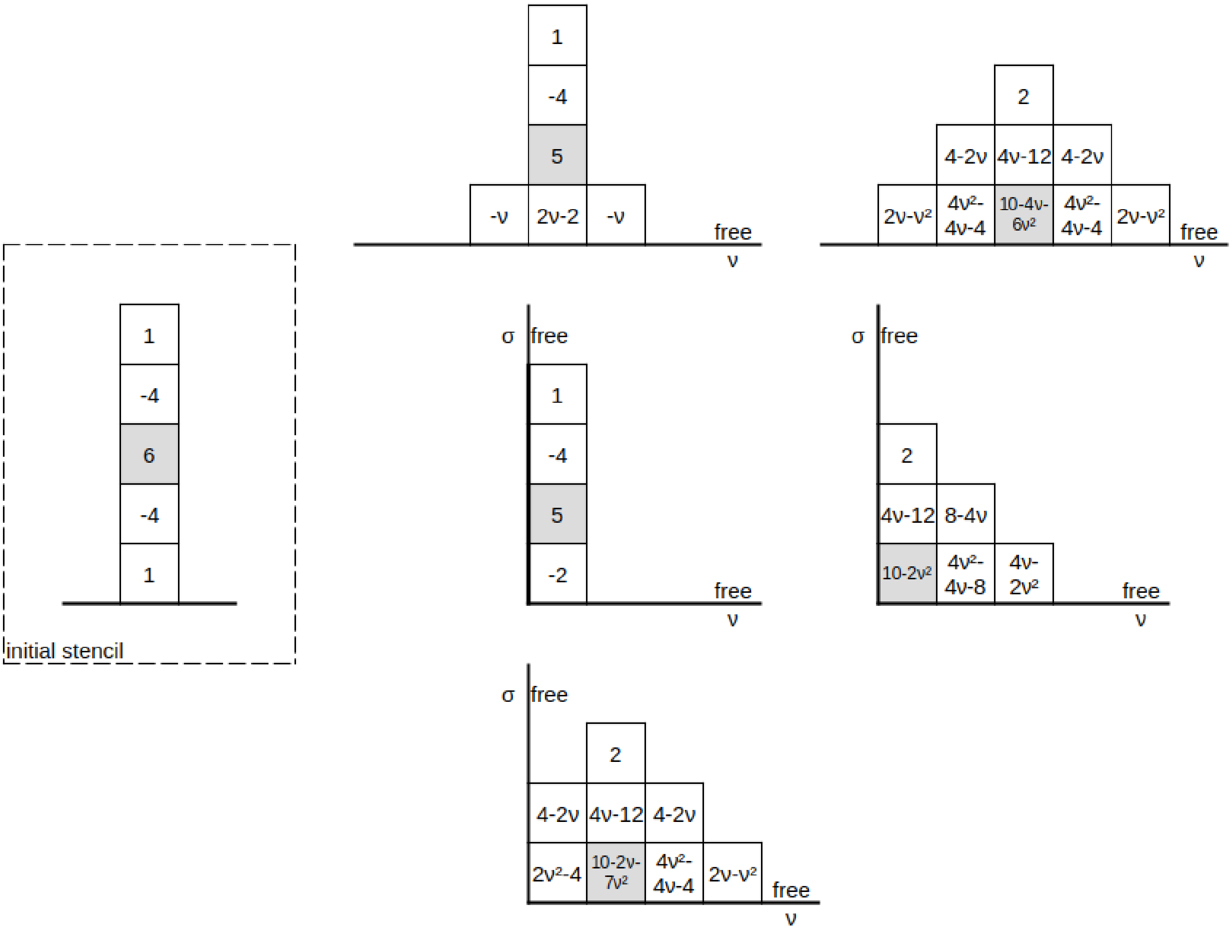

Figure 3: d4w/dx4, d4w/dy4

6 $\frac{\partial^4 w}{\partial x^2 y^2}$ stencils

Same as before, the points out of the plate are replaced thanks to the substitution schematics. **See figure 4**

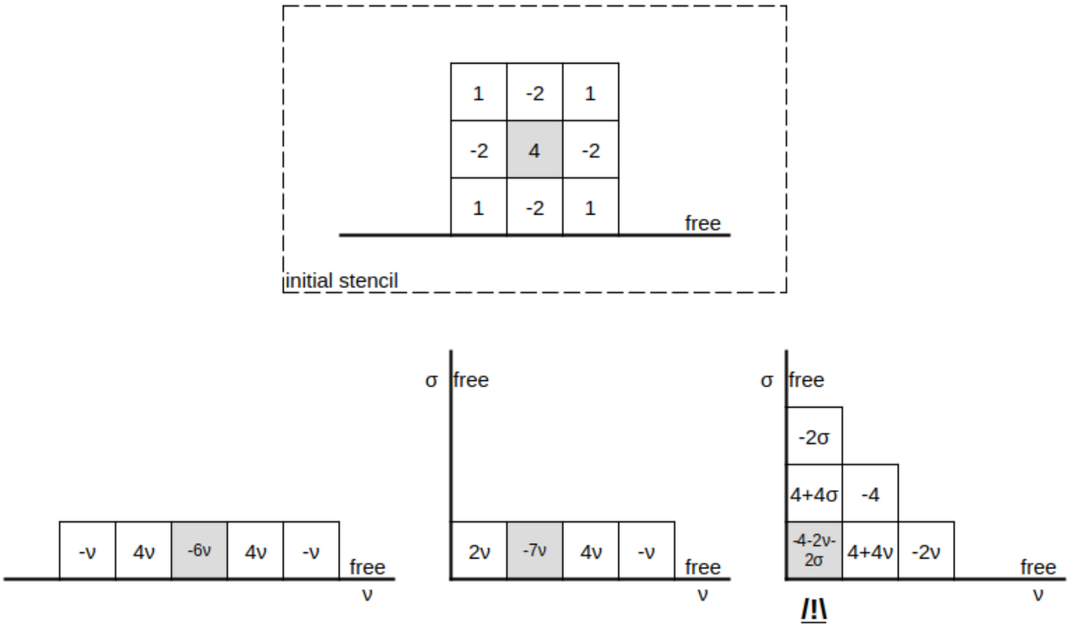

Figure 4: d4w/dx2dy2

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