The University of Nottingham

School of Mathematical Sciences

A LEVEL 1 MODULE, SPRING 2019-2020

ANALYTICAL AND COMPUTATIONAL FOUNDATIONS

COURSEWORK

Submission Deadline: 4pm June 5th 2020

Candidates' submission status will be recorded after collection, then all submission will be passed to relevant module convenor for marking.

Candidates' MATLAB scripts should be clearly set out, with comment statements.

To submit your coursework, please compress all the relevant files into a zip file.

Problem 1: [7 marks]

a) Find all the possible real solutions to

$$x_{n+1} = 4x_n^3 - 3x_n$$

by fixpoint iteration method. You should write your fixpoint iteration method separately as an M-function, which should include the starting point x_0 and the stopping criterion ε as function inputs and have the approximation to the solution and the number of iterations to be the function outputs (It is not necessary to print out the number of iterations). Choose a suitable interval based on your observation that includes all the possible real solutions and then use the M-function to search and identify those solutions.

Note: Let $\varepsilon = 10^{-5}$; you could include more inputs and outputs in the M-function if necessary.

b) Use Newton-Raphson method to find all the possible real roots for the function $f(x) = 4x^3 - 4x$. You should write your Newton-Raphson method separately as an M-function, which should include the starting point x_0 and the stopping criterion ε as function inputs and have the approximation to the solution and the number of iterations to be the function outputs. Choose a suitable interval based on your observation that includes all the possible real solutions and then use the M-function to search and identify those solutions.

Note: Let $\varepsilon = 10^{-5}$; you could include more inputs and outputs in the M-function if necessary.

c) Look at the question a) again. Remember in your lecture slides (`Chapter 7 - Chaos`

`and Contractions`), if your starting value $x_0 = 1/\sqrt{2}$ or $x_0 = -1/\sqrt{2}$, there will be a periodic cycle with no limit. Write an M-function to illustrate whether you could obtain exactly the same result. If not, write up your code to illustrate the reasons.

Problem 2: [7 marks]

a) Consider the ODE:

$$\frac{dy}{dx} = y^2 - y^3, \quad y(0) = 0.01$$

Approximates $y(x)$ using the Runge-Kutta 4th order (RK4) method and plots $y(x)$ where $0 \leq x \leq 200$ using `plot`. You should include the initial value $y(0)$ and step size h in the inputs and have (y_i) in the outputs. Note: RK4 method can be described as follows:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

where

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$$
$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$$
$$k_4 = f(x_i + h, y_i + k_3 h)$$
$$x_{i+1} = x_i + h$$
$$f(x, y) = \frac{dy}{dx}$$

Note: y_i is the RK4 approximation of $y(x_i)$.

b) You are appointed to the forest management office of a Christmas-tree supplier com-

pany. There are three types of trees: seedlings, mid-size trees, big trees.

Every next year

a) 80% of the seedlings grow into mid-size and 20% are lost. These mid-size trees won't be sold at the current year.

b) 20% of the mid-size trees remain mid-size, 50% of them grow to big trees, 20% are sold at the price 30$ each, and the remaining 10% are lost.

c) 50% of the big trees are sold at the price 50$ each, 30% at the price 30$ each and 20% at the price 1$ each.

Seedlings cost you 5$ each. You invest 25000$ and start with 5000 seedlings in year 0. In each further year you buy and plant 7000 new seedlings.

Write a MATLAB script which plots the graphs of the number of mid-size trees, big trees (same coordinate system different colors) and the net income (= annual revenue − investment/cost) per year (separate coordinate system) in the next 10 years.

After how many years will you see the first (positive) net income? How much do you need to invest each year before that? After how many years will you achieve the maximum net income level? What will be the maximum net income?

How much will your maximal net income grow/sink if you buy more expensive seedlings at 6$ each and reduce the number of lost seedlings to 10%?

Note: Use vectors and matrix multiplication to calculate the number of the next year's trees. In addition, use *plot*, *subplot*, *xlabel*, *ylabel*, *title* and *legend* commands for plotting.

Professional marks will be awarded for the format, style and structure of your answers. [1

marks]

END OF THE COURSEWORK