Distribution	Range	pmf or pdf or cDf	Expectation	Variance
0-1	$0<p<1$	$P\{X=k\} = p^k(1-p)^{1-k}$, $k=0,1$	p	$p(1-p)$
Binomial	$n\geq 1$, $0<p<1$	$P\{X=k\} = \binom{n}{k}p^k(1-p)^{n-k}$, $k=0,1,\cdots n$	np	$np(1-p)$
Geometric	$0<p<1$	$P\{X=k\} = (1-p)^{k-1}p$, $k=1,2,\cdots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative Binomial	$r\geq 1$ $0<p<1$	$P\{X=k\} = \binom{k-1}{r-1}p^r(1-p)^{k-r}$, $k=r,r+1,\cdots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$
Hypergeometric	N, M, n $(M\leq N)(n\leq N)$	$P(X=k) = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{k}}$	$\frac{nM}{N}$	$\frac{nM}{N}(1-\frac{M}{N})(\frac{N-n}{N-1})$
Poisson	$\lambda>0$	$P\{X=k\} = \frac{\lambda^k}{k!}e^{-\lambda}$, $k=0,1,2\cdots$	λ	λ
Exponential	$\lambda>0$	$f_X(x)=\begin{cases}\lambda e^{-\lambda x} & x\geq 0\\ 0 & x<0\end{cases}$ $F_X(x)=\begin{cases}1-e^{-\lambda x} & x\geq 0\\ 0 & x<0\end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Uniform (Discrete)	$b>a$	$P_X(x) = \frac{1}{n}$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$
Uniform (Continuous)		$f_X(x) = \begin{cases}\frac{1}{b-a}, & a<x<b\\ 0, & \text{otherwise}\end{cases}$ $F_X(x)=\begin{cases}0 & x<a\\ \frac{x-a}{b-a} & a\leq x<b\\ 1 & x\geq b\end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	μ, $\sigma>0$	$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$	μ	σ^2
Standard Normal	$\mu=0$, $\sigma=1$	$f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$	0	1
Gamma	$\alpha>0$, $\beta>0$	$f_X(x) = \begin{cases}\frac{1}{\Gamma(\alpha)}\beta^\alpha x^{\alpha-1}e^{-\beta x} & x>0\\ 0 & x\leq 0\end{cases}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Beta	$\alpha>0$ $\beta>0$	$f_\alpha(x) = \begin{cases}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1} & 0<x<1\\ 0 & \text{otherwise.}\end{cases}$ $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$ $\Gamma(\alpha) = (\alpha-1)!$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Sums of variables

Let X, Y be any two discrete random variables, taking values in $\{0,1,2\cdots\}$

$Z=X+Y$. Then for $n=0,1,\cdots$

$\boxed{P_Z(n) = \sum_{k=0}^{n} P_{X,Y}(k, n-k).}$

Specially:

$\boxed{P_Z(n) = \sum_{k=0}^{n} P_X(k)\cdot P_Y(n-k)}$

proved by total probability theorem.

Summary

5

George Willsons

'=' 扩大

'无=' 缩

Continuity correction

$P(15 < x < 21) \approx P(15.5 < x < 20.5)$

$P(15 \le x \le 21) \approx P(14.5 < x < 21.5)$

$P(15 < x \le 21) \approx P(15.5 < x \le 21.5)$

$P(15 \le x < 21) \approx P(14.5 < x < 20.5)$

$P(X = x) \approx P(x - 0.5 < Y < x + 0.5) \quad Y \sim N(np, np(1-p))$

Remember $\Leftarrow$ $\boxed{P(-1.96 < Z < 1.96) = 0.95}$

Summary

10 . .

Probability.

- 几何分布与负二项分布
- 读表
- 中心极限
- correction
- 平均数值 区间.
- 样本均值.

(18) A fair die is thrown until a six appears so the number of throws required has a Geometric dis. Throwing continues unil 25 sixes have appeared. The total number of throws is a random variable S.

27.39 ← 15. What's the standard deviation of S

0.0144 ← 16. In order to calculate approximate probabilities associated with S. the Central limit Theorem with continuity correction is used. What is the probability that $S=150$?

0.0326 ← 17. What is the approximate probability that more than 200 throws are required to obtain 25 sixes?

0.0288 ← 18. What is the approximation probability that between 140 and 160 throws (inclusive) are required?

461 ← 19. Suppose the die is thrown until n sixes are obtained and $\bar{x}$ is the mean number of throws required to obtain a 6. What is the value of n for which the approximate probability that $5.5 < \bar{x} < 6.5$ is equal to 0.95? Ignore the continuity correction.

当这个题的第17问我就搞了很久.因为我觉得200和Sn有关.但是好像这个n已经确定为25,毕竟这是一个负二项分布.

Summary

每个x是一个几何分布.

Let X be the number of throws needed for a six.

$X \sim Geom(1/6)$ $\mu = E(X) = \frac{1}{p} = 6$ $\sigma^2 = var(X) = \frac{1-p}{p^2} = 30$

S is the number of throws needed for n sixes.

$S = X_1 + X_2 + \cdots + X_n$, $X_i \sim X$, independent. 15.

特殊性.

$E(S) = 6n$ $var(S) = 30n$. $n=25 \Rightarrow$ $var(S) = 750$. sd. $= \boxed{27.39}$

$n=25$. $E(S) = 150$. $P(S=150) = P(149.5 < S < 150.5)$

$= P(149.5-150/\sqrt{750} < S-150/\sqrt{750} < 150.5-150/\sqrt{750})$

16.

$= P(-0.0183 < Z < 0.0183) = \boxed{0.0146}$ $[Z \sim N(0,1)]$.

$P(S > 200) = P(S > 200.5) = P[S-150/\sqrt{750} > 200.5-150/\sqrt{750}]$

17.

$\approx P(Z > 1.844) = \boxed{0.0326}$

$139.5 < S < 160.5$

$P(140 \le S \le 160) = P[139.5-150/\sqrt{750} < S-150/\sqrt{750} < 160.5-150/\sqrt{750})$

18.

$\approx P(-0.383 < Z < 0.383) = \boxed{0.298}$

$\bar{X} = \frac{S}{n}$. $\bar{X} \approx N(6, 30/n)$. Let $P(5.5 < \bar{X} < 6.5) = 0.95$

$P(5.5 < \bar{X} < 6.5) = P[5.5-6/\sqrt{30/n} < Z < 6.5-6/\sqrt{30/n}] = 0.95$

$P(-1.96 < Z < 1.96) = 0.95 \Rightarrow 6.5-6/\sqrt{30/n} = 1.96 \Rightarrow n = 461$.

probability.

(19) If twenty random numbers are selected independently from the interval $(0,1)$, what is the probability that their sum is at least eight?

·借在连续性均匀分布的方差和期望计算.

这个题还得回归最基础的每一个X_i:

Let X_i be the i^{th} number selected $X \sim U(0,1)$.

$\boxed{E(X_i) = \frac{1}{2} \quad var(X_i) = \frac{1}{12}}$ Let $S_{20} = X_1 + \cdots + X_{20}$

$E(S_{20}) = 10$ and $var(S_{20}) = 20/12$.

Thus. $P(X_1 + \cdots + X_{20} \ge 8) = P[S_{20}-10/\sqrt{20/12} \ge 8-10/\sqrt{20/12}]$

$\approx 1 - \Phi(-1.549) = 0.9393$.

Summary

discrete Unifor. $Pr(x) = \frac{1}{n}$. $E(X) = \frac{n+1}{2}$ $Var(X) = \frac{n^2-1}{12}$

continuous uniform $f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise.} \end{cases}$ $F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x < b \\ 1 & x \ge b \end{cases}$ $\boxed{E(X) = \frac{a+b}{2}}$ $var(X) = \boxed{\frac{(b-a)^2}{12}}$

Probability.

(10) 某种电子器件寿命(小时)具有数学期望 μ (未知) 方差 $\sigma^2=400$, 为了估计 μ. 随机地取 n 只这种器件. 在 $t=0$ 投入测试 (测试是相互独立) 直到失效, 测其寿命 $X_1, X_2, \cdots X_n$, 以 $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ 作为 μ 的估计, 为使 $P\{|\bar{X}-\mu|<1\}\geq 0.95$, 问 n 至少为多少?

$S_n = X_1 + X_2 + \cdots + X_n \sim N(n\mu, n\sigma^2)$

$\therefore P\{|\bar{X}-\mu|<1\}\geq 0.95$

把 $\bar{X}$ 用 S_n 表示. 因为可事先得到 S_n 的分布.

$P\{|\frac{S_n}{n}-\mu|<1\}\geq 0.95$

$P\{-1<\frac{S_n}{n}-\mu<1\}\geq 0.95$

$P\{n(\mu-1)<S_n<n(\mu+1)\}\geq 0.95$

$P\{\frac{n(\mu-1)-n\mu}{\sqrt{400n}}<Z<\frac{n(\mu+1)-n\mu}{\sqrt{400n}}\}\geq 0.95.$

$\frac{n(\mu+1)-n\mu}{\sqrt{400n}}=1.96 \rightarrow n=1536.64$

$\therefore n=1537.$

Summary

Probability.

其实最关键的是这个设未知数的过程.

(101) Let X, Y be independent Poisson random variables with parameter λ and μ respectively. Thus, for example, the probability mass function of X is:

$$p_X(k) = \frac{\lambda^k}{k!} e^{-\lambda} \quad k = 0, 1, \cdots$$

Show that X+Y has a Poisson distribution with parameter $\lambda+\mu$.

$$p(x+y=n) = \sum_{k=0}^{n} p(k, n-k)$$

$$= \sum_{k=0}^{n} p_X(k) \cdot p_Y(n-k)$$

$$= \sum_{k=0}^{n} \cdot \frac{\lambda^k e^{-\lambda}}{k!} \cdot \frac{\mu^{n-k} e^{-\mu}}{(n-k)!}$$

$$= \frac{e^{-(\lambda+\mu)}}{n!} \boxed{\sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \lambda^k \cdot \mu^{n-k}} \text{=二项式定理}$$

$$= \frac{e^{-(\lambda+\mu)}}{n!} \cdot (\lambda+\mu)^n$$

Summary

Sums of random variable.

$$P_Z(n) = \sum_{k=0}^{n} P_{X,Y}(k, n-k), \quad Z = X+Y, \quad n = 0, 1, \cdots \quad X, Y \text{ two r.v. taking } \{0, 1, 2, \cdots\}$$

Specially:

$$P_Z(n) = \sum_{k=0}^{n} P_X(k) P_Y(n-k) \quad Z = X+Y$$

probability.

- 概率论公式灵活运用

$p(A|B\cap F) = \frac{P(A\cap B|F)}{p(B|F)}$

$p(A^c|F) = 1 - p(A|F)$

die1 { 4 red, 2 blue

die2 { 4 blue, 2 red.

问的是前两次都为红的情况下第三次也为红

$p(R_3 | R_1 \cap R_2)$

(102) Two dice are available, one colored red on four faces and blue on two faces, the other coloured blue on four faces and red on two faces.

(i) One of these dice is chosen uniformly at random and rolled. The uppermost face is red. What is the probability that the die with four red faces was chosen?

(ii) The same die is rolled again, and its uppermost face is still red. The same die is then rolled for a third time. What is the probability that its uppermost face is still red?

可能是die1，也可能是die2. 相当于die1，或die 2 连续两次红色.

(i). Let. $E_1 = \{$die with 4 red faces chosen$\}$

$E_2 = \{$die with 4 blue faces chosen$\}$

$R = \{$uppermost red face$\}$. Then.

$p(R|E_1) = \frac{2}{3}$ $\quad p(R|E_2) = \frac{1}{3}$ $\quad p(E_1) = p(E_2) = 1/2$

$$p(E_1|R) = \frac{p(R|E_1)p(E_1)}{p(R|E_1)p(E_1) + p(R|E_2)p(E_2)} = \frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{2}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{2}} = \frac{2}{3}.$$

(ii). Let. $R_2 = \{$die shows red 1st, 2nd$\}$.

$R_3 = \{$die shows red 3rd$\}$

$p(R_2|E_1) = (\frac{2}{3})^2 = \frac{4}{9}$ $\quad p(R_2|E_2) = (\frac{1}{3})^2 = \frac{1}{9}$

$$p(E_1|R_2) = \frac{p(R_2|E_1)}{p(R_2|E_1)p(E_1) + p(R_2|E_2)p(E_2)} = \frac{\frac{4}{9}\cdot 1/2}{\frac{4}{9}\cdot 1/2+\frac{1}{9}\cdot\frac{1}{2}} = \frac{4}{5}.$$

$$\therefore p(R_3|R_2) = p(R_3\cap E_1|R_2) + p(R_3\cap E_2|R_2)$$

$$= p(R_3|E_1\cap R_2)\cdot p(E_1|R_2) + p(R_3|E_2\cap R_2)\cdot p(E_2|R_2).$$

$$= \frac{2}{3}\cdot\frac{4}{5} + \frac{1}{3}\cdot\frac{1}{5} = \frac{3}{5}$$

Summary

5 . .

Probability

审题：a pair of dice.
outcome：两个骰子的数值相加.

(103) Independent trials consisting of rolling a pair of fair dice are performed. What is the probability that an outcome of 5 appears before an outcome of 7 when the outcome of a roll is the sum of the dice?

Method 1:

一旦第一次出现5，就算成功了，可以停止实验了。

有点像几何分布额。

E_n denote event that no 5 or 7 appears on first $n-1$ th trials. 5 appears on the n^{th} trial.

$P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$

$P\{5 \text{ on any trial}\} = \frac{4}{36}$ $\{1,4\}\{4,1\}\{2,3\}\{3,2\}$

$P\{7 \text{ on any trial}\} = \frac{6}{36}$ $\{1,6\}\{6,1\}\{2,5\}\{5,2\}\{3,4\}\{4,3\}$

$P(E_n) = \left(1-\frac{10}{36}\right)^{n-1} \frac{4}{36}$

$\therefore P\left(\bigcup_{n=1}^{\infty} E_n\right) = \frac{1}{9} \sum_{n=1}^{\infty} \left(\frac{13}{18}\right)^{n-1} \rightarrow \frac{1-\left(\frac{13}{18}\right)^n}{1-\frac{13}{18}}$ $\rightarrow \infty$

$= \frac{1}{9} \times \frac{1}{1-\frac{13}{18}}$

$= \frac{2}{5}$

Method 2: Conditional probability:

$E \begin{cases} \text{first is 5. stop.} \Rightarrow F \\ \text{first is not 5 or 7 continue.} \rightarrow G \end{cases}$

$P(E) = P(E|F)P(F) + P(E|G)P(G)$

$P(E|F) = 1$ $P(F) = \frac{4}{36}$ $P(E|G) = P(E)$ $P(G) = \frac{26}{36}$

$P(E) = 1 \times \frac{1}{9} + \frac{26}{36} \times P(E) \Rightarrow P(E) = \frac{2}{5}$

第一次不是5 or 7，相当于继续，从头再来。

Summary

Probability

类似于开锁，虽说是随机拨号，但拨错了就不会再去拨相同的号码了。

话

(14) 某人忘了电话号码最后一个数字，因而他随意拨号，求拨号不超过三次而接通所需电话的概率。若已知最后一个数为奇数，那么此概率为多少？

Method 1: A_i 表示第 i 次拨号拨通电话 $i=1,2,3$. A{拨号不超过3次}

$A = A_1 \cup \bar{A}_1 A_2 \cup \bar{A}_1\bar{A}_2 A_3$

$P(A_1) = \frac{1}{10}$

$P(\bar{A}_1 A_2) = P(A_2|\bar{A}_1)P(\bar{A}_1) = \frac{1}{9}\times\frac{9}{10} = \frac{1}{10}$

$P(\bar{A}_1\bar{A}_2 A_3) = P(A_3|\bar{A}_1\bar{A}_2)P(\bar{A}_2|A_1)P(\bar{A}_1)$

$= \frac{1}{8}\times\frac{8}{9}\times\frac{9}{10} = \frac{1}{10}$

$\therefore P(A) = P(A_1) + P(\bar{A}_1 A_2) + P(\bar{A}_1\bar{A}_2 A_3)$

$= \frac{1}{10}+\frac{1}{10}+\frac{1}{10} = \frac{3}{10}$.

最后一个为奇，同理 $\frac{1}{5}+\frac{1}{5}+\frac{1}{5} = \frac{3}{5}$

Method 2:

$P(A) = 1 - P($拨号三次都接不通$)$

$= 1 - P(\bar{A}_1\bar{A}_2\bar{A}_3) = 1 - P(\bar{A}_3|\bar{A}_1\bar{A}_2)P(\bar{A}_2|\bar{A}_1)P(\bar{A}_1)$

$= 1 - \frac{7}{8}\times\frac{8}{9}\times\frac{9}{10} = \frac{3}{10}$

最后为奇. $1 - \frac{2}{3}\times\frac{3}{4}\times\frac{4}{5} = \frac{3}{5}$.

掌握设未知数方法。设 A_i 为第 i 次拨通电话

Summary

Probability

(105) (a) 设甲袋中装有 n 只白球，m 只红球；乙袋中装有 N 只白球，M 只红球，今从甲袋中任取一只放入乙袋中，再从乙袋中任取一只球，问取到的球是白球的概率？

(b) 第一只盒子装有 5 只红球，4 只白球；第二个盒子有 4 只红球，5 只白球，先从第一盒中任取 2 只球放入第二盒中去，然后从第二盒中任取一只球求取到白球概率？

(a) E_1: 甲 → 乙　E_2: 乙 → 取出来

R{甲取出是红球}　W{乙取得白球}

$P(R) = \frac{m}{n+m}$　　$P(\bar{R}) = \frac{n}{n+m}$

甲取出红 } 甲取出白 }

$\boxed{W = SW = (R \cup \bar{R})W = RW \cup \bar{R}W}$　$(RW) \cap (\bar{R}W) = \emptyset$.

实际上是从乙中摸出的球的概率由甲摸出的决定也带出行摸 第二次做这个题感觉更明显了。

$P(W) = P(R \cap W) + P(\bar{R} \cap W)$

$= P(W|R)P(R) + P(W|\bar{R})P(\bar{R})$

$= \frac{N}{N+M+1} \cdot \frac{m}{n+m} + \frac{N+1}{N+M+1} \cdot \frac{n}{n+m}$

$= \frac{n+N(n+m)}{(n+m)(N+M+1)}$

(b) E_1: 甲取 2 个球放入乙　E_2: 乙中取一个球

R_i {表示从甲中取的有 i 只红色}　W{乙中取出为白}

甲 $\begin{cases} 红=5 \\ 白=4 \end{cases}$　乙 $\begin{cases} 红=4 \\ 白=5 \end{cases}$

$W = SW = (R_0 \cup R_1 \cup R_2)W = R_0W + R_1W + R_2W$

$P(W) = P(R_0W) + P(R_1W) + P(R_2W)$

$= P(W|R_0)P(R_0) + P(W|R_1)P(R_1) + P(W|R_2)P(R_2)$

$P(R_0) = \frac{\binom{4}{2}}{\binom{9}{2}} = \frac{1}{6}$　$P(R_2) = \frac{\binom{5}{2}}{\binom{9}{2}} = \frac{5}{18}$　$P(R_1) = \frac{10}{18}$

Summary

$P(W|R_0) = \frac{7}{11}$ → 乙已变为11个球了　$P(W|R_1) = \frac{6}{11}$　$P(W|R_2) = \frac{5}{11}$

$P(W) = \frac{53}{99}$

性质：互换两行(列)，行列式变号

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 4 & 5 & 7 & 8 \\ 8 & 9 & 10 & 12 \end{vmatrix} = -1$$

$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 \\ 4 & 5 & 7 & 8 \\ 8 & 9 & 10 & 12 \end{vmatrix} = -1 \times \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 4 & 5 & 7 & 8 \\ 8 & 9 & 10 & 12 \end{vmatrix} = 1$$

eg:

$$\begin{vmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 0 & 5 & 2 & 4 \end{vmatrix} \xrightarrow[\times(-1)]{\text{行}_1 \leftrightarrow \text{行}_4} \begin{vmatrix} 0 & 5 & 2 & 4 \\ 0 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 3 \end{vmatrix} \xrightarrow[(-1)\times(-1)]{\text{行}_2 \leftrightarrow \text{行}_3} \begin{vmatrix} 0 & 5 & 2 & 4 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{vmatrix}$$

$$\xleftrightarrow[(-1)\times(-1)\times(-1)]{\text{行}1 \leftrightarrow \text{行}2} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 2 & 4 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{vmatrix} = -1 \times -1 \times -1 \times 1 \times 5 \times 3 \times 3 = -45$$

Summary

求逆矩阵. $(A \vdots E^{-1}) \xrightarrow{\begin{array}{l} ①\text{换行} \\ ②\text{某行乘上一个数} \\ ③\text{一行加上或减去另一行} \end{array}} (E \vdots A^{-1})$.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 5 & 7 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 4 & 5 & 7 & 0 & 0 & 1 \end{array}\right) \xrightarrow{\text{一行变换}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & -2 & 5 & -2 \\ 0 & 0 & 1 & 2 & -3 & 1 \end{array}\right) \quad A^{-1}$$

行列式.

性质：某行(列)加上或减去另一行(列)的几倍
行列式不变

eg:

3阶

$$\begin{vmatrix}1&2&3\\2&3&4\\6&5&7\end{vmatrix} \xrightarrow{行_2-2行_1} \begin{vmatrix}1&2&3\\2-2\times1&3-2\times2&4-2\times3\\4&5&7\end{vmatrix} = \begin{vmatrix}1&2&3\\0&-1&-2\\4&5&7\end{vmatrix}$$

$$\xrightarrow{行_3-4行_1} \begin{vmatrix}1&2&3\\0&-1&-2\\4-4\times1&5-4\times2&7-4\times3\end{vmatrix} = \begin{vmatrix}1&2&3\\0&-1&-2\\0&-3&-5\end{vmatrix}$$

使对角线下面为0

$$\xrightarrow{行_3-3行_2} \begin{vmatrix}1&2&3\\0&-1&-2\\0&-3-3\times(-1)&-5-3\times(-2)\end{vmatrix} = \begin{vmatrix}1&2&3\\0&-1&-2\\0&0&1\end{vmatrix}$$

$$= 1\times(-1)\times1$$

$$= -1$$

性质：某行(列)乘K，等于K乘此行列式.

2倍

$$\begin{vmatrix}1&2&3&4\\2&3&4&5\\4&5&7&8\\8&9&10&12\end{vmatrix} = -1 \qquad \begin{vmatrix}2&4&6&8\\2&3&4&5\\12&15&21&24\\8&9&10&12\end{vmatrix} = 2\times3\times\begin{vmatrix}1&2&3&4\\2&3&4&5\\4&5&7&8\\8&9&10&12\end{vmatrix}$$

3倍.

$$= -6$$

Summary

矩阵可逆条件

① 方阵 (m=n).

② |A|≠0 或 存在一个方阵B. 满足 AB=I 或 BA=I.