10

(a) Two sides test:

reject H_0 if $z = \left| \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \right| > z_{0.025} = 1.96$

$z = 1.8$, so we accept $H_0: \mu = 0$ at the 5% significance level.

(b) One side test:

reject H_0 if $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} > z_{0.05} = 1.645$

Since $z = 1.8$ we reject $H_0: \mu = 0$ at the 5% significance level.

(c) from a: we reject H_0 if

先找出拒绝 H_0 所在的 $\bar{x}$ 的范围

$|\bar{x}| > \mu_0 + z_{0.025}\frac{\sigma}{\sqrt{n}} = 0 + 1.96 \times \frac{2}{8} = 0.49$

Power $= P(\text{Reject } H_0 \mid \mu = \mu_1)$

$\mu_1 = -1: P(|\bar{x}| > 0.49)$ $\quad \bar{x} \sim N(\mu_1, \frac{\sigma^2}{n})$

$= 1 - P(|\bar{x}| < 0.49)$ $\quad \bar{x} \sim N(-1, \frac{1}{16})$

双边

$= 1 - P\left(\frac{-0.49-(-1)}{1/4} < Z < \frac{0.49-(-1)}{1/4}\right)$

$= 1 - P(2.04 < Z < 5.96) = 0.9793$ $\quad Z \sim N(0,1)$

Similarly:

$\mu = -1/2$ $\quad$ Power $= 1 - P(0.04 < Z < 3.96) = 0.5160$

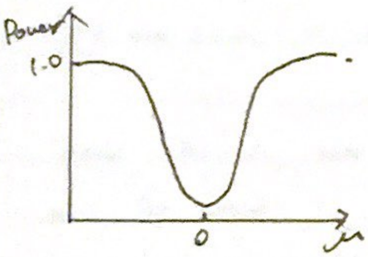
$\mu = 1/2$ $\quad$ Power $= 1 - P(-3.96 < Z < -0.04) = 0.5160$

$\mu = 1$ $\quad$ Power $= 1 - P(-5.96 < Z < -2.04) = 0.9793$.

at $\mu = 0$ $\quad$ Power $= \alpha = 0.05$.

at $\mu \to -\infty$ $\quad$ Power $\to 1$

at $\mu \to \infty$ $\quad$ Power $\to 1$



Summary

$\boxed{\text{Power}}$ $P(\text{reject } H_0 \mid H_0 \text{ false}) = 1 - \beta$ 两总体参数存在差异时，使用的统计检验能够发现这种差异的概率.

$P(\text{Type I error}) = P(\text{reject } H_0 \mid H_0 \text{ true}) = \alpha$

$P(\text{Type II error}) = P(\text{accept } H_0 \mid H_0 \text{ false}) = \beta$

H_1 true.

Sta

(17) In a medical study of patients given a drug and a placebo. Sixteen patients were paired up (by allocating people of the same sex and similar age to each pair) One of each pair ~~p~~ received a drug and the other the placebo. The response score ~~of~~ for each patient was found.

Pair number	1	2	3	4	5	6	7	8
Given number	0.16	0.97	1.57	0.55	0.62	1.12	0.68	1.69
Given placebo	0.11	0.13	0.77	1.19	0.46	0.41	0.40	1.28
Difference	0.05	0.84	0.08	-0.64	0.16	0.71	0.28	0.41

Are the response for the drug and the placebo significantly different?

Match Pairs: Model: $d_i \sim N(\mu_{d_i}, \sigma_d^2)$

$\bar{d} = 0.326$ $s_d^2 = 0.2401$ $s_d = 0.490$. $n=8$.

$H_0: \mu_d = 0$ vs $H_1: \mu_d \neq 0$

Reject H_0 if $|t| > t_{7, \alpha/2}$

$|t| = \left|\frac{\bar{d}}{s_d/\sqrt{n}}\right| = \left|\frac{0.326}{0.49/\sqrt{8}}\right| = 1.88$ $t_{7, 0.05} = 1.89$

so do not reject H_0 at 10% level.

Summary

Sta

$\boxed{\text{Test for existence of regression}}$

(18) For each car of seven cars the weight x (lbs) and fuel consumption y (gallon/100 miles) are given below:

Weight x: 3600 3800 4100 2200 2600 2900 2000

Fuel Consumption y: 5.5 5.9 6.5 3.3 3.6 4.6 2.9

Note that $\sum x_i = 21000$ $\sum x_i^2 = 66820000$

$\sum x_i y_i = 102530$ $\sum y_i = 32.3$

$\sum y_i^2 = 160.73$ $n=7$

Obtain the regression line for fuel consumption given weight and perform a test for the existence of regression.

$$t = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}/\sqrt{(n-1)S_x^2}} \sim t_{n-2}.$$

c.i.: $100(1-\alpha)\%$

$$\hat{\beta}_1 \pm t_{n-2,\alpha/2} \frac{\hat{\sigma}}{\sqrt{(n-1)S_x^2}}$$

1. $S_x^2 = \frac{1}{n-1}(\sum x_i^2 - \frac{1}{n}(\sum x_i)^2) = \frac{1}{6}(66820000 - \frac{(21000)^2}{7}) = 636666.67$

$S_y^2 = \frac{1}{n-1}(\sum y_i^2 - \frac{1}{n}(\sum y_i)^2) = \frac{1}{6}(160.73 - \frac{(32.3)^2}{7}) = 1.9481$

$S_{xy} = \frac{1}{n-1}(\sum x_i y_i - \frac{1}{n}(\sum x_i)(\sum y_i)) = \frac{1}{6} \times (102530 - \frac{(21000)(32.3)}{7}) = 1105.$

$\hat{\beta}_1 = \frac{S_{xy}}{S_x^2} = 0.0017356.$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 4.6143 - 0.0017356 \times 3000 = -0.5925$

so the equation of the fitted line is

$\hat{Y} = -0.5925 + 0.0017356\,x$

Test for existence of regression:

$H_0: \beta_1 = 0$ vs $H_1: \beta_1 \neq 0$ reject H_0 $100\alpha\%$ level if

$$|t| = \left| \frac{\hat{\beta}_1}{\hat{\sigma}/\sqrt{(n-1)S_x^2}} \right| > t_{n-2,\alpha/2}$$

$t_{5,0.025} = 2.571$

Summary

$$\hat{\sigma}^2 = \frac{(n-1)}{(n-2)}\left[S_y^2 - \hat{\beta}_1^2 S_x^2\right] = 0.036317$$

$$|t| = \left| \frac{0.0017356}{\sqrt{0.036317/(6 \times 636666.6)}} \right| = 17.800 > t_{5,\,0.025} = 2.571$$

Very strong evidence that the true slope is non-zero.

Sta

(1.78) The following data give the frequency of distribution of the size of casual groups of people on a spring afternoon in a park

Size of group	1	2	3	4	5	≥ 6
Frequency	1486	694	195	37	10	1
					11	
Expected Frequency	1502.6	668.7	198.4	44.1	7.9	1.3
					9.2	

$\Rightarrow \Sigma = 2423$

A suggest model for the probability P_r, of a group size r is

r is infine.

$$P_r = \frac{\mu^r e^{-\mu}}{r!(1-e^{-\mu})}, \quad r = 1, 2, 3, \cdots$$

where μ is estimated to be 0.89 for this data set. Does this give a good fit to the data?

total number of group: $1486 + 694 + 195 + 37 + 10 + 1 = 2423$

Combine 5-6. together

Degrees of freedom:

$$\underset{\text{Groups}}{5} - \underset{\text{Constraint}}{1} - \underset{\text{Estimate } \mu}{1} = 3$$

$$\chi^2_{obs} = \frac{(1486 - 1502.6)^2}{1502.6} + \cdots + \frac{(11 - 9.2)^2}{9.2} = 2.712$$

reject H_0 if $\chi^2_{obs} \geq \chi^2_{3,\,0.05} = 7.815$ at 5%.

so we would not reject H_0 at 5% level.

It's a good fit to the data.

*: The use of the χ^2 disti. is only valid for large samples. χ^2 is valid when all $e_i \geq 5$; if some $e_i < 5$, then collect some groups together so that all expected frequency are > 5.

Summary

5 .

Sta.

(179) In order to test the lifetime of small batteries used to power clocks. 40 batteries were chosen at random and tested. Their times (in mths) to failure were:

18	11	25	36	40	72	33	51	1	12
46	28	87	75	24	11	23	13	45	2
40	29	14	59	1	7	39	54	16	3
8	2	52	20	9	6	7	26	31	38

The manufacturer claims that the lifetimes have an exponential dist. with mean 30 months. If we assume this, calculate a. b. c and d s.t.

$$p(0<x<a) = p(a<X<b) = p(b<X<c) = p(c<X<d) = p(d<X<\infty) = \frac{1}{5}$$

Construct a table of expected and observed frequency for the above five intervals and hence test the manufacturer's claim by using a goodness-of-fit test at 5% level.

$x \sim Exp(\lambda)$

cdf : $F(x) = 1 - e^{-\lambda x} = p$. ← 累积概率

$\lambda = 1/30$ we take $p = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ 等

$\Rightarrow x = -\lambda^{-1}\ln(1-p) = -30\ln(1-p)$

$\therefore a = 6.7 \quad b = 15.3 \quad c = 27.5 \quad d = 48.3$

Summary

	0 - 6.7	6.7 - 15.3	15.3 - 27.5	27.5 - 48.3	48.3 +
obs. (count the number.)	6	9	7	10	8
Exp.	$\frac{1}{5} \times 40 = 8$	8	8	8	8

$$\chi^2 = \sum \frac{(O_i - e_i)^2}{e_i} = 1.25$$

degree of freedom: $5-1=4$ $\chi^2_{4,0.05}=9.488$

so don't reject H_0. No reason to support that the manufacturer's claim is incorrect.

Change of basis.

LM.

(180) $\mathbb{R}^2$: $B_1 = (e_1, e_2)$ $e_1 = \begin{pmatrix}1\\0\end{pmatrix}$ $e_2 = \begin{pmatrix}0\\1\end{pmatrix}$

$B_2 = ((1,1), (1,-1))$

$v = (2,3)$ write the coordinates of v with respect to basis B_2.

Procedure of finding $M_B^{B'}$

1. Form $[B' \mid B]$
2. row operation
3. $[I \mid M_B^{B'}]$

① Find $M_{B_1}^{B_2}$

② Find Mv.

Write as linear combination:

$e_1 = \begin{pmatrix}1\\0\end{pmatrix} = k_1 b_1 + k_2 b_2 = \frac{1}{2} b_1 + \frac{1}{2} b_2$

$e_2 = \begin{pmatrix}0\\1\end{pmatrix} = k_1' b_1 + k_2' b_2 = \frac{1}{2} b_1 + (-\frac{1}{2}) b_2$

顺序反了.

$M_{B_1}^{B_2} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

↳ 线代老师上课讲错了

$Mv = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ -\frac{1}{2} \end{pmatrix}$

$v = \frac{5}{2} b_1 - \frac{1}{2} b_2 = 2e_1 + 3e_2$

批注：这个做法手格象数上有问题。应当用 e_1, e_2 去表示 B_2，则其才说明是 $B_1 \Rightarrow B_2$ 的.

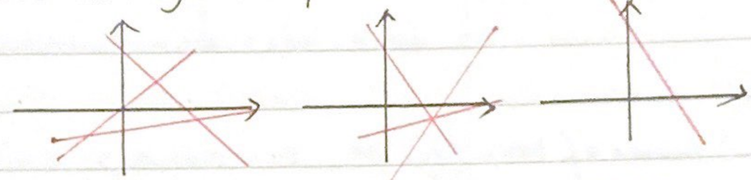
$B_1 \xrightarrow{M_{B_1}^{B_2}} B_2$

$\xleftarrow{{M_{B_1}^{B_2}}^{-1}}$ 逆矩阵可以变换回去.

Summary

Consistency of linear system.

Geometrically interpretation of consistency.



no solution   unique solution   infinite solutions

$\boxed{\text{Theorem}}$: A linear system is consistent if and only if the row echelon form of its augmented matrix contains no row of the form $\boxed{[0, \cdots 0 \mid b]}$ where $b \neq 0$.

判断出 consistent ←
之后，观察有无
free variables.

$Ax = b$

- consistent
  - unique solution
  - infinite solution
- inconsistent

$\boxed{\text{Proposition:}}$ Any homogenous system $Ax = 0$

$\#\{\text{variables}\} = \#\{\text{pivot position of } A\} + \#\{\text{free variables}\}$

Theorem: If A is a matrix with n columns then $\text{rank}(A) + \text{nullity}(A) = n$.

$[\text{number of leading variables}] + [\text{number of free variables}] = n.$

Summary

5

I.M.

Rank-nullity theorem

$\dim(\operatorname{Ker}(T)) =$ nullity.

$\dim(R(T)) =$ rank

$\operatorname{rank}(A) + \operatorname{nullity}(A) = n$

(18) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

(i) Find $\operatorname{Ker}(A)$

(ii) find all solutions to $Ax = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix}$, given that $x_0 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ is one such solution.

(iii) Verify ran-nullity for A.

$\dim(\operatorname{Ker}(A)) + \operatorname{rank}(A) = n$.

(i) Gaussian elimination: $Ax = 0$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$B^1 - 2B^2 + B^3 = 0$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 0 \quad \begin{pmatrix} \alpha = \gamma \\ \beta = -2\gamma \\ \gamma = \gamma \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$x = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

(ii) We can get $x = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ $t \in \mathbb{R}$

Theorem: solving general linear equations via translation. $Ax = b$ if $Ax_0 = b$, then the general solution is $x = x_0 + v$, $v \in \operatorname{Ker}(A)$.

proof: $Ax = b \iff Ax - Ax_0 = b - b = 0$

$\iff A(x - x_0) = 0$

$\iff x - x_0 \in \operatorname{Ker}(A)$

$\iff x = x_0 + v$, $v \in \operatorname{Ker}(A)$.

(iii) from (i) $\dim(\operatorname{Ker}(A)) = 1$

echolen form $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \operatorname{rank} = 2$

the number of pivot columns in reduced echelon form.

Summary

5

LM.

(18a) $W = \left\{ \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} \middle| \begin{matrix} \alpha, \beta, \gamma \in R \\ \underline{\alpha+\beta+\gamma=0} \end{matrix} \right\} \subseteq M_{3,3}(R)$

& Tr(A) = 0

$V_1 = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \beta_1 & 0 \\ 0 & 0 & \gamma_1 \end{pmatrix} \quad V_2 = \begin{pmatrix} \alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \gamma_2 \end{pmatrix}$

$V_1 + kV_2 = \begin{pmatrix} \alpha_1 + k\alpha_2 & 0 & 0 \\ 0 & \beta_1 + k\beta_2 & 0 \\ 0 & 0 & \gamma_1 + k\gamma_2 \end{pmatrix} = \begin{pmatrix} \alpha' & 0 & 0 \\ 0 & \beta' & 0 \\ 0 & 0 & \gamma' \end{pmatrix}$

$\alpha' + \beta' + \gamma' = \alpha_1 + k\alpha_2 + \beta_1 + k\beta_2 + \gamma_1 + k\gamma_2$

$= \alpha_1 + \beta_1 + \gamma_1 + k(\alpha_2 + \beta_2 + \gamma_2)$

$= 0 + k \cdot 0 = 0$

$\therefore V_1 + kV_2 = \begin{pmatrix} \alpha' & 0 & 0 \\ 0 & \beta' & 0 \\ 0 & 0 & \gamma' \end{pmatrix}$ & $\alpha' + \beta' + \gamma' = 0$

$\therefore W \leq M_{3,3}(R)$

$\underline{V} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & -\alpha-\beta \end{pmatrix} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & -\beta \end{pmatrix}$

$= \alpha \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}} + \beta \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}$

$\in W$ Trace = 0. Both

$\therefore W = \left\langle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \right\rangle$

$k_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} + k_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad k_1 = k_2 = 0$

$B = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \right\}$ is linearly independent spanning set. Hence a basis.

$\dim(W) = 2$.

Summary

Summary

STATISTICS FORMULA SUMMARY

5

SAMPLE STATISTICS

$$S_x^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2 = \frac{1}{n-1}\left\{\sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2\right\}$$

$$S_y^2 = \frac{1}{n-1}\sum (y_i - \bar{y})^2 = \frac{1}{n-1}\left\{\sum y_i^2 - \frac{1}{n}\left(\sum y_i\right)^2\right\}$$

$$S_{xy} = \frac{1}{n-1}\sum (x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{n-1}\left\{\sum x_i y_i - \frac{1}{n}\left(\sum x_i\right)\left(\sum y_i\right)\right\}$$

$$S^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i - 1} = \frac{1}{n-1}\left\{\sum f_i x_i^2 - \frac{1}{n}\left(\sum f_i x_i\right)^2\right\}.$$

$\sum f_i$

Lower Quartile: $\frac{n+1}{4}$

Upper Quartile: $\frac{3(n+1)}{4}$

Interquartile Range: $IQR = Q_3 - Q_1$

Regression

$\hat{\beta}_1 = \frac{S_{xy}}{S_x^2}$ $\quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ $\quad$ Predict Y from X.

$\beta_1^* = \frac{S_{xy}}{S_y^2}$ $\quad \beta_0^* = \bar{x} - \beta_1^* \bar{y}$ $\quad$ Predict X from Y

Symmetry and Skewness

$$g = \frac{1}{n}\sum \left(\frac{x_i - \bar{x}}{s}\right)^3$$

QQ Plot:

1. Rank: $y_1 \le y_2 \le \dots y_n$
2. Find x_i s.t.: $F(x_i) = \frac{i - \frac{1}{2}}{n}$ $\quad$ $F(\,)$ is $\boxed{\text{cdf.}}$
3. Plot y_i against x_i.

				Summary
μ	one	$z = \dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$	$\bar{x} \pm z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$	
		$t = \dfrac{\bar{x}-\mu_0}{s/\sqrt{n}} \sim t_{n-1}$	$\bar{x} \pm t_{n-1,\alpha/2} \dfrac{s}{\sqrt{n}}$	
	two	$z = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$	$\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	
		$t = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t_{n_1+n_2-2}$	$\bar{x}_1 - \bar{x}_2 \pm t_{n_1+n_2-2,\alpha/2} \sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}$	$s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$
		$t = \dfrac{\bar{D} - \mu_d}{s_d/\sqrt{n}} \sim t_{n-1}$	$\bar{d} \pm t_{n-1,\alpha/2} \dfrac{s_d}{\sqrt{n}}$	$d_i = x_{1i} - x_{2i}$
σ^2		$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$	$\left(\dfrac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} , \dfrac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}} \right)$	
		$F = \dfrac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \sim F_{n_1-1,\, n_2-1}$	$F > F_{n_1-1, n_2-1, \alpha/2}$ or $F < F_{n_1-1, n_2-1, 1-\alpha/2} = \dfrac{1}{F_{n_2-1, n_1-1, \alpha/2}}$	
correlation	linear	$t = r\sqrt{\dfrac{n-2}{1-r^2}} \sim t_{n-2}$	$\rho = 0$ $\rho \neq 0$	$r = \dfrac{S_{xy}}{S_x S_y}$
	no-linear	$r_s = 1 - \dfrac{6\sum d_i^2}{n(n^2-1)}$		
Regression		$t = \dfrac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}/\sqrt{(n-1)s_x^2}} \sim t_{n-2}$	$\hat{\beta}_1 \pm t_{n-2,\alpha/2} \dfrac{\hat{\sigma}}{\sqrt{(n-1)s_x^2}}$	$\boxed{\hat{\sigma}^2 = \dfrac{(n-1)}{(n-2)}\left[S_y^2 - \hat{\beta}_1^2 S_x^2\right]}$
Y		$\hat{\beta}_0 + \hat{\beta}_1 x_0 \pm t_{n-2,\alpha/2}\, \hat{\sigma} \sqrt{\dfrac{1}{n} + \dfrac{(x_0-\bar{x})^2}{(n-1)s_x^2}}$		
individual Y		$\hat{\beta}_0 + \hat{\beta}_1 x_0 \pm t_{n-2,\alpha/2}\, \hat{\sigma} \sqrt{1 + \dfrac{1}{n} + \dfrac{(x_0-\bar{x})^2}{(n-1)s_x^2}}$		
Propation	one propation	$z = \dfrac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1)$	$p \pm z_{\alpha/2} \sqrt{\dfrac{p(1-p)}{n}}$	$p = \dfrac{x}{n}$
	two propation	$z = \dfrac{p_1 - p_2}{\sqrt{\hat{\pi}(1-\hat{\pi})(\frac{1}{n_1}+\frac{1}{n_2})}} \sim N(0,1)$		$\hat{\pi} = \dfrac{x_1 + x_2}{n_1 + n_2}$
		$\chi^2_{obs} = \sum \dfrac{(O_i - e_i)^2}{e_i}$		

5 · 18th

LM.

- Eigenvector and eigenvalue.

- repeated value.

$$Av = \lambda v.$$
$$(A - \lambda I)v = 0$$

重根最好不用 $B^1\ B^2\ B^3$.

批注：也可表示为 $\beta\begin{pmatrix}1\\1\\0\end{pmatrix} + \gamma\begin{pmatrix}-1\\0\\2\end{pmatrix}$ 最后的空间都是一样的

Summary

(18) $A = \begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix}$

Characteristic: $c(\lambda) = \det(A - \lambda I)$

$$c(\lambda) = \begin{vmatrix} 1-\lambda & -4 & 2 \\ -4 & 1-\lambda & -2 \\ 2 & -2 & -2-\lambda \end{vmatrix} = -(3+\lambda)^2(\lambda-6)$$

$\lambda_1 = 6. \quad \lambda_2 = \lambda_3 = -3$

① $\lambda_1 = 6.$ $v_1 = \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}$

$$\begin{pmatrix} -5 & -4 & 2 \\ -4 & -5 & -2 \\ 2 & -2 & -8 \end{pmatrix}\begin{pmatrix}\alpha_1\\\beta_1\\\gamma_1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -5 & -4 & 2 & 0 \\ -4 & -5 & -2 & 0 \\ 2 & -2 & -8 & 0 \end{array}\right)$$

$B^1 - B^2 = -\frac{1}{2}B^3 \qquad B\begin{pmatrix}2\\-2\\1\end{pmatrix} = 0_3$

$v_1 = t\begin{pmatrix}2\\-2\\1\end{pmatrix} \qquad E(A;1) = \langle(2,-2,1)\rangle.$

② $\lambda_2 = \lambda_3 = -3 \qquad v = \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}$

$$\begin{pmatrix} 4 & -4 & 2 \\ -4 & 4 & -2 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

Augmented Matrix: $\left(\begin{array}{ccc|c} 4 & -4 & 2 & 0 \\ -4 & 4 & -2 & 0 \\ 2 & -2 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 4 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$4\alpha - 4\beta + 2\gamma = 0 \qquad \gamma = -2\alpha + 2\beta.$

$$v = \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\\-2\alpha+2\beta\end{pmatrix} \qquad \alpha \neq 0 \quad \beta \neq 0$$

$$= \begin{pmatrix}\alpha\\0\\-2\alpha\end{pmatrix} + \begin{pmatrix}0\\\beta\\2\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\\-2\end{pmatrix} + \beta\begin{pmatrix}0\\1\\2\end{pmatrix}$$

Eigenspace: $\begin{pmatrix}x\\y\\z\end{pmatrix} = \alpha\begin{pmatrix}1\\0\\-2\end{pmatrix} + \beta\begin{pmatrix}0\\1\\2\end{pmatrix} \quad \forall \alpha, \beta \in \mathbb{R}$

$x = \alpha. \quad y = \beta. \quad z = -2\alpha + 2\beta$

$2x - 2y + z = 0.$

求 Range 也可这样

Diagonalisation of A.

$\lambda_1 = 6.\quad \underline{v_1} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

$\lambda_2 = \lambda_3 = -3.\quad \underline{v_2} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \underline{v_3} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

linearly independent.

take $P = (v_1 ; v_2 ; v_3) = \begin{pmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & -2 & 2 \end{pmatrix}$

$$D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

Then $P^{-1}AP = D$.

Caculate P^{-1}

$$\begin{pmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & -2 & 2 \end{pmatrix} \to \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 1 & -2 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$P^{-1} = \frac{1}{9}\begin{pmatrix} 2 & -2 & 1 \\ 5 & 4 & -2 \\ 4 & 5 & 2 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 6 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix} = D$$

Orthogonal

$\{v_1\ v_2\ v_3\}$ Three orthogonal Eigenvectors

Normalised: $\hat{v_i} = \dfrac{v_i}{\|v_i\|}$

$\{\hat{v_1}\ \hat{v_2}\ \hat{v_3}\}$ three orthonormal eigenvectors

$\hat{v_i}^T \hat{v_j} = 0 \quad i \neq j$

$\hat{v_i}^T \hat{v_j} = 1 \quad i = j$

Summary

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Diagonalise matrix P

$P = [\ \hat{v}_1 \ ; \ \hat{v}_2 \ ; \ \hat{v}_3\ ]$

since $P^{-1}AP = D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

But $P^TP = I$

$\Rightarrow \quad P^{-1} = P^T$

$\Rightarrow \quad P^TAP = D$

LINEAR TRANFORMATION

MAPPINGS BETWEENS VECTOR SPACES V AND W

- $W = V$ is possible
- Usually described by Matrix.

Definition: $T: V \to W$ is a linear transformation if for All $v_1\ v_2 \in V$, $k \in \mathbb{R}$

$\boxed{T(v_1 + v_2) = T(v_1) + T(v_2)}$ & $\boxed{T(kv_1) = kT(v_1)}$

Equivalently:

$T(v_1 + kv_2) = T(v_1) + kT(v_2)$

$T: V \to W$

$v_1 \in V \Rightarrow T(v_1) \in W$

$T(v_1 \underset{\text{in } V}{+} kv_2) = T(v_1) \underset{\text{in } W}{+} kT(v_2) \quad \forall\ v_1, v_2 \in V$

Notice: $T(\underset{V}{0}) = \underset{W}{0}$

Summary

5

Test for linear Transformation

eg: $T = \mathbb{R}^2 \to \mathbb{R}^2$

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+y+1 \\ x-y \end{bmatrix}$$

$$T(0) = T\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2\cdot 0+0+1 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ T cannot be a linear transformation.

eg: $V = \mathbb{R}$ $W = M_{2\times 2}(\mathbb{R})$

$$T_1(x) = \begin{bmatrix} x & 0 \\ 0 & 1 \end{bmatrix} \quad T_2(x) = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix}$$

In V, "$\underline{0}$" $= 0$. In W, "$\underline{0}$" $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

先排除0

$$T_1(\underline{0}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \underline{0}$$

$\therefore T_1$ is not a L.T. Let $\underline{v}_1 = x_1$ $v_2 = x_2$

$T_2(v_1 + kv_2) = T_2(x_1 + kx_2)$

$$T_2(\underline{v}_1 + kv_2) = T_2(x_1 + kx_2) = \begin{bmatrix} x_1 + kx_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & 0 \\ 0 & 0 \end{bmatrix} + k\begin{bmatrix} x_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= T_2(x_1) + kT_2(v_2)$$

$$T_2(\underline{v}_1 + kv_2) = T_2(v_1) + kT_2(v_2)$$

T_2 is a L.T.

eg: $V = \mathbb{R}^3$ $W = \mathbb{R}$

$$T_3\begin{bmatrix} x \\ y \\ z \end{bmatrix} = xyz \quad T_4\begin{bmatrix} x \\ y \\ z \end{bmatrix} = x+y+z$$

Both cases: $T(0) = 0$

T_3: take $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\underline{v} + \underline{v} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ $T_3(v+v) = 2\times 2\times 2 = 8$

But $T_3(\underline{v}) + T_3(\underline{v}) = 2T_3(v) = 2T_3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 2 \neq 8$

$\therefore T_3(v+v) \neq T_3(\underline{v}) + T_3(\underline{v})$

Summary

T_3 is not a L.T.

$$T_4(\underline{v}_1 + kv_2) = (x_1 + kx_2) + (y_1 + ky_2) + (z_1 + kz_2)$$

$$= (x_1 + y_1 + z_1) + k(x_2 + y_2 + z_2)$$

$\therefore T_4$ is a L.T.