Coursework Title

Report 3: RC Circuits

Office use only		
	Class (E.g. C26-C27)	B28
	Class Group (E.g. Group 1)	Group 5
	Grade awarded	
	Penalty	
	Final grade	
	Marker's Signature	

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

COURSEWORK SUBMISSION FORM

TO THE STUDENT: Please complete the boxes, and submit the form together with two copies of your essay/coursework (one printed copy and one electronic copy).

Name (s)	**ID Number (s)**	**Date Handed In**
Mingdong He **Yu'ang Li** **Tianyu Sun**	**20126521** **20126362** **20126367**	**2018/12/21**
Module Title: Foundation Science A		**Module Convenor** Neil Arnold
Coursework Title: RC Circuits		**Module Code** CELEN039

Compulsory

I/We have read the section relating to plagiarism in the University's Regulations and confirm that the attached submission is my/our own work. I/We understand that 5 marks per working day will be deducted from the final mark for lateness, unless an extension form has been authorised and is attached, e.g. a mark of 42 minus 5 marks changes to 37.
The deadline for handing in coursework is 4pm.

Signature (s)

..

..

Optional

I/We give permission for the attached piece of work to be used for future research and training purposes.

Signature (s)

....

--
OFFICE USE ONLY
Copies received 1 or 2

Date & Time Received	**Penalty**	**Extenuating Circumstances**	**Evidence Attached**
Late: YES / NO		**YES / NO**	**YES / NO**

SUMMARY:

The voltage of the capacitor and the time taken for the potential difference across the capacitor to drop to the voltage values was measured to investigate the hypothesis that the change of voltage across the capacitor with respect to time can be accurately modelled with the equation. The voltage from the power supply was adjust to 20V. After the capacitor charged up, disconnected the power supply from the circuit and the time taken for the potential difference across the capacitor to drop to the voltage values was measured by a stopwatch consistently. The procedure was repeated for three times. The predicted times were agreed with the measured time for all of the results, when reasonable uncertainties from measurement and modelling assumptions were considered and therefore these experimental results could be used to support the hypothesis.

OBJECTIVES:

The objective of this experiment is to example how the voltage across a capacitor changes with time, which could be modelled by the following equation, which states that;

The voltage across the capacitor is equal to the initial voltage across the capacitor times e to the power of minus time divided by the product of resistance and capacitance, where e is the natural logarithm.

$$V_C = V_0 e^{-t/RC}$$

Eqn.1

Where
V_C= the voltage across the capacitor (V)
V_0= the voltage across the capacitor when t= 0s (V)
t = time from the beginning of discharge (s)
R= resistance in the circuit (Ω)
C= capacitance of the capacitor (F)

INTRODUCTION:

In 1845, Gustav Robert Kirchhoff proposed the Kirchhoff's Rules, which can be used to deal with complicated circuits. In this experiment, we investigated whether the change of voltage across the capacitor with respect to time can be accurately modelled with the equation:

$$V_C = V_0 e^{-t/RC} \quad \text{Eqn.1}$$

When a capacitor was already charged, and it was then allowed to discharge as shown above. The voltage across the resistor at any instant equals that across the capacitor:

$$IR = \frac{Q}{C} \quad \text{Eqn.2}$$

Since the charge left the capacitor

$$I = -\frac{dQ}{dt} \quad \text{Eqn.3}$$

Then we can rewrite the Eqn.2

$$-\frac{dQ}{dt} R = \frac{Q}{C} \quad \text{Eqn.4}$$

Rearranging the above

$$\frac{dQ}{Q} = -\frac{dt}{RC} \quad \text{Eqn.5}$$

Therefore we can integrate the Eqn.5 from $t = 0$ when the charge on the capacitor is Q_0, to some time t later when the charge is Q

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = \int_0^t -\frac{dt}{RC} \quad \text{Eqn.6}$$

Then we can get

$$ln\frac{Q}{Q_0} = -\frac{t}{RC} \quad \text{Eqn.7}$$

$$Q = Q_0 e^{-t/RC} \quad \text{Eqn.8}$$

Since

$$V = \frac{Q}{C} \quad \text{Eqn.9}$$

Therefore the voltage across the capacitor as a function of time is

$$V_C = V_0 e^{-t/RC} \quad \text{Eqn.1}$$

Thus, the charge on the capacitor, and the voltage across it, decrease exponentially in time with a time constant RC, so the time constant $\tau = \mathrm{RC}$ give us a measure of in an RC circuit.

In order to depict the measurements, we rearranged the Eqn.1, which made the time as the dependent value and the voltage across the capacitor as the independent value.

$$t = RC(-ln\frac{V_c}{V_0}) \quad \text{Eqn.10}$$

To model this system, the following modelling assumptions were made:

- The resistance and capacitor remain constant
- The temperature of the circuit is not changeable
- The resistance of the circuit can be ignored

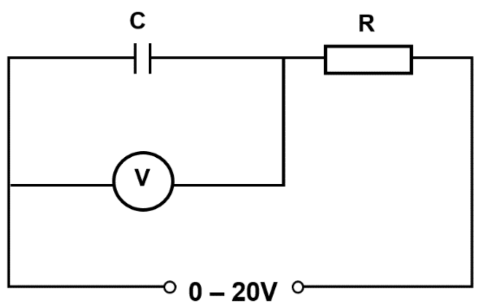

Figure 1. Circuit diagram of the RC circuit

APPARATUS

- 22 kΩ Resistor
- 2200 μF Capacitor
- Prototype board
- Voltmeter
- Power supply
- stopwatch

PROCEDURE

The apparatus was assembled, with a resistor and a capacitor in series. Adjusted the power supply to 20 V, and the capacitor was charged up, indicated by the analogue voltmeter having a reading greater than 6 V. Then disconnected the power supply from the circuit, and the time taken for the potential difference across the capacitor to drop to the voltage values was measured by a stopwatch consistently. The procedure was repeated for three times.

RESULTS:

As an example, the natural log of the voltage ratio was calculated by using the following equation when the voltage across the capacitor was 5.5 V

$$\ln\left(\frac{\mathbf{V}_c}{\mathbf{V}_0}\right) = \ln\left(\frac{5.5}{6}\right) = -0.087 \quad \text{Eqn.11}$$

The measured time was the average of the time measured in three times, where the measurements were 2.80 s, 2.74s, 2.91s, respectively.

$$\mathrm{t_m} = \frac{\boldsymbol{t_1+t_2+t_3}}{3} = \frac{\mathbf{2.82+2.74+2.91}}{3} = 2.82\ s \quad \text{Eqn.12}$$

The predicted time was calculated by using the Eqn.10

$$t = RC\left(-ln\frac{V_c}{V_0}\right) = 1.49 \times 10^4 \times 0.0022\left(-ln\frac{5.5}{6}\right) = 2.85\ s \quad \text{Eqn.13}$$

Table 1. Time taken for the capacitor to discharge.

Measurement no	**Voltage $\mathbf{V}_c$ (V)**	$\boldsymbol{ln\left(\frac{V_c}{V_0}\right)}$	**Time (s)** $\boldsymbol{t_1}$	$\boldsymbol{t_2}$	$\boldsymbol{t_3}$	$\boldsymbol{t_m}$	$\boldsymbol{t_p}$
1	6.0	0.00	0.00	0.00	0.00	0.00	0.00
2	5.5	-0.087	2.80	2.74	2.91	2.82	2.85
3	5.0	-0.182	6.34	6.03	5.70	6.02	5.97
4	4.5	-0.288	9.50	9.03	9.34	9.29	9.44
5	4.0	-0.405	14.54	13.65	13.15	13.78	13.28
6	3.5	-0.539	18.26	18.37	17.85	18.16	17.67
7	3.0	-0.693	23.10	23.37	23.13	23.20	22.72
8	2.5	-0.875	30.27	29.34	29.15	29.59	28.69
9	2.0	-1.099	38.04	36.57	35.73	36.78	36.03
10	1.5	-1.386	48.41	46.40	45.54	46.78	45.43
11	1.0	-1.792	62.00	57.49	60.00	59.83	58.74

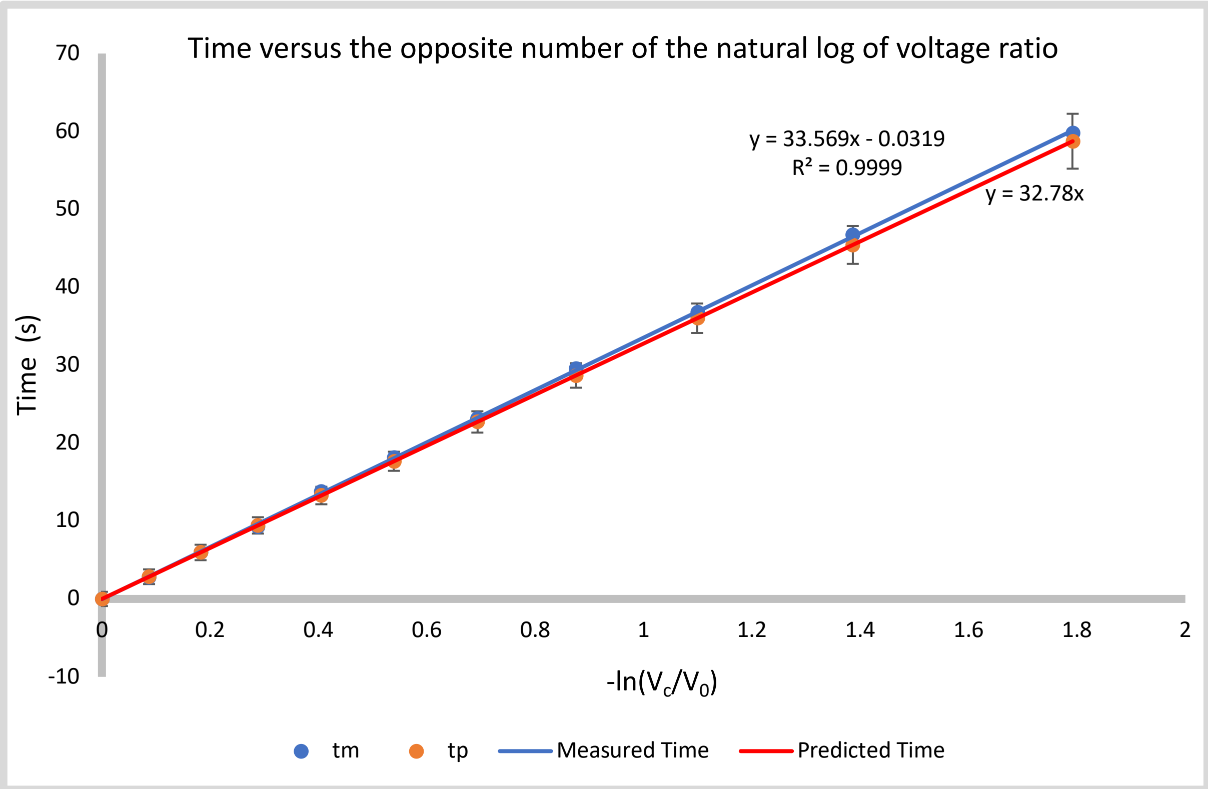

Figure.2

UNCERTAINTY ANALYSIS:

The uncertainty associated with measurements of voltage δV can be calculated using the following equation;

$$\delta V_0 = \sqrt{\left(\frac{0.25}{\sqrt{6}}\right)^2 + (0.01V)^2} = \sqrt{\left(\frac{0.25}{\sqrt{6}}\right)^2 + (0.01 \times 6)^2} = 0.1184 \text{ V} \qquad \text{Eqn.14}$$

For example, for measurement 2, the uncertainty can be calculated to be

$$\delta V_C = \sqrt{\left(\frac{0.25}{\sqrt{6}}\right)^2 + (0.01V)^2} = \sqrt{\left(\frac{0.25}{\sqrt{6}}\right)^2 + (0.01 \times 5.5)^2} = 0.1159\, V \qquad \text{Eqn.14}$$

The uncertainty associated with the resistance of a resistor is 1% of the value of the component.

$$\delta R = 0.01R = 149 \;\; \Omega \qquad \text{Eqn.15}$$

The uncertainty associated with the capacitance of a capacitor is 1% of the value of the component.

$$\delta C = 0.01C = 0.000022 \;\; \text{F} \qquad \text{Eqn.16}$$

Let $q = ln\left(\frac{V_c}{V_0}\right)$, so we need to calculate the δq. According to the Final Law of Uncertainty, for measurement 2

$$\frac{\partial q}{\partial V_C} = \frac{1}{\frac{V_C}{V_0}} \times \frac{1}{V_0} = \frac{1}{V_C} \qquad \text{Eqn.17}$$

$$\frac{\partial q}{\partial V_0} = \frac{1}{\frac{V_C}{V_0}} \times \frac{V_C}{{V_0}^2} = \frac{1}{V_0} \qquad \text{Eqn.18}$$

Therefore

$$\delta q = \delta ln\left(\frac{\mathrm{V}_c}{\mathrm{V}_0}\right) = \sqrt{(\frac{\partial q}{\partial V_C}\delta V_C)^2 + (\frac{\partial q}{\partial V_0}\delta V_0)^2} = \sqrt{(\frac{\delta V_C}{V_C})^2 + (\frac{\delta V_0}{V_0})^2} \qquad \text{Eqn.19}$$

$$\delta ln\left(\frac{\mathrm{V}_c}{\mathrm{V}_0}\right) = \sqrt{(\frac{\delta V_C}{V_C})^2 + (\frac{\delta V_0}{V_0})^2} = \sqrt{(\frac{0.1159}{5.5})^2 + (\frac{0.1184}{6.0})^2} = 0.028 \qquad \text{Eqn.20}$$

The uncertainty associated with the measured time, t, can be calculated using the following equation;

$$\delta t = \sqrt{\left(\frac{0.005}{\sqrt{3}}\right)^2 + (0.1)^2 + (0.001t)^2} = \sqrt{\left(\frac{0.005}{\sqrt{3}}\right)^2 + (0.1)^2 + (0.001 \times 2.82)^2} \qquad \text{Eqn.21}$$

$$\delta t = 0.1001\ s$$

According to the Final Law of uncertainty, for Eqn.10,

$$\frac{\partial t_p}{\partial R} = C(-ln\frac{V_C}{V_0}) \qquad \text{Eqn.22}$$

$$\frac{\partial t_p}{\partial C} = R(-ln\frac{V_C}{V_0}) \qquad \text{Eqn.23}$$

$$\frac{\partial t_p}{\partial V_C} = -RC\frac{1}{V_C} \qquad \text{Eqn.24}$$

$$\frac{\partial t_p}{\partial V_0} = -RC\frac{1}{V_0} \qquad \text{Eqn.25}$$

Therefore,

$$\delta t_p = \sqrt{(\frac{\partial t_p}{\partial R}\delta R)^2 + (\frac{\partial t_p}{\partial C}\delta C)^2 + (\frac{\partial t_p}{\partial V_C}\delta V_C)^2 + (\frac{\partial t_p}{\partial V_0}\delta V_0)^2} \qquad \text{Eqn.26}$$

$$\delta t_p = \sqrt{(C(-ln\frac{V_C}{V_0})\delta R)^2 + (R(-ln\frac{V_C}{V_0})\delta C)^2 + \left(-RC\frac{1}{V_C}\delta V_C\right)^2 + \left(-RC\frac{1}{V_0}\delta V_0\right)^2} = 0.947\ s \qquad \text{Eqn.27}$$

The standard uncertainty was calculated to be +/- 0.947, and is shown in the second row of column 5 of Table 2. All figures in Table 2 are given to four decimal places.

Table 2. Uncertainties associated with measured and predicted values.

Measurement no	δV (V)	$\delta ln\left(\frac{V_c}{V_0}\right)$	Time (s) δt_m	δt_p
1	0.1184	0.0280	0.1000	0.9150
2	0.1159	0.0290	0.1001	0.9470
3	0.1136	0.0300	0.1002	0.9900
4	0.1115	0.0320	0.1005	1.0470
5	0.1096	0.0340	0.1010	1.1230
6	0.1079	0.0370	0.1017	1.2260
7	0.1064	0.0410	0.1027	1.3680
8	0.1051	0.0460	0.1043	1.5750
9	0.1040	0.0560	0.1066	1.8930
10	0.1032	0.0720	0.1104	2.4320
11	0.1025	0.1040	0.1166	3.5230

DISCUSSION:

It is clear from examining Table 1&2 all of the measured time values comes within the corresponding the predicted time value range. Figure 2. Shows a graphical definitive trend that as the value of the natural log of voltage ratio increases the measured time becomes greater and all the measured time values are greater than the predicted values. This demonstrates there is an issue with either the mathematical model used for this experiment or an error in the experimental technique. When reasonable uncertainties from measurement and modelling assumptions were considered and therefore these experimental results could be used to support the hypothesis. More than half of the results are larger than expected, therefore the most likely causes of this discrepancy are:

- The circuit has resistance which we ignored in our assumption
- As the experiment went on, the temperature of the resistor increased, therefore the whole resistance increased.

For Figure.2 we can obtain the equation which can best depict the measured values

$$y = 33.569x - 0.0319 \qquad \text{Eqn.28}$$

Compare this to equation (10)

$$t = RC(-ln\frac{V_c}{V_0}) \qquad \text{Eqn.10}$$

As $y = t$ and x=$(-ln\frac{V_c}{V_0})$ has been plotted the theoretical straight line equation is

$$y = 32.78x \qquad \text{Eqn.29}$$

Table 3. Measured and predicted values for the trendline coefficients.

	Predicted Value	**Measured Value**
Gradient (units)	32.7800 ± 1.9463	33.5694± 1.3961
Intercept (units)	0.0000± 22.0479	-0.0319± 1.0132

Obviously, the predicted and measured values all correspond one another so this experiment can be used to support our initial hypothesis. However, the value of the coefficient of determination, R^2 is 0.9999 and the product moment correlation coefficient, r, is 0.9999, which implies that there is a very strong relationship between the x and y values. The modelled for this relationship is what was expected, there is a clear relationship between time and the voltage across the capacitor.

CONCLUSION:

To conclude, in this experiment, the Kirchhoff's Rules was investigated. All of the results obtained was found to be within the corresponding predicted time. The values of R^2 exactly suggested a strong relationship between the variable drawn in Figure.2.and these experimental results could be used to support the hypothesis. when reasonable uncertainties from measurement and modelling assumptions were considered and these experimental results could be used to support the hypothesis. The key factor which affected the results is most likely to be the temperature of the circuit increased as the experiment went on so that the whole resistance increased. The experiment could be further improved by;

- using a capacitor which have a smaller capacitance to shorten the time of discharging.
- ensuring there is enough space between the two measurements to remain the temperature of the circuit be the same.