Chapter Three

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Exercise One

Question

Verify the expressions given for the mean and variance in (3.2) for statistics based on sums S_+ or S_- of signed scores $s_i, 1 = 1, 2, ..., n$ where the sign associated with each s_i is equally likely to be plus or minus.

Solution

Exercise Two

Question

In Comment 4 on Example 3.10 we asserted that the usual t-statistic could be used in place of S_+ as the test statistics for the Pitman test because there was a one-to-one correspondence between the ordering of the two statistics. Establish that this is so. (Hint: show that the denominator of the t-statistic is invariant under all permutations of the signs of the derivations d_i.)

Solution

Exercise Three

Question

In Comment 3 on Example 3.2 we suggested that for a variety of reasons one should be cautious about extending inferences about heartbeat rates for female students to the population at large. What might some of these reasons be?

Solution

Exercise Four

Question

Using the data in in Table 3.1 for the distribution of the Wilcoxon S when $n = 7$, construct a bar chart like that in Figure 3.1 showing the probability function for S. Discuss the similarity, or lack of similarity, to a normal distribution probability density function.

Solution

Exercise Five

Question

Establish that the permutation distribution of the Wilcoxon signed-rank statistic for testing the hypothesis $H_0 : \theta = 6$, given the observations

$$4 \quad 4 \quad 8 \quad 8 \quad 8 \quad 8 \quad 8$$

has a distribution equivelant to that fo rthe sign test of the same hypothesis. Would this equivelance hold if the null hypothesis was changed to $H_0 : \theta = 7$?

Solution

Exercise Six

Question

Establish nominal 95 percent confidence intervals for the median based on the Wilcoxon signed-rank test for the following data sets. If an appropriate computer program is available use it to comment on the discontinuities at the end points of your estimated intervals based on Walsh averages.

Set I	1	1	1	1	1	3	3	5	5	7	7
Set II	1	2	2	4	4	4	4	5	5	5	7

Solution

Exercise Seven

Question

Form a table of Walsh Averages for the Fisher sentence length data given in Example 3.6, and use it to obtain 95 and 99 percent cofidence intervals.

Solution

Exercise Eight

Question

The numbers of pages in the sample of 12 books given in Exercise 2.5 were

$$126 \quad 142 \quad 156 \quad 228 \quad 245 \quad 246 \quad 370 \quad 419 \quad 433 \quad 454 \quad 478 \quad 503$$

Use the Wilcoxon signed-rank test to test the hypothesis that the mean number of pages in the statistics books in the library from which the sample was taken is 400. Obtain a 95 percent confidence interval for the mean number of pages based on this Wilcoxon test and compare it with the interval obtained using a t-test under an assumption of normality.

Solution

Exercise Nine

Question

Apply the sign test to the data in Example 3.2 for the hypotheses considered there.

Exercise Ten

Question

For the sample of 20 in Example 3.7 if θ is the population median test the hypothesis $H_0 : \theta = 9$ against the alternative $H_1 : \theta \neq 9$ using the sign test by computing any relevant binomial pobabilities directly from the binomial probability formula. It is not necessary to determine the complete distribution to obtain the relevant P value. To perform the test for $H_0 : \theta = 7.5$ against the alternative $H_1 : \theta > 7.5$ additional terms in the distribution will be needed. Either calculate these, or use tables or computer software to carry out the appropriate test.

Exercise Eleven

Question

Before treatment with a new drug 11 people with sleep problems have a median sleeping time of 2 hours per night. A drug is administered and it is known for good scientific reasons that it ahs an effct it will increase sleeping time but some doctors doubt it will have any effect. Are their doubts justified if the hours per night slept by these individuals after taking the drug are:

3.1 1.8 2.7 2.4 2.9 0.2 3.7 5.1 8.3 2.1 2.4

Exercise Twelve

Question

Kimura and Chikuni (1987) give data for lengths of Greenland turbot of various ages sampled from commercial catches in the Bering Sea as aaged and measured by the Northwest and Alaska Fisheries Center. for 12-year-old turbot the numbers of each length were:

Length (cm)	No. of Fish
64	1
65	2
66	1
67	1
68	4
69	3
70	4
71	5
72	3
73	3
75	1
77	6
78	1
83	1

Would you agree with someone who asserted that, on this evidence, the median length of 12-year-old Greenland Turbot was almost certanly between 69 and 72 cm?

Exercise Thirteen

Question

Use the Wilcoxon signed-rank test to test the hypothesis that the median length of 12-year-old turbots is 73.5 using the data in Exercise 12.

Exercise Fourteen

Question

The first application listed in Section 3.7 involved British insurance claims. The 2005 median was £1570. A random sample of 14 claims from a large batch received in the first quarter of 2006 were for the following amounts (in £):

1175 1183 1327 1581 1592 1624 1777 1924 2483 2642 2713 3419 5250 7615

What test do you consider appropriate fo ra shift in median relative to the 2005 median? Would a one-tail test be appropriate? Obtain a 95 percent confidence interval for the median based upon these data. If all amounts were converted to, say, Euros or to $US, would your conclusions be the same?

Exercise Fifteen

Question

Exercise Sixteen

Question

Exercise Seventeen

Question