

Chapter Two

Ryan B. Honea

Exercise One

Question

A new type of intensive physiotherapy is developed for individuals who have undergone spinal surgery. Due to limited hospital resources it can only be given to 3 out of 10 patients. The patients are aged:

15 21 26 32 39 45 52 60 70 82

Explain how a permutation test could be used to investigate whether use of physiotherapy is related to patient age, (i.e.) whether there is a policy to give treatment to younger as opposed to older groups or *vice versa*). If the patients aged 15, 26, and 32 have the intensive physiotherapy find the P -value for a two-tailed test of an appropriate null hypothesis. Comment on your findings.

Solution

Exercise Two

Question

Suppose that the new drug under test in Example 2.1 has all the ingredients of a standard drug at present in use and an additional ingredient that has proved to be of use for a related disease, so that it is reasonable to assume that the new drug will do at least as well as the standard one, but may do better. Formulate the hypotheses leading to an appropriate one-tail test. If the post-treatment ranking of the patents receiving the drug is 1,2,3,6 asses the strength of the evidence against the relevant H_0 .

Solution

Exercise Three

Question

An archaeologist numbers some artifacts 1 to 11 in the order he discovers them. He selects at random a sample of 3 of them. What is the probability that the sum of the numbers on the items he selects is less than or equal to 8? (You do not need to list all combinations of 3 items from 11 to answer this question).

If the archaeologist believed that items belonging to the more recent of two civilizations were more likely to be found earlier in his dig and of his 11 items 3 are identified as belong to that more recent civilization (but the remaining 8 come from an earlier civilization) does a rank sum of 8 for the 3 matching the more recent civilization provide reasonable support for this theory?

Solution

Exercise Four

Question

A library has on its shelves 114 books on statistics. I take a random sample of 12 and want to test the hypothesis that the median number of pages, θ , in all 114 books is 225. In the sample of 12, I note that 3 have less than 225 pages. does this justify the retention of the hypothesis that $\theta = 225$? What should I take as an appropriate alternative hypothesis? What is the largest critical region for a test with $P \leq 0.05$ and what is the corresponding exact P -level?

Solution

Exercise Five

Question

The numbers of pages in the sample of 12 books in Exercise 2.4 were:

126 142 156 228 245 246 370 419 433 454 478 503

Find a confidence interval at a level not less than 95 percent for the median θ /

Solution

Exercise Six

Question

In Sect1.4.1 we associated a confidence interval with a two-tail test. As well as such two-sided confidence intervals, one may define a one-sided confidence interval composed of all parameter values that would not be rejected in a one-tail test. Follow through such an argument to obtain a confidence interval at level not less than 95 percent based on the sign test criteria for the 12 book sample values given in Exercise 2.5 relevant to a test of $H_0 : \theta = \theta_0$ against a one-sided alternative $H_1 : \theta > \theta_0$.

Solution

Exercise Seven

Question

From 6 consenting patients requiring a medical scan, 3 are chosen at random to undergo a positron emission tomography (PET) scan, the others receiving a magnetic resonance imaging (MRI) scan. Image quality is ranked in order by a hospital consultant from 1 (best) to 6 (worst). Describe how you would test H_0 : *scan quality is unrelated to scan method* against (i) H_1 : *PET scans are better* (ii) H_1 : *the scans differ in quality depending on whether they are from PET or MRI*. Interpret the finding that the consultant rates the three PET scans as the three highest quality images.

Solution

Exercise Eight

Question

In Example 2.4 we remarked that a situation could arise where we might reject H_0 for the wrong reason. Explain how this is possible in that example.

Solution

Exercise Nine

Question

State appropriate null and alternative hypotheses for the example from the book of Daniel about the diet in Section 1.1.3. How could you use the ranks to calculate the probability that the four receiving the diet of pulses were ranked 1,2,3,4? Calculate this probability assuming that there were 20 young men involved altogether.

Solution

Exercise Ten

Question

A sample of 12 is taken from a continuous uniform distribution over the interval (0,1). What is the probability that the largest sample value exceeds 0.95? (Hint: Determine the probability that any sample values exceeds 0.95. The condition is met if at least one value exceeds 0.95.)

Solution

Exercise Eleven

Question

A sample of 24 is known to come either from a uniform distribution over the interval (0,10) or else from a symmetric triangular distribution over the same interval (0,10). The sample values are

4.17	8.42	3.02	2.89	9.77	6.06	2.72	5.12	6.00	4.78	2.61	7.20
1.61	5.92	7.25	8.01	4.76	5.36	5.34	7.59	0.66	7.27	3.39	1.40

Use appropriate graphical or other EDA techniques to get an indication as to which of these distributions is the more likely source of the sample.

Solution