Time Series Homework 4
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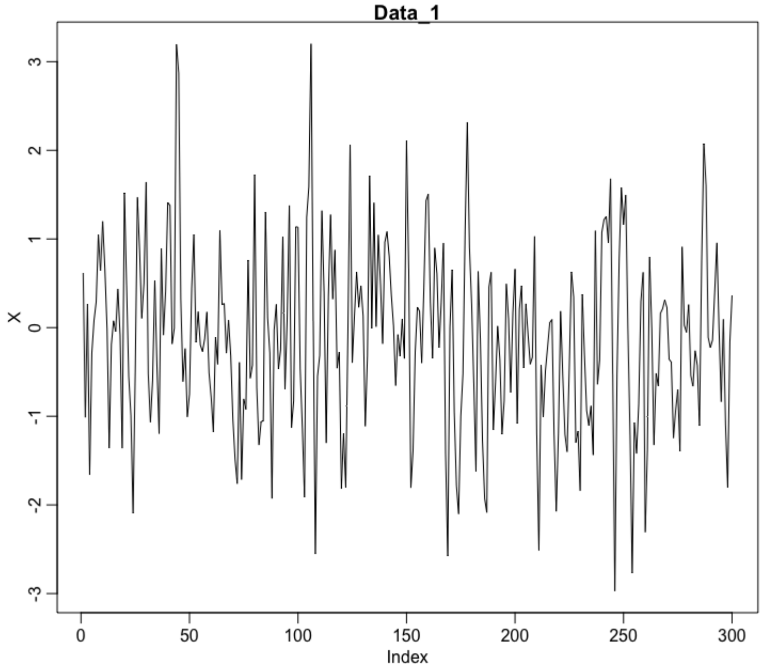
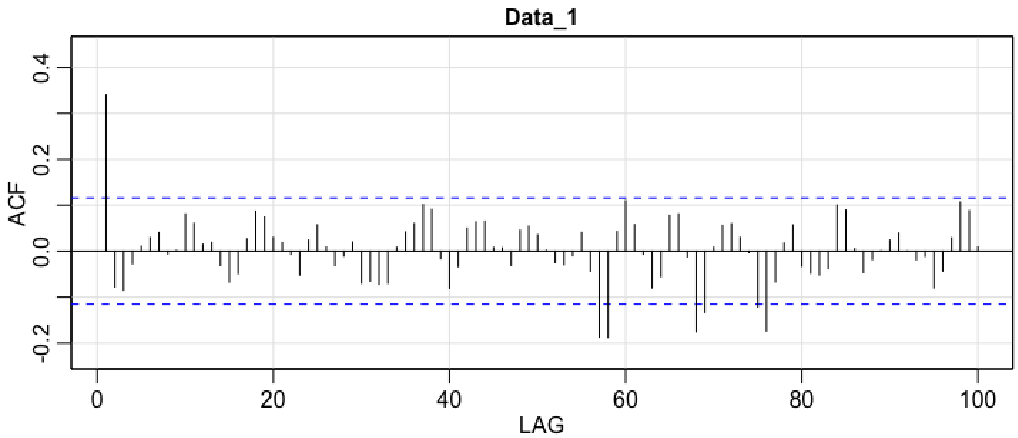
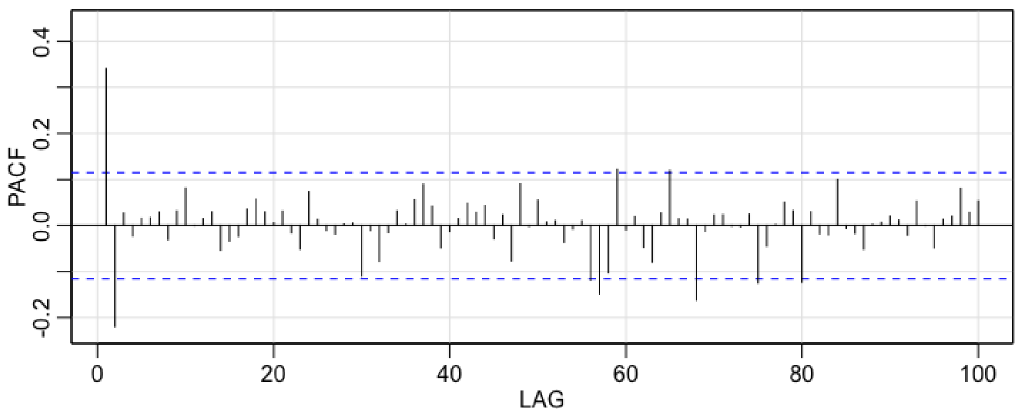
1. Download data sets from D2L. For each data sets plot the ACF and ACVF. Using those plot guess appropriate time series model/models, also provide explanation to support your choice. Maximum two guesses are allowed.

   (a) Data-1

Solution: Based on the initial data plot which shows stationarity, we can do traditional analysis of the ACF and PACF plots.



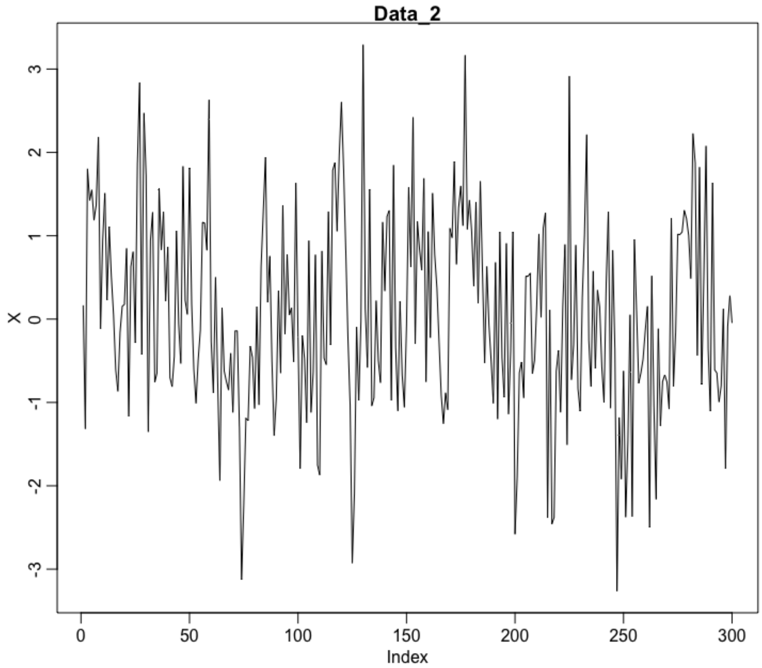


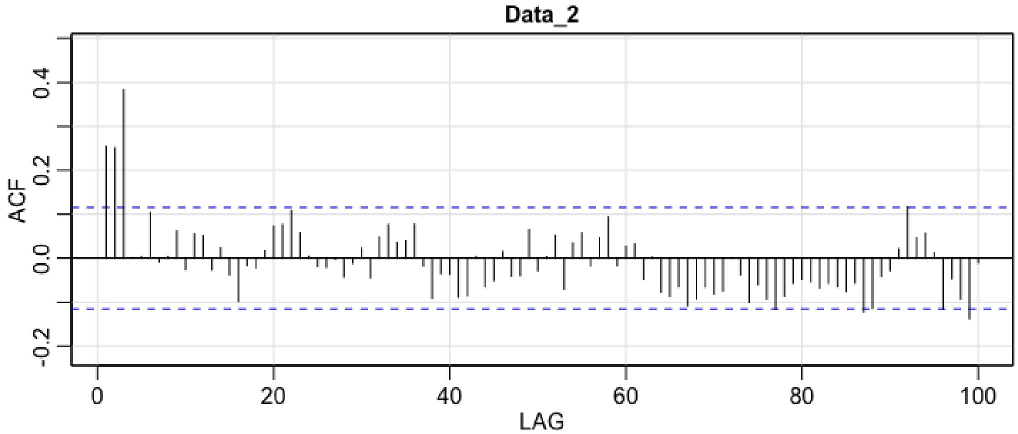


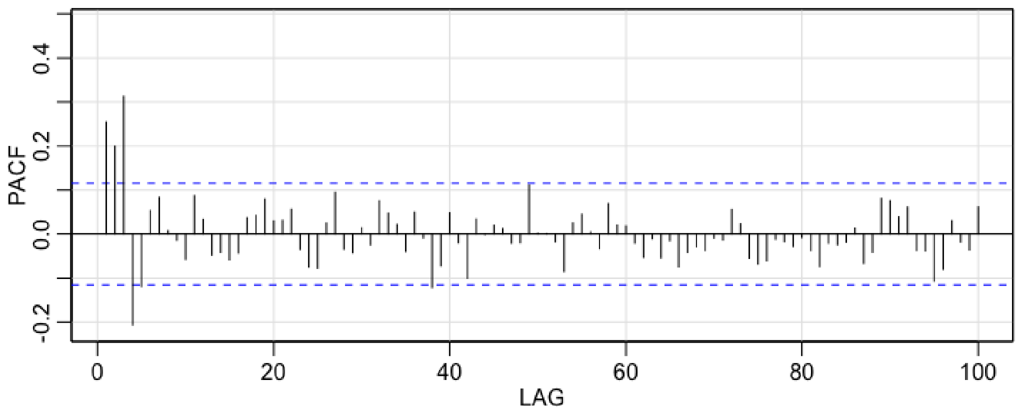
Based on the diminishing result in the ACF plot after lag 1, and the diminishing result in the PACF plot after lag 2, I'd argue that the model is $ARMA(2,1)$. I could also potentially see it being $AR(2)$.

(b) Data-2

Solution: Based on the initial data plot which shows stationarity, we can do traditional analysis of the ACF and PACF plots.



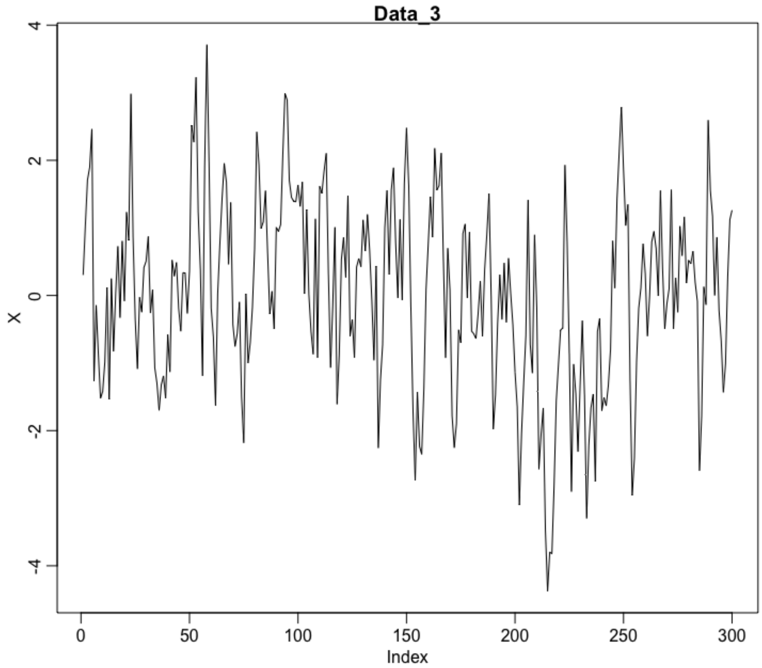


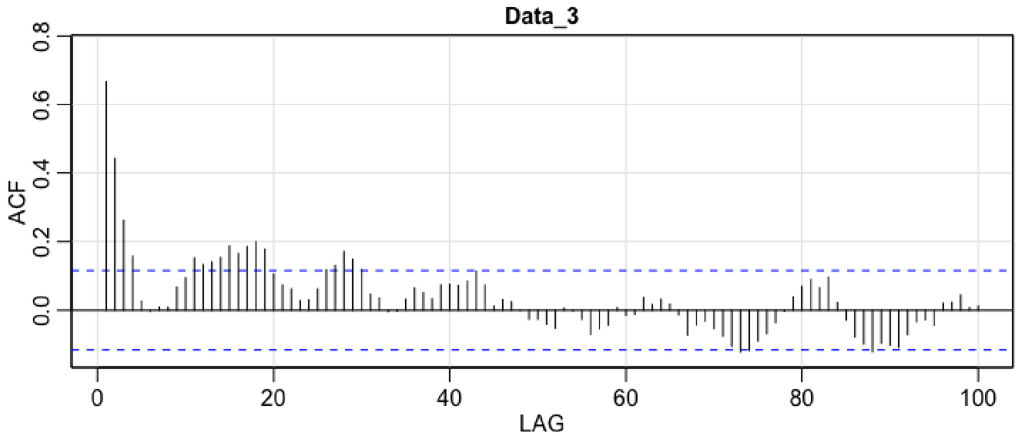


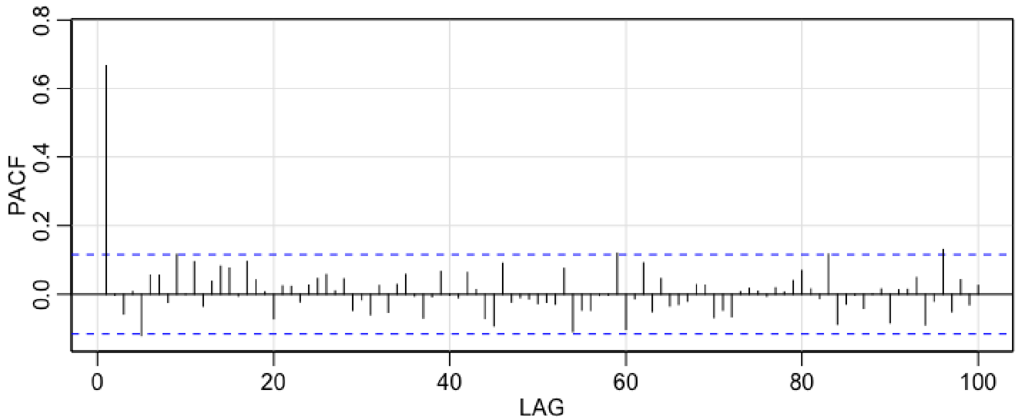
Based on the plots significant lags, with logic similar to on Data-1, I'd argue that the data is $Arma(5, 3)$. However, the significant results in the ACF plot could be due to the AR process and so my second guess would be $AR(5)$.

(c) Data-3

Solution: The initial data plot is not clearly stationary, but with an ADF test with p-value .01, we assume stationarity and so traditional analysis of ACF and PACF plots is appropriate..



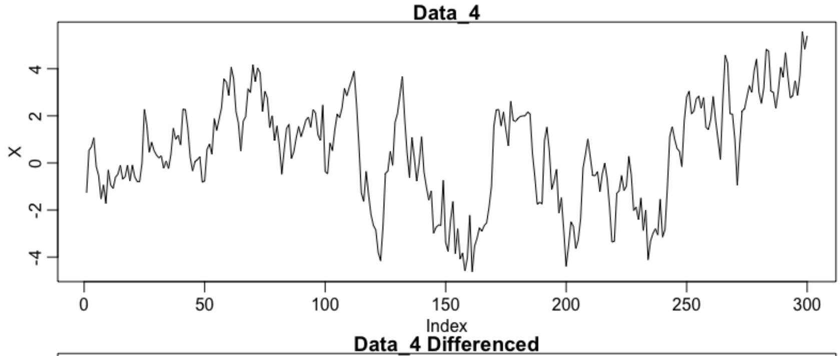


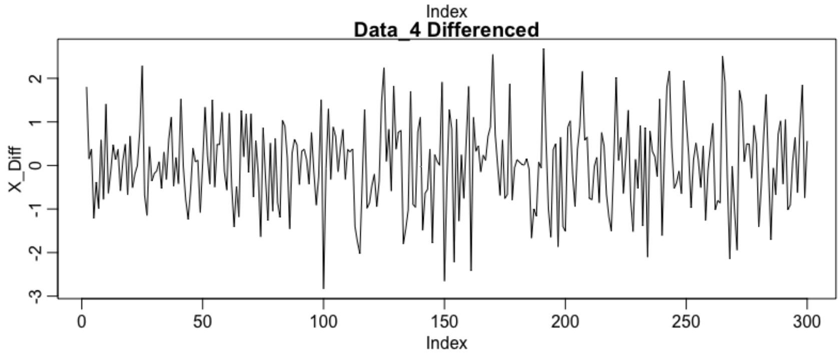


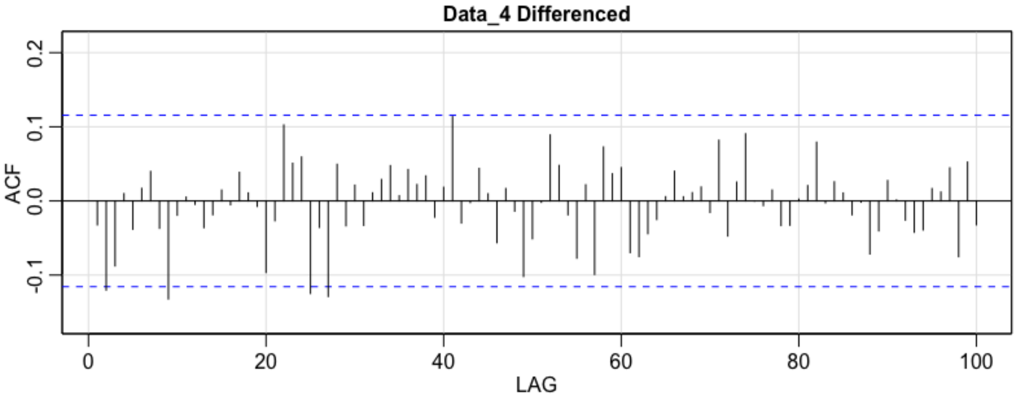
Since the ACF seems to be changing pretty geometrically for the most part and the PACF only jumps at 1, I'd say this is an $AR(1)$ process. It could potentially be $AR(1,1)$.

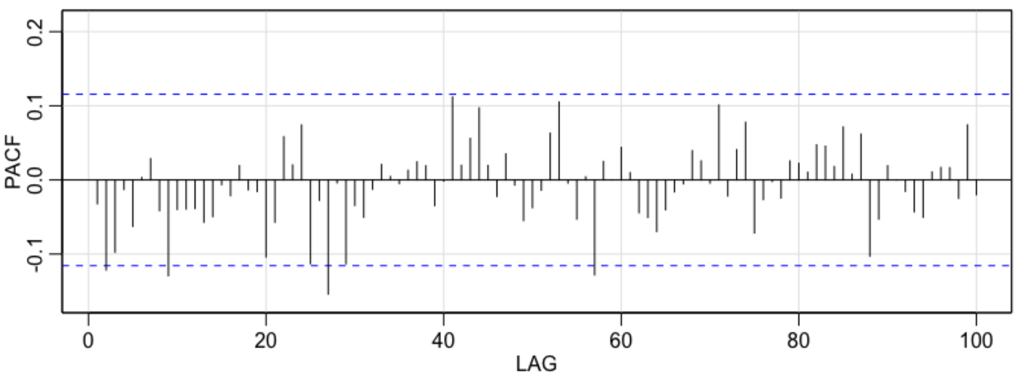
(d) Data-4

Solution: The initial plot is clearly not stationarity and this is verified by an ADF test, so we difference it once and achieve stationarity.





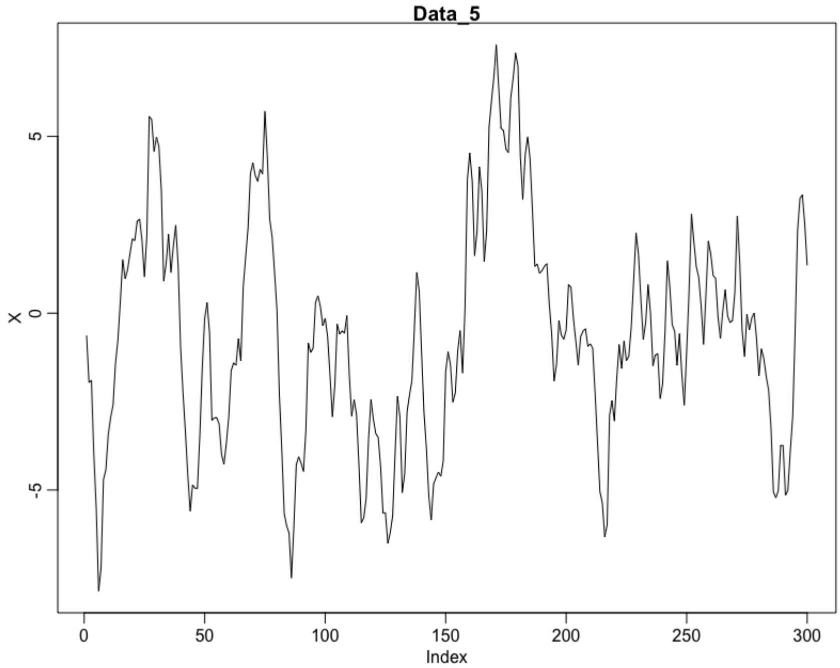


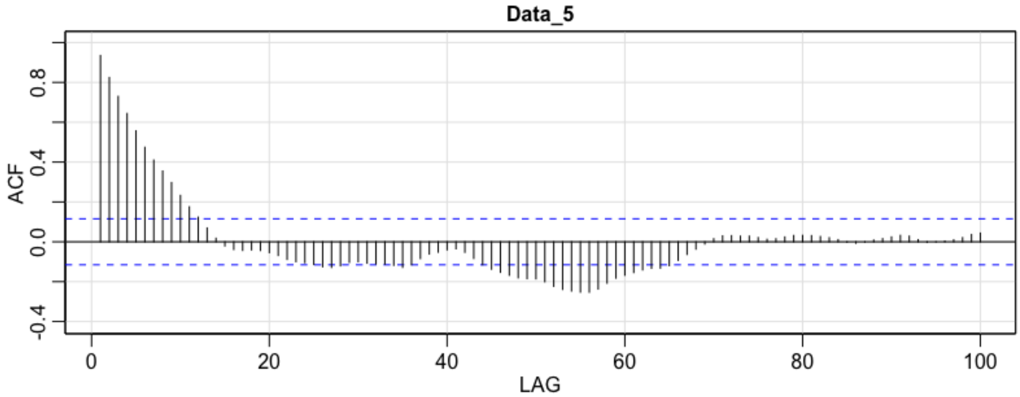


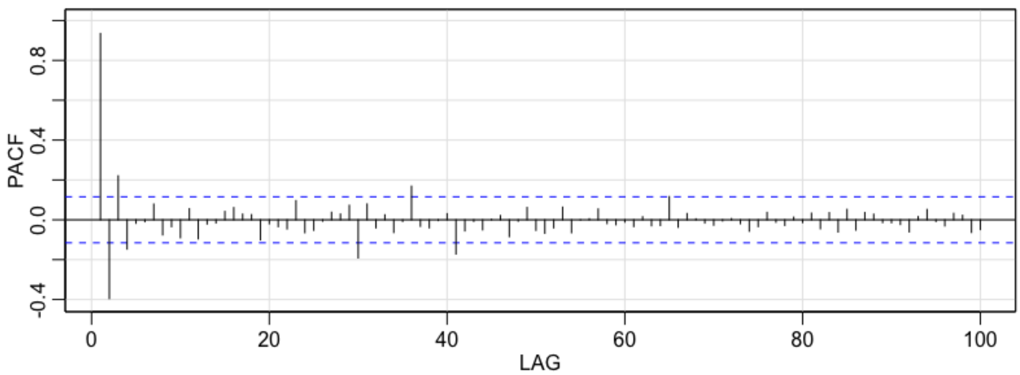
This doesn't appear to have any autocorrelation or partial autocorrelation, so perhaps the data is $ARIMA(1,1,0)$. Another guess is $ARIMA(1,0,1)$.

(e) Data-5

Solution: The initial plot is not obviously stationary but an ADF test on the data shows that it likely is. So traditional analysis should work.



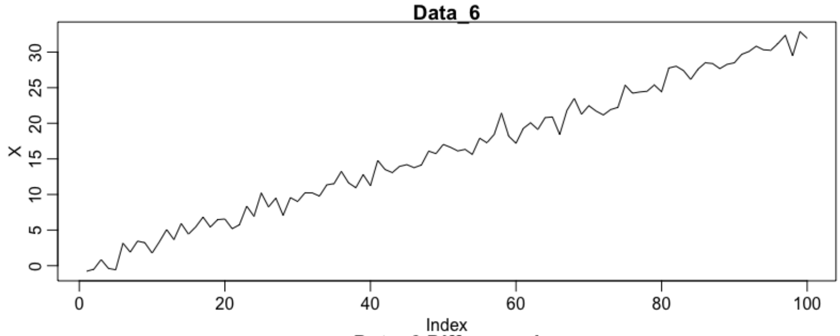


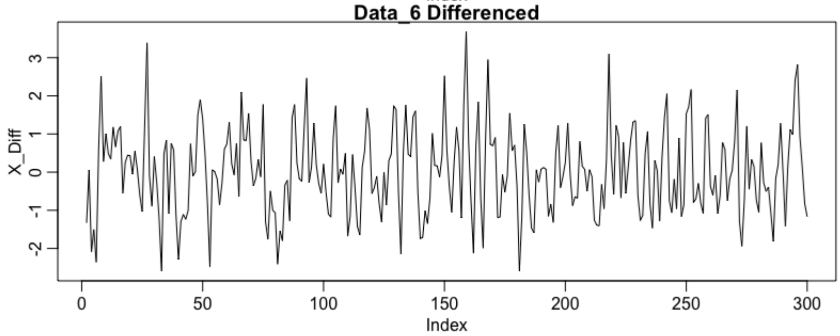


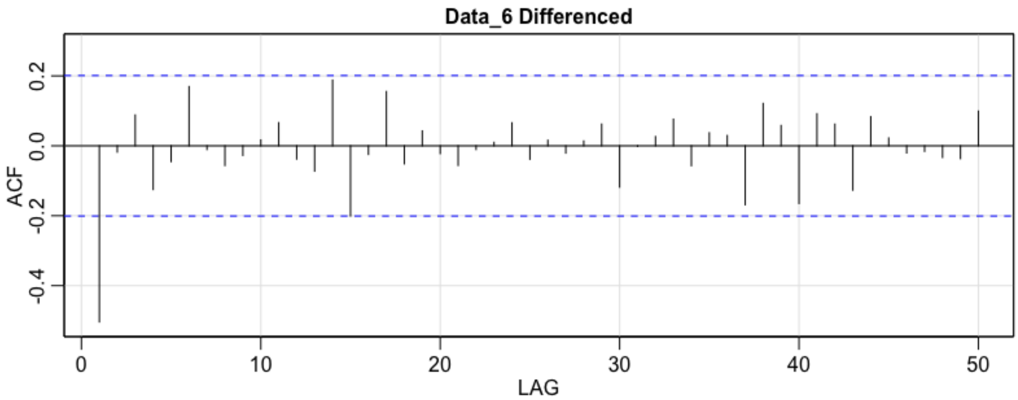
Based on the nearly geometric progression of the ACF, I would say that most of the effect comes from an $AR(4)$ process based on the PACF.

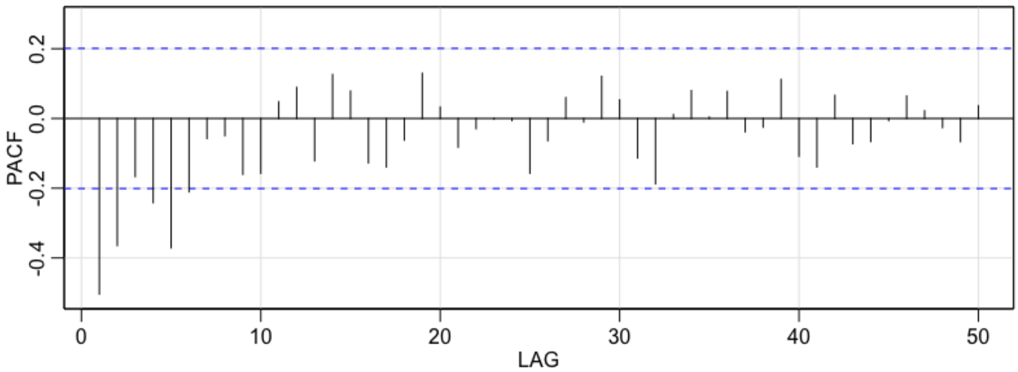
(f) Data-6

Solution: Without even testing, it's clear that this model is not stationary and needs differencing.





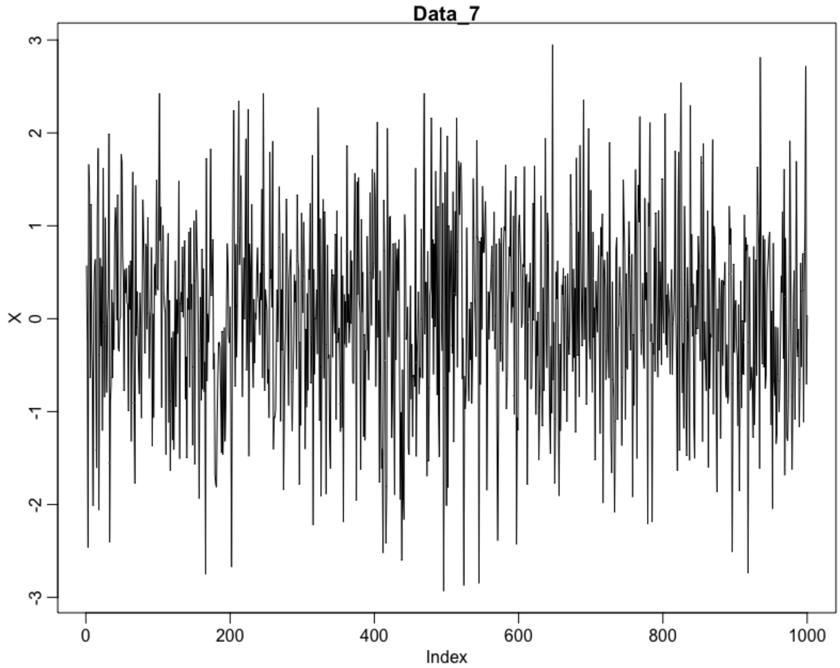


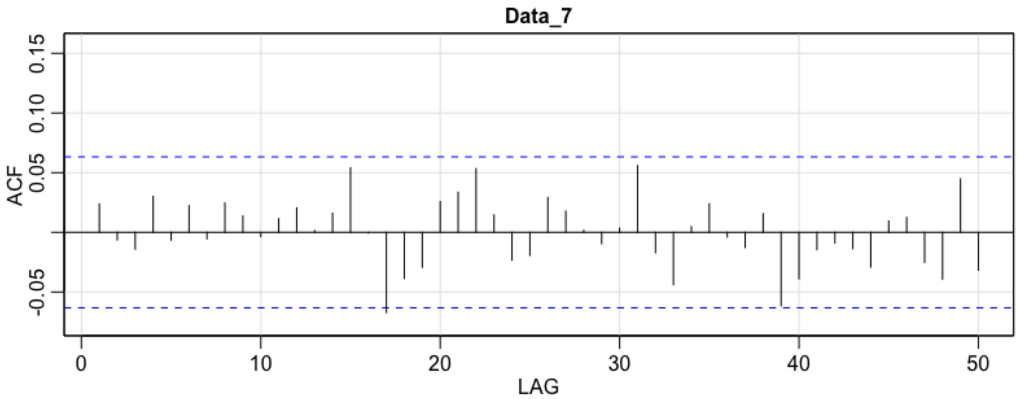


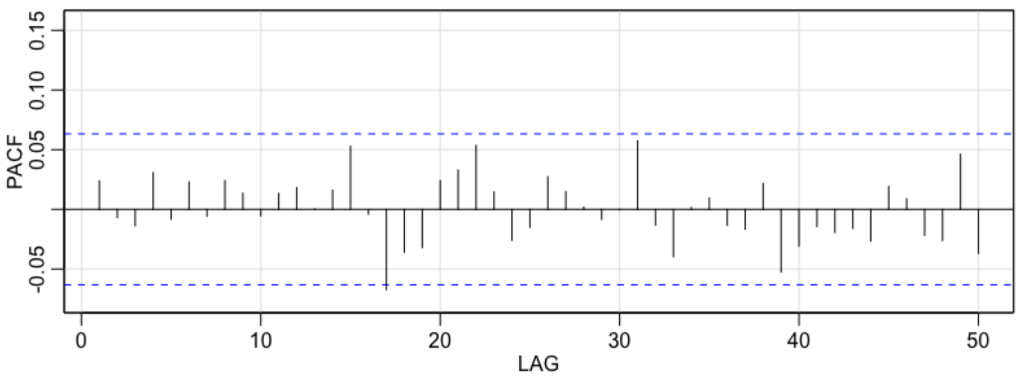
Based on these plots, I would say that this is $ARIMA(5,1,1)$ or maybe $ARIMA(2,1,1)$.

(g) Data-7

Solution: The initial plot shows stationarity and is verified by an ADF test with p-value less than .01. So analysis of the plots should be sufficient.



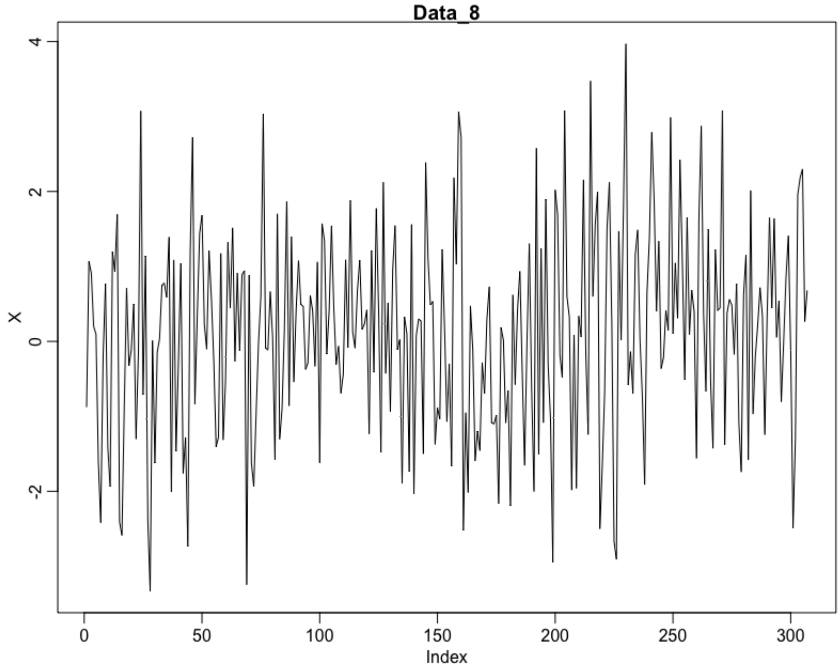


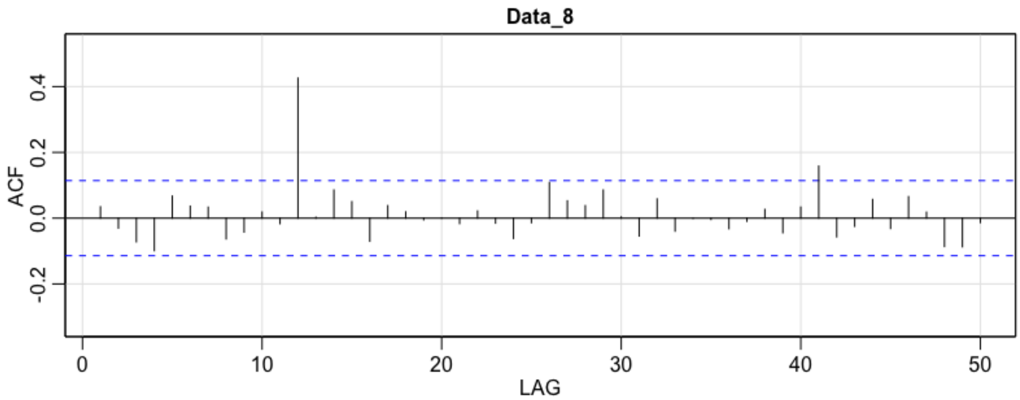


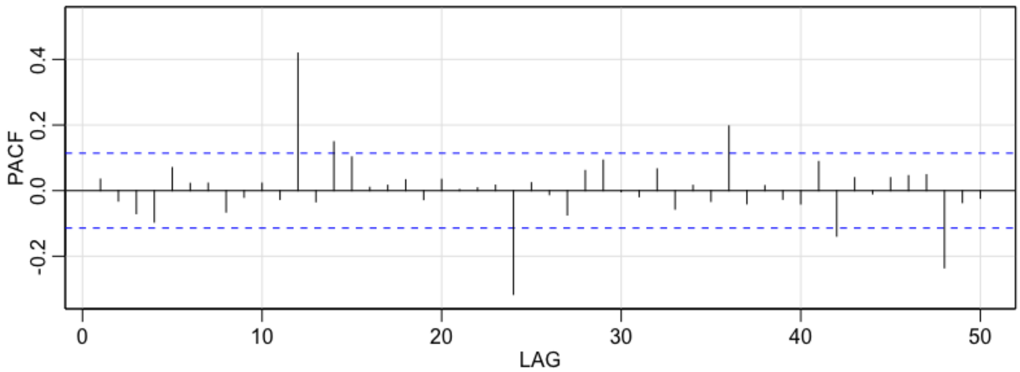
This is just white noise based on these plots.

(h) Data-8

Solution: The data appears stationary and ADF test verifies this.



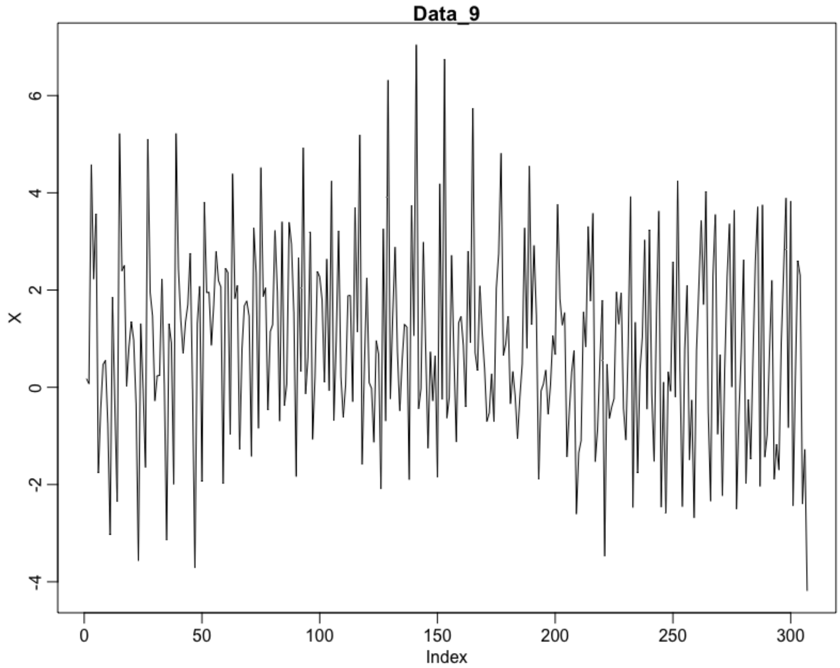


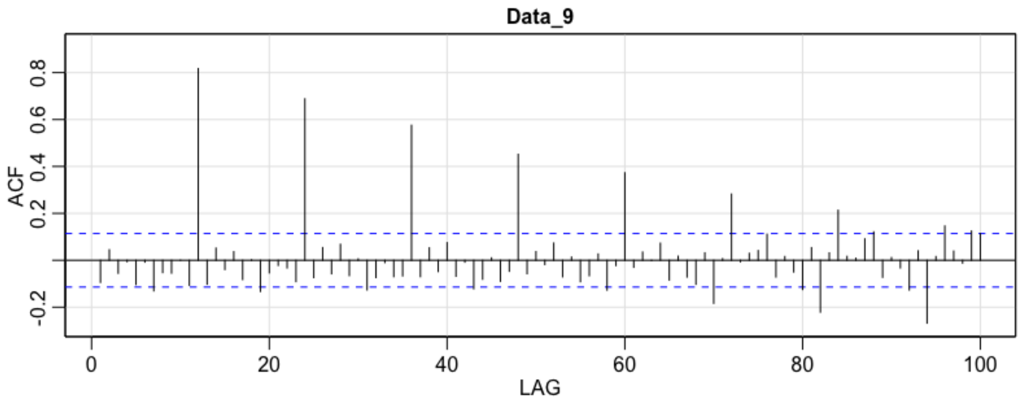


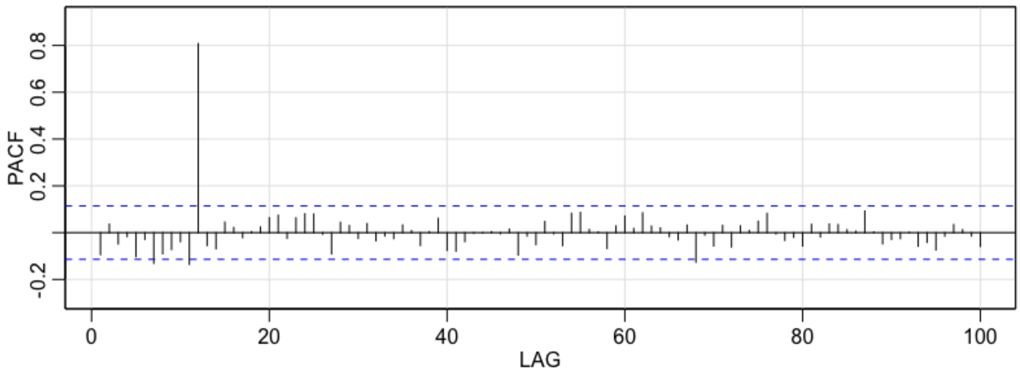
Based on the jumps in ACF and PACF at lag 12, this is definitely seasonal. This is perhaps $ARMA(0,0) \times (2,1)_{d=12}$ or perhaps $ARMA(0,0) \times (2,0)_{d=12}$.

(i) Data-9

Solution: The data is clearly stationary, but an ADF verifies that it likely is.



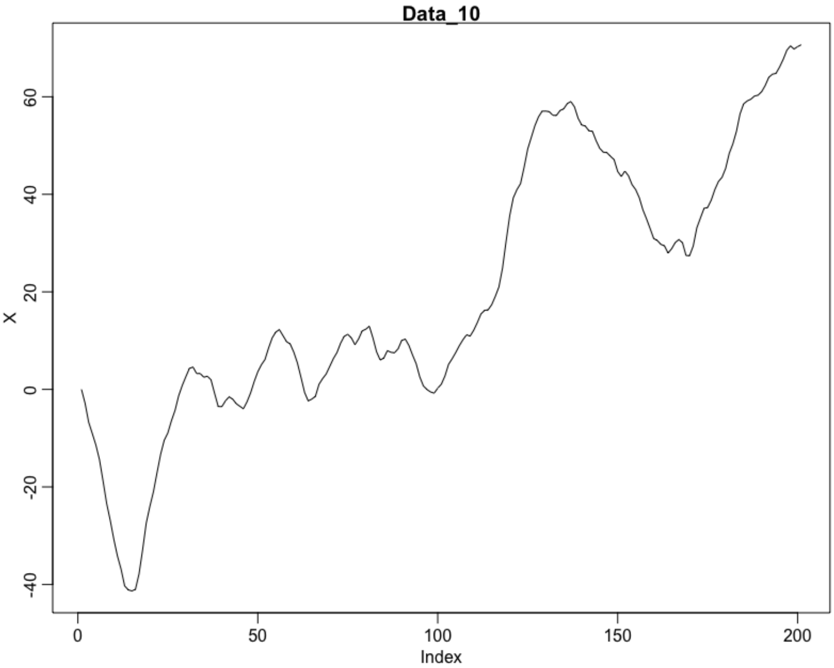


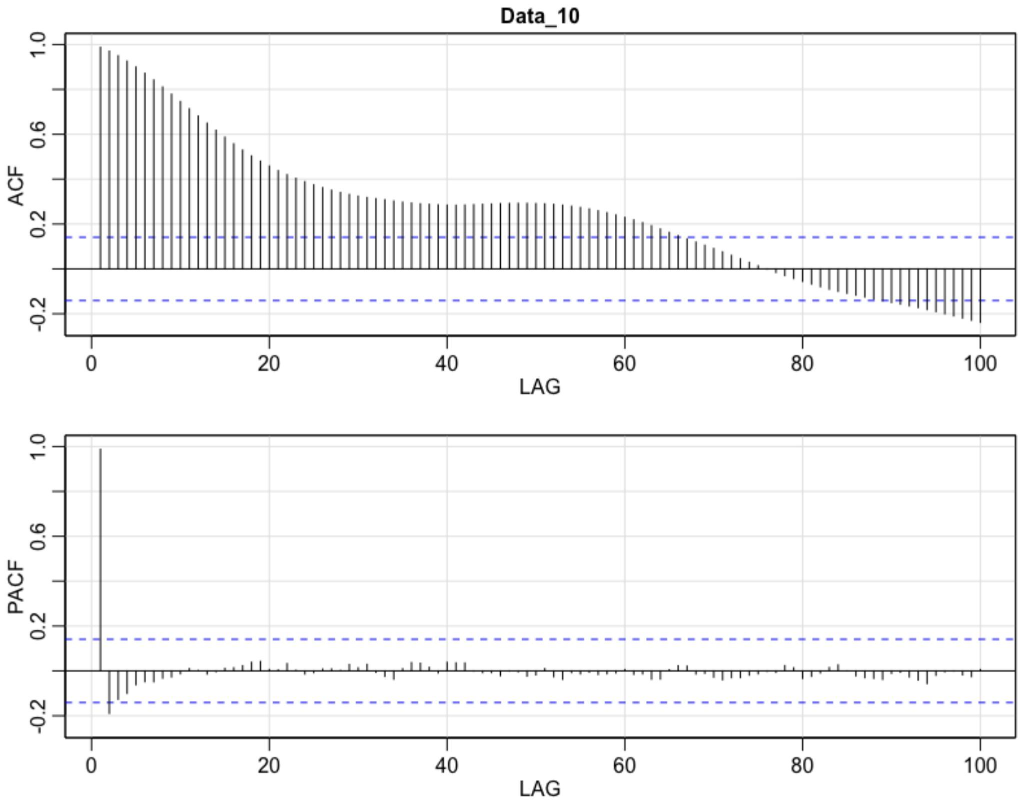


This one isn't so easy, but I would argue that most of the MA effect comes from the seasonal effect within the AR part of the process. I think this could be $ARMA(0,0) \times (1,1)_{d=12}$ or maybe the MA could be dropped leaving just $ARMA(0,0) \times (1,0)_{d=12}$.

(j) Data-10

Solution: Wildly enough, while the plot appears to not be stationary, an ADF test argues that it is. I will believe it, but highly doubt it.





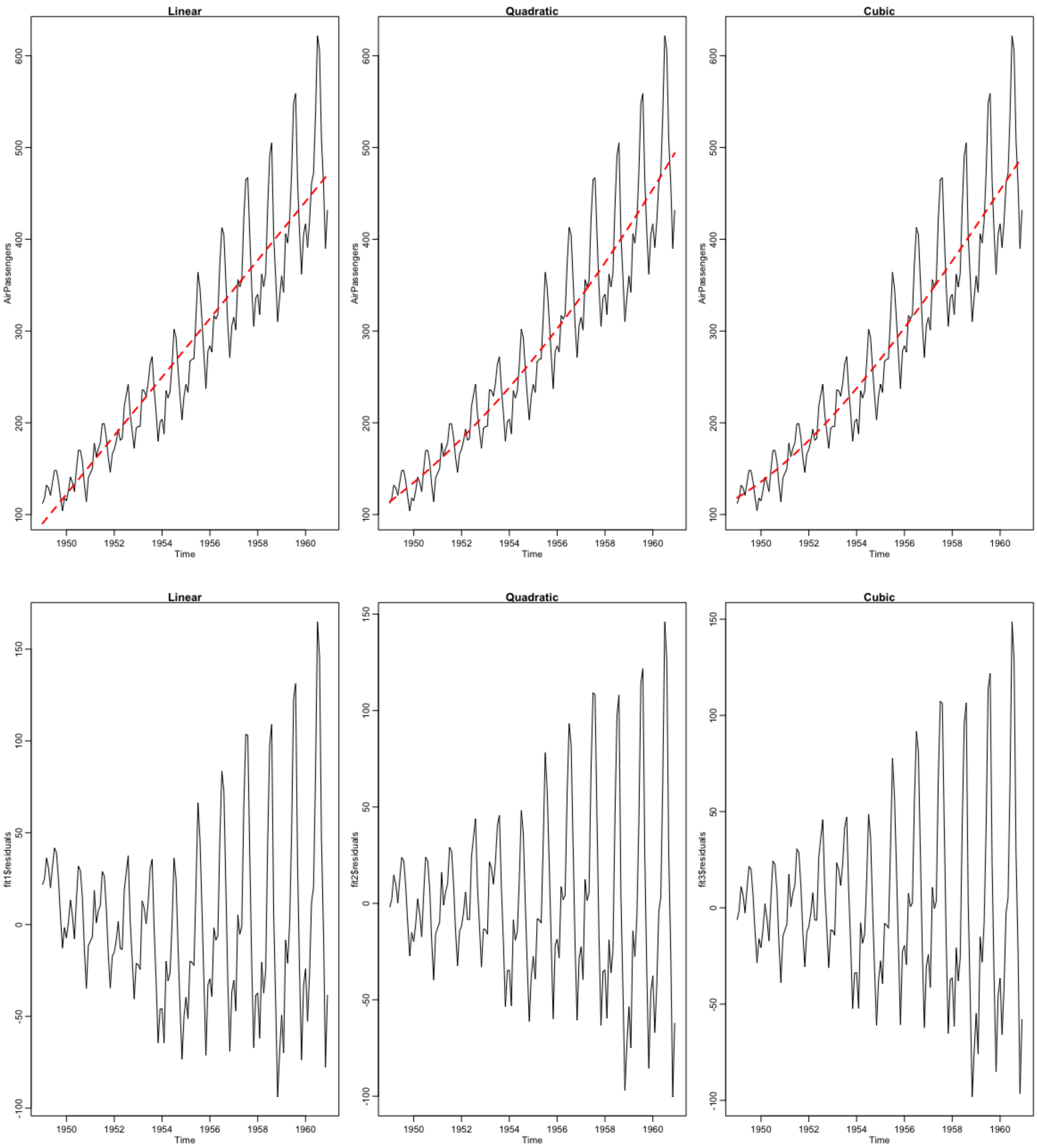
The best I can come up with this strange plot is $AR(2)$ or maybe $AR(2,1)$. The MA effects could be mostly from the AR effect, but this is not decreasing geometrically which is why the effect from an MA process might be necessary.

2. Use the "AirPassenger" data and fit a linear, quadratic, and cubic trend and for each trend you fit plot the residuals. Extract trend seasonality using non parametric methods. Provide R code and the plots.

Solution: Using the below code, graphs are obtained.

```
fit1 <- tslm(AirPassengers ~ trend)
fit2 <- tslm(AirPassengers ~ poly(trend,2))
fit3 <- tslm(AirPassengers ~ poly(trend,3))
par(mfrow = c(1,3))
plot.ts(AirPassengers, main = "Linear", cex.main=1.5)
lines(fitted(fit1), lwd=2, col=2, lty = 2)
plot.ts(AirPassengers, main = "Quadratic", cex.main=1.5)
lines(fitted(fit2), lwd=2, col=2, lty = 2)
plot.ts(AirPassengers, main = "Cubic", cex.main=1.5)
lines(fitted(fit3), lwd=2, col=2, lty = 2)

plot(fit1$residuals, main = "Linear")
plot(fit2$residuals, main = "Quadratic")
plot(fit3$residuals, main = "Cubic")
```

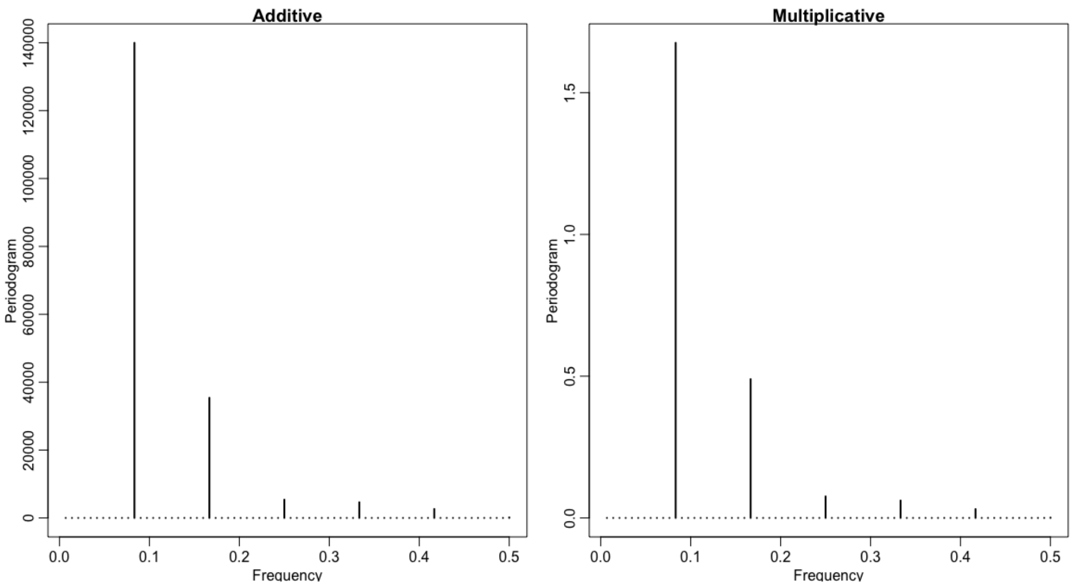


Solution: We can extract the seasonality by decomposition and use of a periodogram. I utilize the Additive and Multiplicative nonparametric methods to extract seasonality and that is done below.

```
data(AirPassengers)
xt=AirPassengers
par(mfrow=c(1,2))

#Additive Effect
decomp.x.ad=decompose(xt,type = "additive")
periodogram(decomp.x.ad$seasonal, main = "Additive")

#Multiplicative Effect
decomp.x.mul=decompose(xt,type = "multiplicative")
periodogram(decomp.x.mul$seasonal, main = "Multiplicative")
```



The highest peak appears to be around $\frac{1}{12}$ with the second highest being around $\frac{1}{6}$, so seasonality of period 6 and 12 could perhaps be observed.

3. Write the complete model for the following:

(a) $AR(P=2)_{d=12}$

Solution:

$$X_t - \Phi_1 X_{t-12} - \Phi_2 X_{t-24} = e_t$$

(b) $MA(Q=2)_{d=12}$

Solution:

$$X_t = e_t + \Theta_1 e_{t-12} + \Theta_2 e_{t-24}$$

(c) $ARMA(P=1, Q=2)_{d=12}$

Solution:

$$X_t - \Phi X_{t-12} = e_t + \Theta_1 e_{t-12} + \Theta_2 e_{t-24}$$

(d) $ARMA(P=2, Q=0)_{d=12}$

Solution:

$$X_t - \Phi_1 X_{t-12} - \Phi_2 X_{t-24} = e_t$$

(e) $ARMA(p=0, q=2) \times (P=1, Q=2)_{d=12}$

Solution:

$$\begin{aligned}
X_t - \Phi X_{t-12} &= e_t(1 + \theta_1 B + \theta_2 B^2)(1 + \Theta_1 B^{12} + \Theta_2 B^{24}) \\
X_t - \Phi X_{t-12} &= e_t + \Theta_1 e_{t-12} + \Theta_2 e_{t-24} \\
&\quad + \theta_1 e_{t-1} + \theta_1 \Theta_1 e_{t-13} + \theta_1 \Theta_2 e_{t-25} \\
&\quad + \theta_2 e_{t-2} + \theta_2 \Theta_1 e_{t-14} + \theta_2 \Theta_2 e_{t-26}
\end{aligned}$$

(f) $SARIMA(p=1, d=1, q=1) \times (P=1, D=1, Q=1)_{d=12}$

Solution:

$$\begin{aligned}
(1-B)(1-B^{12})(1-\phi B)(1-\Phi B^{12})X_t &= (1+\theta B)(1+\Theta B^{12})e_t \\
(1-B-B^{12}+B^{13})(1-\phi B)(1-\Phi B^{12})X_t &= (1+\theta B+\Theta B^{12}+\theta\Theta B^{13})e_t \\
\Big[(1-B-B^{12}+B^{13}-\phi B+\phi B^2+\phi B^{13}-\phi B^{14}) & \\
(1-\Phi B^{12})\Big]X_t &= e_t + \theta e_{t-1} + \Theta e_{t-12} + \theta\Theta e_{t-13} \\
\Big[1-B-B^{12}+B^{13}-\phi B+\phi B^2+\phi B^{13}-\phi B^{14}-\Phi B^{12}+\Phi B^{13} & \\
+\Phi B^{24}-\Phi B^{25}+\phi\Phi B^{13}-\phi\Phi B^{14}-\phi\Phi B^{25}+\phi\Phi B^{26}\Big]X_t &= e_t + \theta e_{t-1} + \Theta e_{t-12} + \theta\Theta e_{t-13} \\
X_t - X_{t-1} - X_{t-12} + X_{t-13} - \phi X_{t-1} + \phi X_{t-2} + \phi X_{t-13} & \\
-\phi X_{t-14} - \Phi X_{t-12} + \Phi X_{t-13} + \Phi X_{t-24} - \Phi X_{t-25} & \\
+\phi\Phi X_{t-13} - \phi\Phi X_{t-14} - \phi\Phi X_{t-25} + \phi\Phi X_{t-26} &= e_t + \theta e_{t-1} + \Theta e_{t-12} + \theta\Theta e_{t-13}
\end{aligned}$$