

Proofs

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Theorem One

The limiting distribution of the $\text{Poisson}(\lambda)$ distribution as $\lambda \rightarrow \infty$ is normal.

Pf. Let $X \sim \text{Poisson}(\lambda)$ which has the probability mass function

$$f_X(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad x = 0, 1, 2, \dots$$

and moment generating function

$$M_X(t) = e^{e^t - 1}$$

We will specifically consider the standardized Poisson random variable X

$$\frac{X - \lambda}{\sqrt{\lambda}}$$

which has the Moment Generating Function

$$\begin{aligned} M_{(X-\lambda)/\sqrt{\lambda}}(t) &= \mathbb{E} \left[\exp \left(t * \frac{X - \lambda}{\sqrt{\lambda}} \right) \right] \\ &= \exp(-t\sqrt{\lambda}) * \mathbb{E} \left[\exp \left(\frac{tX}{\sqrt{\lambda}} \right) \right] \\ &= \exp(-t\sqrt{\lambda}) * \exp(e^{t/\sqrt{\lambda}} - 1) \end{aligned}$$