Chapter Six

NAME HERE

10/2/2017

Exercise Four

Question

It is well known that the concentration of cholesterol in blood serum increase with age, but it is less clear whether cholesterol level is also associated with body weight. The table below shows for thirty women serum cholesterol, (millimoles per liter), age (years) and body mass index (weight divided by height squared, where weight was measured in kilograms and height in meters). Use multiple regression to test whether serum cholesterol is associated with body mass index when age is already included in the model.

CHOL	Age	BMI	CHOL	Age	BMI
5.94	52	20.7	6.48	65	26.3
4.71	46	21.3	8.83	76	22.7
5.86	51	25.4	5.1	47	21.5
6.52	44	22.7	5.81	43	20.7
6.8	70	23.9	4.65	30	18.9
5.23	33	24.3	6.82	58	23.9
4.97	21	22.2	6.28	78	24.3
8.78	63	26.2	5.15	49	23.8
5.13	56	23.3	2.92	36	19.6
6.74	54	29.2	9.27	67	24.3
5.95	44	22.7	5.57	42	22
5.83	71	21.9	4.92	29	22.5
5.74	39	22.4	6.72	33	24.1
4.92	58	20.2	5.57	42	22.7
6.69	58	24.4	6.25	66	27.3

Solution

Exercise Five

Question

The table below shows plasma inorganic phospate levels (mg/dl) one hour after a standard glucose tolerance test for obese subjects, with or without hyperinsulinemia, and controls (data from Jones 1987).

Hyperinsulinemic obese	Non-hyperinsulinemic obese	Controls
2.3	3	3
4.1	4.1	2.6
4.2	3.9	3.1
4	3.1	2.2
4.6	3.3	2.1
4.6	2.9	2.4
3.8	3.3	2.8
5.2	3.9	3.4
3.1		2.9
3.7		2.6
3.8		3.1
		3.2

Solution

(a): Perform a one-factor analysis of variance to test the hypothesis that there are no mean differences among the three groups. What conclusions can you draw?

(b): Obtain a 95% confidence interval for the difference in means between the two obese groups.

(c): Using an appropriate model, examine the standardized residuals for all the observations to look for any systematic effects and to check the Normality assumption.

Exercise Seven

Question

For the balanced data in the table below (Table 6.10), the analyses in Section 6.4.2 showed that the hypothesis tests were independent. An alternative specification of the design matrix for the saturated model (6.9) with the corner point constraints $\alpha_1 = \beta_1 = (\alpha\beta)_{12} = (\alpha\beta)_{21} = (\alpha\beta)_{31} = 0.$ so that

$$\boldsymbol{\beta} = \begin{bmatrix} \mu \\ \alpha_2 \\ \alpha_3 \\ \beta_2 \\ (\alpha\beta)_{22} \\ (\alpha\beta)_{32} \end{bmatrix} \quad \text{and} \quad \boldsymbol{X} = \begin{bmatrix} 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix},$$

where the columns of $\boldsymbol{X}$ corresponding to the terms $(\alpha\beta)_{jk}$ are the products of columns corresponding to terms α_j and β_k.

Levels of Factor A	Levels of Factor B		
	B_1	B_2	Total
A_1	6.8, 6.6	5.3, 6.1	24.8
A_2	7.5, 7.4	7.2, 6.5	28.6
A_3	7.8, 9.1	8.8, 9.1	34.8
Total	45.2	43	88.2

Source of variation	Degrees of freedom	Sum of Squares	Mean square	F
Mean	1	648.2700		
Levels of A	2	12.7400	6.3700	25.82
Levels of B	1	0.4033	0.4033	1.63
Interactions	2	1.2067	0.6033	2.45
Residual	6	1.4800	0.2467	
Total	12	664.1000		

Year of Graduation	Faculty: Medicine		Arts		Science		Engineering	
	S	T	S	T	S	T	S	T
1938	18	22	16	30	9	14	10	16
1939	16	23	13	22	9	12	7	11
1940	7	17	11	25	12	19	12	15
1941	12	25	12	14	12	15	8	9
1942	24	50	8	12	20	28	5	7
1943	16	21	11	20	16	21	1	2
1944	22	32	4	10	25	31	16	22
1945	12	14	4	12	32	38	19	25
1946	22	34			4	5		
1947	28	37	13	23	25	31	25	35
Total	177	275	92	168	164	214	103	142

Solution

(a): Show that $\boldsymbol{X}^T\boldsymbol{X}$ has the block diagonal form described in Section 6.2.5. Fit the model (6.9) and also the models (6.10) and (6.12) and verify that the results in the second table above (Table 6.12) are the same for this specification of $\boldsymbol{X}$.

(b): Show that the estimates for the means of the subgroup with the treatments A_3 and B_2 for two different models are the same as the values given at the end of Section 6.4.2.