Strongly interacting dark matter with real gauge group representations

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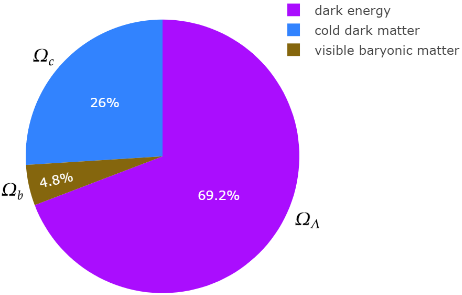
Strongly interacting dark matter (SIDM)

Let's start with the usual story

There is a non-negligible non-visible matter component in the universe



No experimentally verified description on the fundamental level so far

Dark matter evidence

Evidence for dark matter on various scales

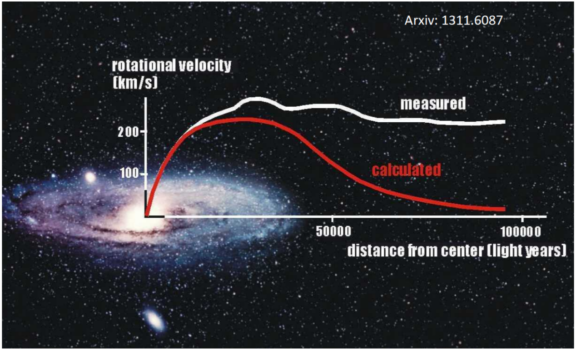
- Galaxy scale: Rotational curves
- Galaxy cluster scale: Visible mass to little to hold together coma cluster
- Cosmological scales: CMB anisotropies

More evidence from Gravitational Lensing, Numerical Simulations, BBN, Galaxy correlation functions, ...

Evidence from rotational curves

Visible matter suggests
$v \approx \frac{1}{r}$

We observe that
$v \approx \text{const.}$



Problems of **C**old **D**ark **M**atter (CDM)

- **Missing satellite problem:**
  Simulations predict significantly more DM-subhalos than the number of satellite galaxies of the Milky-Way we observe.
- **Too big to fail problem:**
  Simulations predict too much mass in the central region of the halo. The simulation results are in conflict with the number of observed satellites of Milky-Way and Andromeda.
- **Cusp vs. Core problem:**
  Simulations suggest cuspy density profiles for DM halos, while observations point towards more cored profiles.

Cusp vs. Core problem

Data from the DDO 154 dwarf galaxy

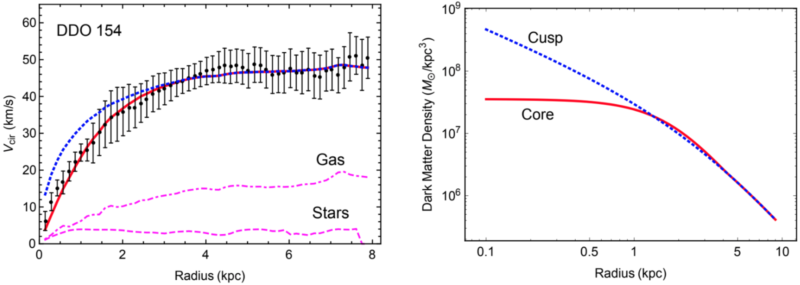


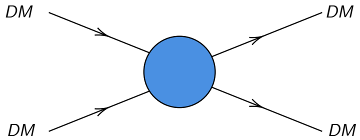
Figure: Taken from the talk on Dark QCD of [Murayama (2022)]

Self interacting dark matter

Introduction of self interactions within the dark sector may solve these problems as shown by N-body simulations. [arXiv:astro-ph/9909386v2]

- Required self interaction cross section:

$$\frac{\sigma}{m} = 0.1 - 1.0 \ \frac{\mathrm{cm}^2}{g}$$



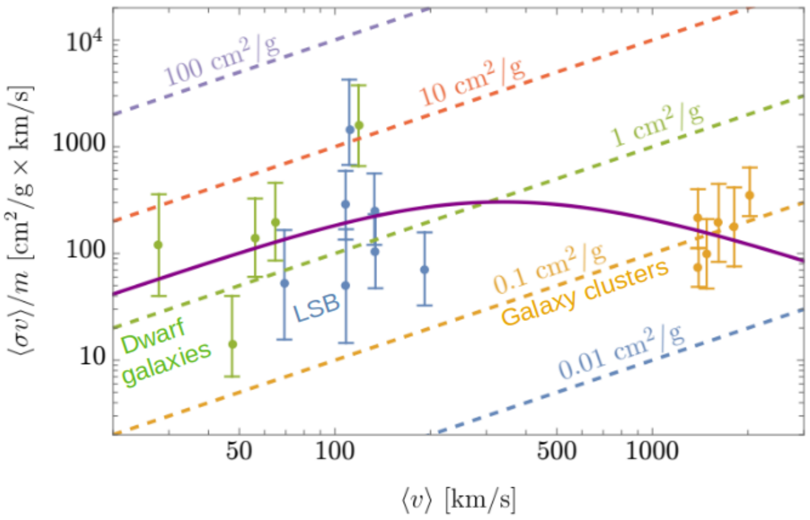
Constraints:

- Bullet cluster constraint: $\frac{\sigma}{m} \lesssim 0.7 \ \frac{\mathrm{cm}^2}{g}$ [arXiv:astro-ph/0704.0261]

DM self interaction cross section

arXiv:hep-ph/2205.08088

arXiv:hep-ph/1508.03339



What is a strongly interacting gauge theory?

We look at gauge theories with a non-abelian gauge group and fermionic matter such that:

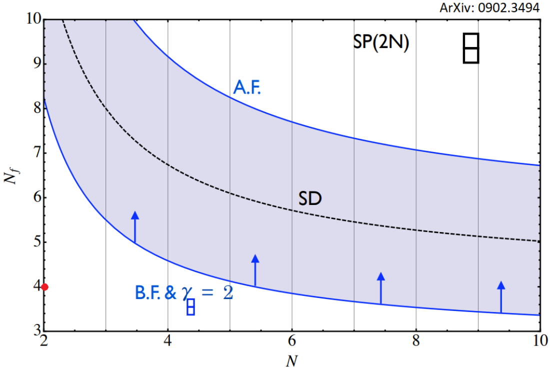
- The theory is asymptotically free
  $\Rightarrow$ Gaussian UV fixed point
- The theory has a rich infrared phenomenology
  $\Rightarrow$ No IR fixed point

We expect that at low temperatures such theories are in a chirally broken phase and elementary degrees of freedom confine into bound states.

Conformal window and the confining phase

arXiv:hep-ph/0902.3494

Above: Asymptotic freedom lost / no trivial UV fixed point



Below: Chiral symmetry breaking / no IR fixed point

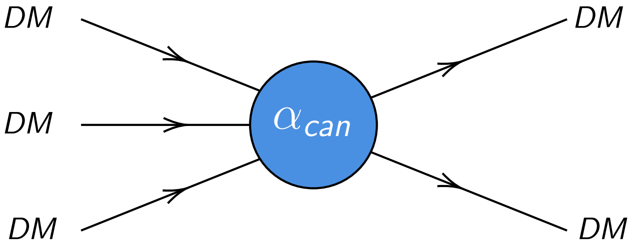
Strongly interacting dark matter

- Cold dark matter from bound states of a QCD like dark sector
- Larger $\frac{\sigma}{m}$ due to nature of underlying strongly interacting force
- Might provide velocity dependent $\frac{\sigma}{m}$
- Effective field theory description of IR and dimensional suppression of interactions
- UV symmetries may constrain effective field theory description, leading to sufficient stability of DM

SIMP mechanism - Dark matter freeze out

arXiv:hep-ph/1402.5143

Dark matter depletes via a $3 \rightarrow 2$ cannibalization process

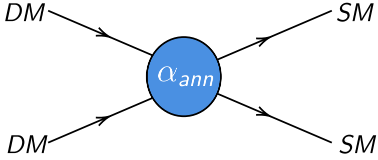


- Problem: If DM is not coupled to SM, dark sector heats up
- Opportunity: Additional motivation, besides detectability, for SM coupling

SIMP mechanism - Thermal equilibrium

arXiv:hep-ph/1402.5143

Dark sector dumps heat into SM via $2 \leftrightarrow 2$ processes



- Implementation via a dark photon e.g. $U_D(1)$ - gauge symmetry and kinetic mixing
- Implementation of 4-point vertex via Higgs-portal

Summary - Strongly interacting DM

- CDM as bound states of strongly interacting sector (Theory below conformal window)
- Natural implementation of sufficiently large $\frac{\sigma}{m}$ with potential velocity dependence
- SIMP mechanism gives potential freeze out mechanism and additional motivation for SM coupling
- Rich (IR) phenomenology and spectrum

QCD-like Lagrangian with real representations

Weyl fermions

Muller-Kirsten and Wiedemann (2010)

Left-handed Weyl

$$\psi_L \in W$$

Lorentz transformation Λ:

$$\psi'_L(x') = M[\Lambda]\psi_L(\Lambda x')$$

Right-handed Weyl

$$\psi_R \in \overline{W}^*$$

Lorentz transformation Λ:

$$\psi'_R(x') = (M[\Lambda]^{-1})^\dagger \psi_R(\Lambda x')$$

$$\begin{aligned} \tilde{C} : W &\to \overline{W}^* \\ \psi_L &\mapsto -E\psi_L^* \end{aligned}$$

$$E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Weyl: Spin invariant "scalar" products

Product on W:

$$(\psi_L, \phi_L)_W := \psi_L^\top E \phi_L$$

Product on $\overline{W}^*$:

$$(\psi_R, \phi_R)_{\overline{W}^*} := \psi_R^\top E^{-1} \phi_R$$

What is important ?

i) Invariant under Lorentz transformation
ii) Linear in both components
iii) Symmetric for anticommuting spinors
iv) Non degenerate products

Dirac fermions

Muller-Kirsten and Wiedemann (2010)

If we want parity as a good symmetry we work with Dirac fermions

Dirac bi-spinor $Q \in V_D := W \oplus \overline{W}^*$

$$Q = \begin{pmatrix} Q_L \\ Q_R \end{pmatrix}$$

Lorentz transformation Λ:

$$\begin{pmatrix} Q_L \\ Q_R \end{pmatrix} \mapsto \begin{pmatrix} M[\Lambda] & 0 \\ 0 & (M[\Lambda]^{-1})^\dagger \end{pmatrix} \begin{pmatrix} Q_L \\ Q_R \end{pmatrix}$$

Dirac: Spin invariant "scalar" products

Dirac product

$$\langle Q, P\rangle_D := \left(\tilde{C}^{-1}(Q_R), P_L\right)_W + \left(\tilde{C}(Q_L), P_R\right)_{\overline{W}^*} =: \overline{Q}P$$

- **Anti**-linear in first component
- Parity invariant

Majorana product

$$(Q, P)_M := (Q_L, P_L)_W + (Q_R, P_R)_{\overline{W}^*}$$

- Linear in both components
- **Not** parity invariant

Charge conjugation for Dirac fermions

Charge conjugation $C : V_D \to V_D$

$$\begin{pmatrix} Q_L \\ Q_R \end{pmatrix} \xrightarrow[\textbf{anti}\text{-linear}]{C} \eta_C \begin{pmatrix} \tilde{C}^{-1}(Q_R) \\ \tilde{C}(Q_L) \end{pmatrix} =: Q^C$$

- Interchanges left- and right-handed information
- Self-inverse i.e. $C^2 = \mathbb{1}$
- Compatible with Lorentz group structure

Majorana fermions

Hamilton (2017)

C-Eigenstates

$$C(M) = +M$$

Decomposition

$$Q = M[\psi_L] + \mathrm{i}\; M[\phi_L]$$

Parametrization

$$M[\psi_L] = \begin{pmatrix} \psi_L \\ \tilde{C}(\psi_L) \end{pmatrix}$$

Follows solely from the properties of Charge Conjugation!

(Pseudo) Real representations

Hamilton (2017)

A representation $U : G \to Aut(V)$ is called **(pseudo) real** if there exists an **anti-linear** map $J : V \to V$ such that

"Self" - Inversion

$$J^2 = \begin{cases} +\mathbb{1} & \text{real} \\ -\mathbb{1} & \text{pseudo-real} \end{cases}$$

Equivalence of conjugate representations

$$J \; U \; J^{-1} = U^*$$

Structural consequences

If the gauge-group representation is (pseudo-)**real** there exist a gauge-invariant **bilinear** product $(\ .,\ .)_G : V_C \times V_C \to \mathbb{C}$ on color-space.

If the gauge-group representation is **real** the maps J and C have the **same** properties.

Extending the representation by the gauge-group

$$\begin{tikzcd}[column sep=huge] V_D \arrow[r, "\otimes V_C"] & V_D \otimes V_C \\ (\,.\,,\,.\,)_W \arrow[r, "\otimes(\,.\,,\,.\,)_G"] & (\,.\,,\,.\,) \\ \tilde{C} \arrow[r, "\otimes\ J"] & \tilde{\mathcal{C}} \end{tikzcd}$$

$$\begin{tikzcd}[column sep=huge] \langle\,.\,,\,.\,\rangle_D \arrow[r, "\text{extend by construction}"] & \langle\,.\,,\,.\,\rangle \end{tikzcd}$$

$$\begin{tikzcd}[column sep=huge] \textbf{Charge conjugation}\ C \arrow[r, "\text{extention with subtleties}"] & \textbf{C-Symmetry}\ \mathcal{C} \end{tikzcd}$$

The Lagrangian

Determined by gauge group representation

$$\mathcal{L} = \mathrm{i} \sum_{k=1}^{N_F} \overline{Q^{(k)}} \gamma^\mu D_\mu \left(Q^{(k)}\right)$$

$\overline{Q^{(k)}} := \langle Q^{(k)}, \, . \rangle$

$$- \sum_{k=1}^{N_F} m^{(k)} \overline{Q^{(k)}} Q^{(k)}$$

$$- \frac{1}{2} Tr_C \left\{ \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} \right\}$$

Independent of gauge group representation

Covariant derivative

$$D_\mu(Q) := \partial_\mu Q - g\; U_*[A_\mu](Q)$$

$U_*[A_\mu]$ ... induced Lie-algebra rep of the gauge group G

Example: $\exp\left(-\mathrm{i}A^\alpha \boldsymbol{\tau}_\alpha\right) \in G$

$$\begin{aligned}
\text{Fundamental :} \qquad & U_*[A_\mu](Q) = -\mathrm{i}A_\mu^\alpha \boldsymbol{\tau}_\alpha Q \\
\text{(2,0)-Tensor :} \qquad & U_*[A_\mu](Q) = -\mathrm{i}A_\mu^\alpha \left(\boldsymbol{\tau}_\alpha Q + Q\boldsymbol{\tau}_\alpha^\top\right) \\
\text{Adjoint :} \qquad & U_*[A_\mu](Q) = -\mathrm{i}A_\mu^\alpha \left(\boldsymbol{\tau}_\alpha Q - Q\boldsymbol{\tau}_\alpha\right)
\end{aligned}$$

Rewriting the Lagrangian - Nambu-Gorkov formalism

Decompose N_F Dirac fermions into $N_f = 2N_F$ Majorana fermions, parametrized by N_f Weyl Femions.

$U(2N_F)$ - Flavor symmetry
Pauli - Gürsey - symmetry

$$\mathcal{L}_M = \mathrm{i} \sum_{k=1}^{2N_F} \left(\tilde{C} \left(\psi_L^{(k)} \right), \overline{\sigma}^{\mu} D_{\mu} \left(\psi_L^{(k)} \right) \right)$$

$$- \sum_{k=1}^{2N_F} m^{(k)} \left(\left(\psi_L^{(k)}, \psi_L^{(k)} \right) - \left(\psi_L^{(k)}, \psi_L^{(k)} \right)^{*} \right)$$

SO - Flavor symmetry

Spontaneous symmetry breaking

Peskin, Nuclear Physics B 175 (1980)

Chiral condensate (Order parameter)

$$\Sigma := \sum_{k=1}^{2N_F} \overline{Q^{(k)}} Q^{(k)} = \sum_{k=1}^{2N_F} \left(\psi_L^{(k)}, \psi_L^{(k)}\right) - \left(\psi_L^{(k)}, \psi_L^{(k)}\right)^*$$

Breaking pattern

$$SU(2N_F) - \textbf{Fund.} \xrightarrow{\Sigma \neq 0} SO(2N_F) - \textbf{Fund.}$$

Breaking pattern

$$U(2N_F)$$

$\Big\downarrow$ Axial anomaly

$$SU(2N_F)$$

$\Bigg\downarrow$ Chiral condensate $\Sigma \neq 0$
deg. mass $\quad m^{(k)} = m^{(k+2)} \neq 0$

$$SO(2N_F)$$

$\Big\downarrow$ non deg. mass $\quad m^{(k)} \neq m^{(k+2)} \neq 0$

$$SO(2) \times \cdots \times SO(2)$$

Comparison of 2-flavor theories

arXiv:hep-ph/2202.05191

Complex	*pseudo − real*	*real*
QCD	$Sp(4)$ − Fund.	$Sp(4)$ − Antisymmetric
$U(2) \times U(2)$	$U(4)$	$U(4)$
$\downarrow$	$\downarrow$	$\downarrow$ Axial anomaly
$SU(2) \times SU(2) \times U(1)$	$SU(4)$	$SU(4)$
$\downarrow$	$\downarrow$	$\downarrow$ Chiral condensate $\Sigma \neq 0$ deg. mass $m^{(k)} = m^{(k+2)} \neq 0$
$SU(2) \times U(1)$	$Sp(4)$	$SO(4)$
$\downarrow$	$\downarrow$	$\downarrow$ non deg. mass $m^{(k)} \neq m^{(k+2)} \neq 0$
$U(1) \times U(1)$	$Sp(2) \times Sp(2)$	$SO(2) \times SO(2)$

Spatial reflection parity

Parity operation

$$\hat{\boldsymbol{P}} Q^{(k)}(\vec{x}, t) \hat{\boldsymbol{P}}^\dagger = \eta_P \gamma_0 Q^{(k)}(-\vec{x}, t)$$

Convenient choice of η_P

- QCD choice : $\eta_P = 1$
  $\Rightarrow$ Not all Goldstones are psedoscalars

- D-Parity : $\eta_P = \mathrm{i}$
  $\Rightarrow$ all Goldstones are psedoscalars

C-parity

C-Symmetry for Weyl flavors

$$\mathcal{C}\begin{pmatrix}\psi^{2k-1}\\ \psi^{2k}\end{pmatrix} = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\psi^{2k-1}\\ \psi^{2k}\end{pmatrix}$$

- Every Dirac flavor (k) gives rise to two Weyl flavors of opposite C-Parity.
- Independent of η_C definition.
- Extends $SO(2)$-Flavor symmetry to $O(2)$ symmetry.
- Particles are their own anti-particles.

Scalar bilinear interpolating operators

arXiv:hep-ph/1912.06505

Pairing symmetrizes components

$$\mathcal{O}_s = T_{kj}\,(\chi^{(k)},\chi^{(j)}) \pm T^*_{kj}\,(\chi^{(k)},\chi^{(j)})^*$$

$N_f(N_f+1)/2$ symmetric components

Choice of sign

$\Rightarrow N_f(N_f+1)$ independent scalar operators.

Quantum numbers of Goldstone modes

Goldstone modes are characterized by the quantum numbers of conserved current components j_α^0 corresponding to broken generators $\boldsymbol{\tau}_\alpha$.

$$\begin{aligned} 0 &\neq \langle 0|\, j_A^0 \,|GB\rangle \\ &= \langle 0|\, \hat{\boldsymbol{P}}^\dagger \hat{\boldsymbol{P}} j_A^0 \hat{\boldsymbol{P}}^\dagger \hat{\boldsymbol{P}} \,|GB\rangle \\ &= \eta_P(J)\eta_P(GB) \langle 0|\, j_A^0 \,|GB\rangle \end{aligned}$$

$$\begin{aligned} \eta_P(J) &= 1 \qquad \Rightarrow \qquad \eta_P(GB) = 1 \\ \eta_P(J) &= -1 \qquad \Rightarrow \qquad \eta_P(GB) = -1 \end{aligned}$$

Goldstones in $Sp(4)$ antisymmetric 2-tensor gauge theory

Name	$T_{kj}\left(\chi^{(k)}, \chi^{(l)}\right) + T^*_{kj}\left(\chi^{(k)}, \chi^{(l)}\right)^*$	J^P	J^D
π_1	$\overline{u}\gamma_5 d$	0^-	0^-
π_2	$\overline{d}\gamma_5 u$	0^-	0^-
π_3	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma_5 u - \overline{d}\gamma_5 d\right)$	0^-	0^-
π_4	$d^{\mathcal{C}\dagger}\gamma_5 u^*$	0^+	0^-
$\vdots$	$\vdots$	$\vdots$	$\vdots$
π_9	$d^{\mathcal{C}\top}\gamma_5 u$	0^+	0^-

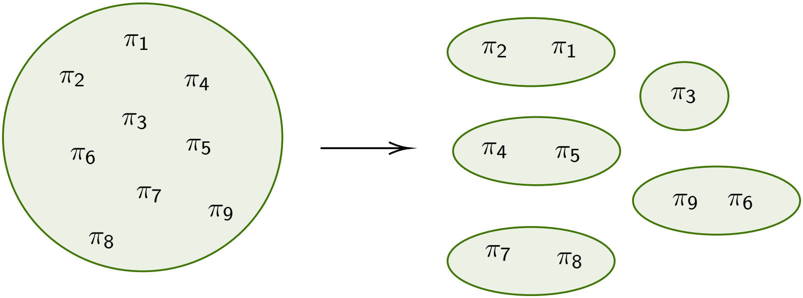
Goldstones in $Sp(4)$ antisymmetric 2-tensor gauge theory

Degenerate mass

$SO(4)$

Non-degenerate mass

$SO(2) \times SO(2)$



Summary - QCD-like Lagrangian with real representations

- For real representations of the gauge group the representation-theoretical features of the ungauged fermions are mostly preserved
- Discussed the breaking patterns occurring in this class of theories
- Discussed C and P/D-parity in this class of theories
- Discussed construction and identification of interpolation operators of the Goldstone bosons

Outlook - Bringing it together

EFT for goldstone bosons

arXiv:hep-ph/2202.05191

Pion field

$$\Sigma = \mathrm{e}^{\mathrm{i}\pi/f_\pi}\ \Sigma_0\ \mathrm{e}^{\mathrm{i}\pi^\top/f_\pi}$$

$$\pi = \sum_{broken} \pi_i \boldsymbol{T}_i$$

$f_\pi \ldots$ Pion decay constant

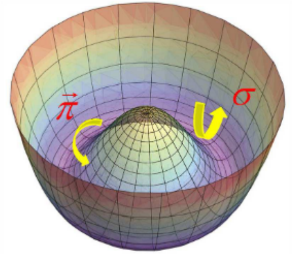


Figure: Taken from [arXiv:hep-ph/1804.05664]

Σ transforms in the symmetric (2,0)-Tensor repr. of $SU(2N_F)$

$$\Sigma \mapsto U\Sigma U^\top$$

Chiral lagrangian

Chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma\right] - \frac{\mu^3}{2}\mathrm{Tr}\left[M\Sigma + h.c.\right] + \dots$$

- Incorporates all UV-symmetries
- Organisation via Weinberg power-counting theorem
- Vacuum alignment Σ_0 minimizes static part

$$\min_{\pi=0}\left(\mathrm{Tr}\left[M\Sigma + h.c.\right]\right) = M\mathrm{Tr}\left[M\Sigma_0 + h.c.\right]$$

Chiral perturbation theory

Expand exponential function and organise terms in powers of momenta $\mathcal{O}\left(p^k\right)$.

Example: Expansion including $\mathcal{O}(p^2)$ of kinetic term

$$\begin{aligned}\mathcal{L}_{Kin} &= \frac{1}{2}\sum_k \partial_\mu \pi_k \partial^\mu \pi_k \\ &+ \frac{1}{2}\sum_{k,n} C_{kn}^{(2,1)} \pi_n^2 (\partial_\mu \pi_k)^2 + C_{kn}^{(2,2)} \pi_n \partial_\mu \pi_k \; \pi_k \partial_\mu \pi_n \\ &+ \mathcal{O}(\frac{\pi^6}{f_\pi^4})\end{aligned}$$

Wess-Zumino-Witten (WZW) Term

arXiv:hep-ph/1411.3727

Additional topological term that **must** be included in the Chiral Lagrangian in order to incorporate effects of the **axial anomaly**.

WZW term in first order Chiral-PT.

$$\mathcal{L}_{WZW} \propto \frac{N_C}{f_\pi^5}\epsilon^{\mu,\nu\rho\sigma}\,\mathrm{Tr}\{\pi\ \partial_\mu\pi\ \partial_\nu\pi\ \partial_\rho\pi\ \partial_\sigma\pi\ \} + \mathcal{O}\left(\frac{\pi^6}{f_\pi^6}\right)$$

- Incorporates $3 \to 2$ processes for SIMP mechanism.
- Non vanishing if $N_f \geq 3$ since [arXiv:hep-ph/1411.3727]

$$\pi_5(SU(N_f)/SO(N_f)) = \mathbb{Z} \ \text{ if } \ N_f \geq 3$$

$U_D(1)$ - Dark photon

Couple the dark sector to the standard model via a dark Photon:

- Charge assignment of Weyl fermions under $U_D(1)$ restricted by anomaly cancellation
- Use Brout-Englert-Higgs mechanism to make dark photon massive
- Couple to standard model photon via kinetic mixing

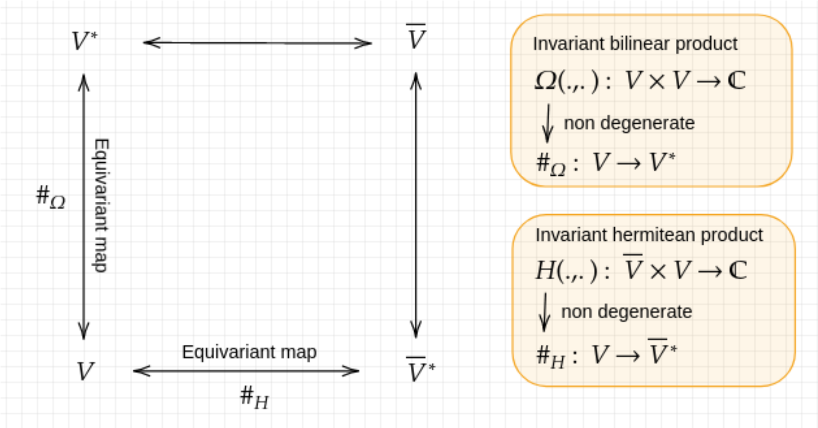
$$\mathcal{L} \supset \frac{\epsilon}{2\cos(\theta_W)} F'_{\mu\nu} F^{\mu\nu}$$

Including further parts of the particle spectrum

- Will include ρ-mesons as massive vector mediators via a gauged copy of the flavor symmetry
  - Covariant derivatives in Chiral Lagrangian
  - Add terms to make WZW term gauge invariant
- What about "Baryons"?
  - Can we build interpolating operators out of 3 or more fields (find gauge singlets)?
  - Are they stable? (Is there a baryon number symmetry?)
  - Are they, besides stability, relevant for IR dynamics?

Bonus Content

Representations over $\mathbb{C}$



Representation theory of Charge Conjugation

$$\begin{tikzcd}[column sep=huge, row sep=huge]
V_D \otimes V_C \arrow[r, dashed, "\mathsf{K}", "\psi^i \mapsto \psi^{i*}"'] \arrow[d, dashed, "\substack{\text{C-Symmetry} \\ \psi \mapsto \psi^C}"'] \arrow[dr, dashed, "\text{Charge conjugation}", "\psi \mapsto C\bar{\psi}^\top"'] & \overline{V_D \otimes V_C} \arrow[d, "\substack{\text{Fermionic C - Matrix} \\ \psi^{i*} \mapsto C^i{}_j \, \psi^{j*}}"] \\
V_D \otimes V_C \arrow[r, leftrightarrow, "\text{Equivariant map}", "\psi \mapsto J(\psi)"'] & V_D \otimes \overline{V_C}
\end{tikzcd}$$