Introduction to dark matter phenomenology

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Topic overview

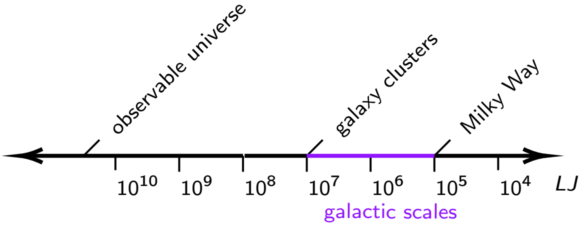
1. Astrophysical aspects
   - Rotational curves
   - The universe in standard cosmology
   - Dark matter in standard cosmology

2. Particle physics aspects
   - Local thermodynamics
   - Boltzmann equation
   - Species and freeze out

Note: We use a system of unit where $c = \hbar = 1$

Sub-cosmological scales

- Galactic scales of 10^4 ly or 1 kpc.
- Visible matter of our galactic disc within 10 kpc.
- Stars are effectively collision free mass points $\Rightarrow$ Use as tracers.
- Newtonian gravity is still a good approximation at this scales.



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Expectation from Newtonian gravity

Balance between gravitation and centrifugal force

$$\frac{G\,M(r)}{r^2} = \frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{G\,M(r)}{r}}$$

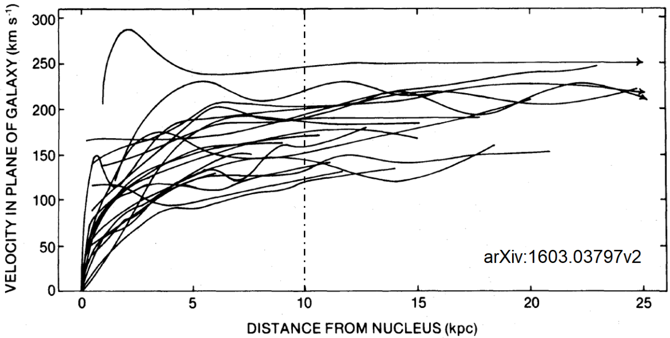
For $r \gtrsim 10$ kpc we expect

$$v \propto \frac{1}{\sqrt{r}}$$

Observation from rotational curves (taken from[1])

For $r \gtrsim 10$ kpc we see

$$v \approx \text{const.}$$



Dark matter halo model

Model assumptions:

1. DM is non dissipative
2. DM is spherically distributed

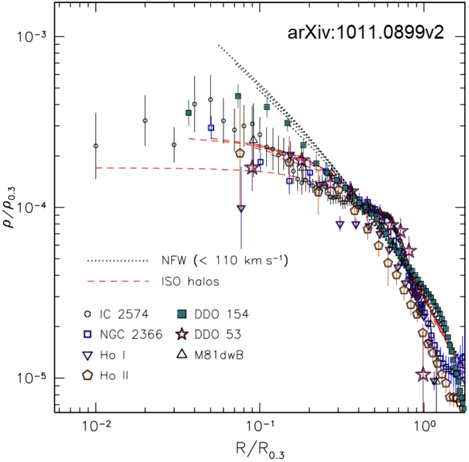
$$\Rightarrow \rho \propto \frac{M(r)}{r^3} \approx \frac{1}{r^2}$$

Dark matter halo of Milky Way:

$$M_{halo} \approx 10^{12}\ \mathrm{M}_\odot$$
$$R_{halo} \approx 100\ \mathrm{kpc}$$
$$< v > \approx 200\ \mathrm{km/s}$$

Cusp vs. Core problem

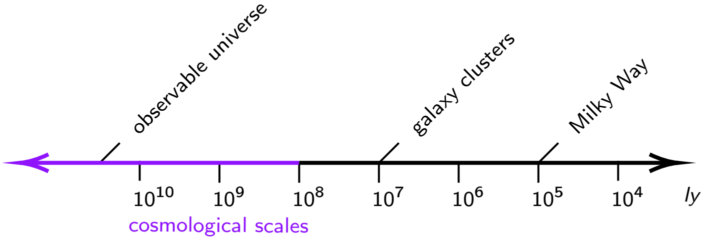
- "Cuspy" profiles favored by "gravitational only" dark matter
- "Cored" profiles favored by Observations of Dwarf galaxies.
- Hint on self-interacting dark matter



Assumptions of standard cosmology

- General relativity holds on macroscopic scales

- Spatial isotropy on cosmological scales
- Spatial homogeneity on cosmological scales



Perfect fluid matter

Energy momentum tensor

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p \, \eta^{\mu\nu}$$

Equation of state

$$p = \omega \rho$$

matter	ω	ρ
radiation	1/3	ρ_{rad}
incoherent matter	0	ρ_m
cosmological constant	-1	ρ_Λ

Symmetry ansatz for metric

Robertson Walker metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2\right)$$

Cosmic scale factor $R(t)$

(Spatial) Curvature parameter $k \in \begin{cases} 1 & \ldots \quad \text{closed} \\ 0 & \ldots \quad \text{flat} \\ -1 & \ldots \quad \text{open} \end{cases}$

Equations describing the universe

Evolution of the universe is described by:

- Einstein equations
- Equations of state for matter contributions
- Matter conservation $\mathrm{D}_\nu T^{\mu\nu} = 0$

Reduce to 2 fundamental equations:

- Friedmann equation for $R(t)$
- First law of thermodynamics

Equation for $R(t)$

Friedmann equation

$$H^2 + \frac{k}{R^2} = \frac{8\pi}{3m_{Pl}^2}(\rho_m + \rho_{rad} + \rho_\Lambda)$$

Hubble parameter:

$$H(t) = \frac{\dot{R}(t)}{R(t)}$$

Dark energy density:

$$\rho_\Lambda = \frac{m_{Pl}^2\,\Lambda}{8\pi}$$

Rewriting Friedmann equation

Relic density

$$\Omega = \frac{\rho}{\rho_{crt}}$$

Critical density $\rho_{crt} := \frac{3m_{Pl}^2 H^2}{8\pi}$

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The Friedmann equation becomes

$$\Omega_m + \Omega_{rad} + \Omega_\Lambda = 1 + \frac{k}{H^2 R^2} =: 1 - \Omega_k$$

Modeling the state of the universe

State of the universe

$$(\Omega_m,\ \Omega_{rad},\ \Omega_\Lambda,\ \Omega_k,\ H,\ \ldots)$$

- The state is time dependent.
  e.g. $H = H(t), \Omega_m = \Omega_m(t), \ldots$
- Not all parameters are independent.
  e.g. Friedmann equation: $\Omega_m + \Omega_{rad} + \Omega_\Lambda = 1 - \Omega_k$
- Some parameters may be negligible.
  e.g. matter dominance at present time $(t_0) : \Omega_{rad}(t_0) << \Omega_m(t_0)$

ΛCDM-Modell (for more see [2])

Relevant matter contributions:

Ω_Λ	...	dark energy
Ω_m	...	incoherent matter
Ω_b	...	visible baryonic matter
Ω_c	...	cold dark matter

Matter composition

$$\Omega_m \approx \Omega_b + \Omega_c$$

Friedmann equation

$$\Omega_m + \Omega_\Lambda \approx 1$$

Further parameters:

t_0	...	Age of the universe
$H(t_0)$	...	Hubble constant
τ	...	Optical depth to reionization
n_s	...	Scalar power-law index
σ_8	...	Amplitude of matter fluctuations

Only 6 parameters are independent.

Some suggestions on parameter estimates

- Matter density of the universe
  - count galaxy (cluster) number density
  - determine galaxy mass from rotation curves
  - cluster mass with virial theorem
  - mass determination by infall method
- Baryonic matter content from
  - primordial deuterium abundance measurements
  - Big Bang Nucleosynthesis (BBN) theory
- Large scale structure formation favors models with
  - a flat universe $\Omega_k \approx 0$
  - a matter content of $\Omega_k \approx 0.3$

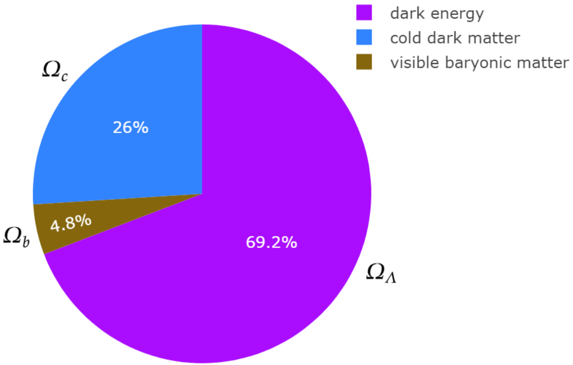
Values of Planck-Collaboration (taken from [3], table 4, column 2)

The values originate from detailed analysis of the CMB anisotropies.

cosmic parameter	value
Ω_Λ	0.693 ± 0.012
Ω_m	0.308 ± 0.012
Ω_b	0.048 ± 0.001
Ω_c	0.258 ± 0.008
H_0	$67.81 \pm 0.92 \ \frac{\mathrm{km\,s^{-1}}}{\mathrm{Mpc}}$
t_0	$13.799 \pm 0.038 \ \mathrm{Gyr}$

Its no real dark matter talk without this plot

Total energy content of the universe



Local thermodynamics (LTD) (for more see [4], Chapter 3.1 - 3.3)

Physical comoving unit volume

$$V := R^3$$

First law of LTD

$$\mathrm{d}\,(\rho V) = -p\,\mathrm{d}\,(V)$$

Conservation of entropy per comoving volume

$$\mathrm{d}S = 0$$

Entropy density of the universe

$$s := \frac{S}{V}$$

Phase-space distribution

$$f\left(\epsilon\left(\vec{p}\right), t\right) \overset{equilibrium}{=} \begin{cases} \text{Fermi-Dirac} \\ \text{Bose-Einstein} \\ \text{Boltzmann} \end{cases}$$

Number and energy density

$$n = g \int f(\vec{p}, t) \, \frac{\mathrm{d}^3 \vec{p}}{(2\pi)^3}$$

$$\rho = g \int \epsilon\left(\vec{p}\right) \; f(\vec{p}, t) \, \frac{\mathrm{d}^3 \vec{p}}{(2\pi)^3}$$

Temperature - Expansion relation

Assume relativistic particles dominate energy density.

$$\text{relativistic gas in equilibrium :} \quad \rho \propto g T^4$$

$$\downarrow$$

$$H^2 = \frac{8\pi}{3 m_{Pl}^2} \rho \propto \frac{8\pi}{3 m_{Pl}^2} g T^4$$

$$\downarrow$$

$$H \propto \frac{\sqrt{g}}{m_{Pl}} T^2$$

Time - Expansion relations

	radiation dominance	matter dominance	vacuum dominance
$p = \omega\rho$	$p = \frac{1}{3}\rho$	$p = 0$	$p = -\rho$
$\rho(R)$ *	$\rho \propto R^{-4}$	$\rho \propto R^{-3}$	$\rho \propto \mathrm{const.}$
$R(t)$ **	$R \propto t^{1/2}$	$R \propto t^{3/2}$	$R \propto \exp(H_0 t)$
$H(t)$	$H \propto t^{-1}$	$H \propto t^{-1}$	$H = H_0 = \mathrm{const.}$

$*$ follows from first law of LTD

$**$ follows from Friedmann equation

Non equilibrium dynamics of $f(\epsilon(\vec{p}), t)$

Boltzmann equation

$$\underbrace{L[f]}_{\text{Liouville operator}} = \underbrace{C[f]}_{\text{Collision operator}}$$

Liouville operator models free dynamics

$$g \int L[f](\vec{p}, t) \, \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} = \frac{\mathrm{d}n}{\mathrm{d}t} + 3H\, n$$

Collision term

- $C[f]$ models collisions of particles
  - with particles of the same species
  - with particles of other species
- If multiple species are involved multiple Boltzmann equations couple.
- Often lots of simplifications needed to get this term under control.

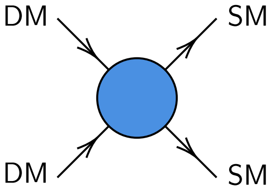
In modeling this term, particle physics comes into play !

2DM → 2SM Scattering

Collision term

$$g \int C[f](\vec{p}, t)\, \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} = -\left\langle \sigma v \right\rangle_{DM\to SM} \left(n_{DM}\right)^2 + \left\langle \sigma v \right\rangle_{SM\to DM} \left(n_{SM}\right)^2$$

$\langle \sigma v \rangle_{DM\to SM}$ ... Thermal crosssection average Møller velocity



Deviation from equilibrium

Assume SM particles are in equilibrium with photon bath.

If DM and SM in equilibrium

$$\langle \sigma v \rangle_{DM \to SM} \left(n_{DM}^{eq}\right)^2 = \langle \sigma v \rangle_{SM \to DM} \left(n_{SM}^{eq}\right)^2$$

Simplified Boltzmann equation for $n = n_{DM}$

$$\frac{\mathrm{d}n}{\mathrm{d}t} + 3H\,n = -\,\langle \sigma v \rangle_{DM \to SM} \left(n^2 - n_{eq}^2\right)$$

Rewrite Boltzmann equation

Eliminate effects of expansion

$$Y := \frac{n}{s}$$

Parametrize time over temperature

$$x := \frac{m_{DM}}{T} \propto \sqrt{t}$$

Reformulated simplified Boltzmann equation

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = -\frac{s \left\langle \sigma v \right\rangle_{DM \to SM}}{x\; H} \left(Y^2 - Y_{eq}^2 \right)$$

Freeze out condition (for more see [4], Chapter 5.2)

Boltzmann equation

$$\frac{x}{Y_{eq}}\frac{\mathrm{d}Y}{\mathrm{d}t} = -\frac{\Gamma}{H}\left(\frac{Y^2}{Y_{eq}^2} - 1\right)$$

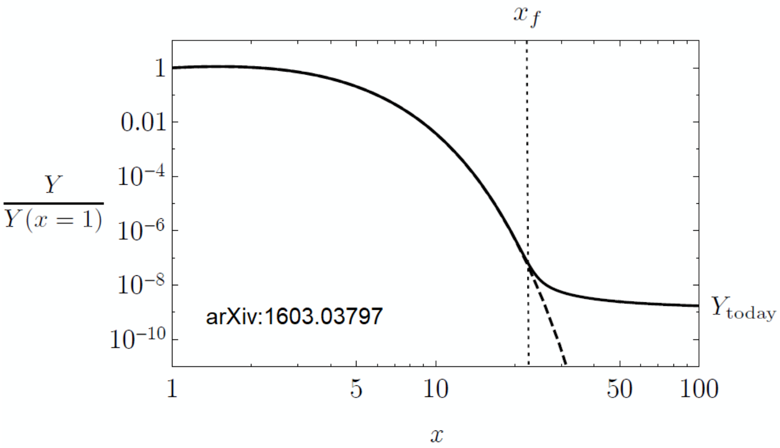
Scattering rate

$$\Gamma := Y_{eq} \ \langle \sigma v \rangle_{DM \to SM}$$

Freeze out condition

$$\Gamma \approx H$$

Freeze out (taken from [1])



WIMP-Miracle (for more see [1])

From solving Boltzmann equation one obtains Y_{Today}, x_f

Relic density

$$\Omega_c = \frac{m_{DM} \; s_{Today} Y_{Today}}{\rho_{crt}}$$

Parametrize thermal average velocity

$$\langle \sigma v \rangle_{DM \to SM} \approx \frac{\alpha^2}{m_{DM}^2}$$

Constraint ratio $\frac{\alpha}{m_{DM}}$

$$1 \approx \frac{\Omega_c H^2}{1000} \approx \frac{10^{-25}\mathrm{cm\ s}^{-1}}{\langle \sigma v \rangle_{DM \to SM}} \approx \left(\frac{0.1}{\alpha}\right)^2 \left(\frac{m_{DM}}{100\ \mathrm{GeV}}\right)^2$$

Summary

- Rotational curves first hinted a non visible, spherically distributed matter component.
- ΛCDM with Friedmann equation $\Omega_b + \Omega_c + \Omega_\Lambda \approx 1$
- Cold dark matter relic density $\Omega_c = 0.258 \pm 0.008$
- Particle physics to model $C[f]$ in Boltzmann equation
- Freeze out condition $\Gamma \approx H$
- WIMP Miracle: $\alpha \approx 0.01$, $m_{DM} \approx 100\ \mathrm{GeV}$

End of the presentation

Thank you for your attention

Any questions ?

Cusp vs. Core problem

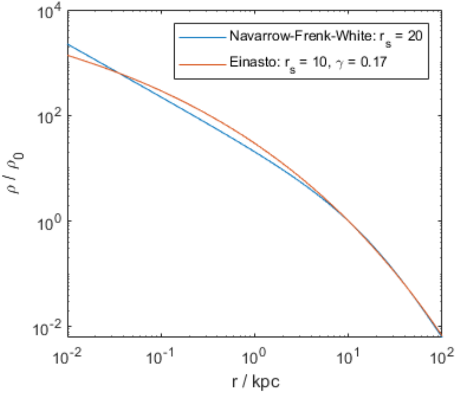
"Cuspy" profiles favored by "dark matter only" simulations.

Navarrow-Frenk-White profile:

$$\rho_{nfw}(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2}$$

Einasto profile:

$$\rho_{ein}(r) = \rho_0 \exp\left(\frac{2}{\gamma}\left(1 - \frac{r^\gamma}{r_s^\gamma}\right)\right)$$



Cusp vs. Core problem

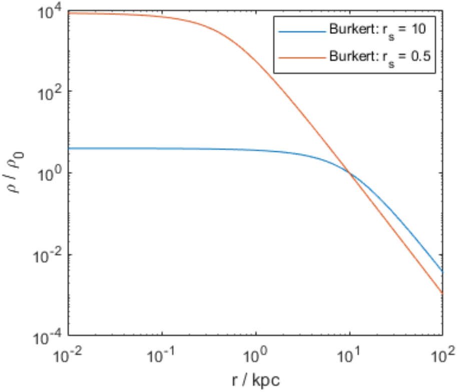
Profiles might be more “cored” than NFW and Einasto profile.

Burkert profile:

$$\rho_{nfw}(r) = \frac{\rho_0}{\left(1 + \frac{r}{r_s}\right)\left(1 + \left(\frac{r}{r_s}\right)^2\right)}$$

Cored profiles favored by

- data from dwarf galaxies
- self interacting models



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