Composite dark matter from non-abelian gauge theories with real representations

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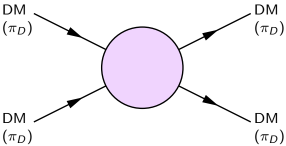
Review: Strongly interacting dark matter

Dark matter from a SM-extension by a **confining, strongly interacting** sector based on dark **gauge group** G_D.

- Dark matter made of bound states, for example **dark pions**.
- Accidental symmetries may explain **stability of dark matter**.
- **Self interactions** may solve structure formation puzzles.

  [Dave et al., The Astrophysical Journal 547 (2001)]



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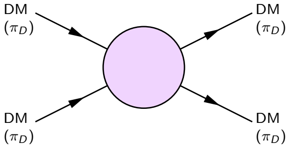

SIMP model: Freeze out via $3 \to 2$ cannibalization process implemented by anomalous Wess Zumino action.

[Hochberg et al., Physical Review Letters 113 (2014)]

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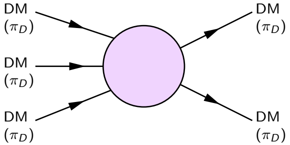


Aim and scope

Research group on strongly interacting $Sp(4)$ dark matter:

- [Kulkarni et al. (2022), arXiv:hep-ph/2202.05191]

Close collaboration with $Sp(4)$ composite Higgs research group:

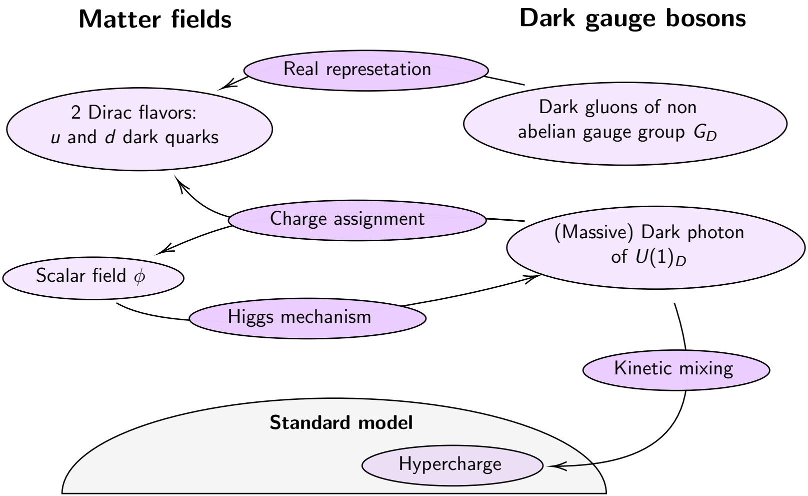
- [Bennett et al., Physical Review D 106 (2022)]

"Combining low energy effective studies and lattice field theory in the context of strongly interacting DM."

- Establish a dark matter model in the UV.
- Study the dark matter IR phenomenology via a low energy effective description.
- Connect UV to IR properties and motivate lattice studies from dark matter phenomenology.

Dark matter model in the UV

Hochberg, Kuflik, and Murayama, Journal of High Energy Physics (2016)



Continuous symmetries

Isolated strong sector:

$U(4)$ — Large N_c

↓ Axial anomaly

$SU(4)$

↓ Chiral quark condensate $\Sigma \neq 0$, Degenerate mass $m_u = m_d \neq 0$

$SO(4)$

Large N_c ↓ Chiral quark condensate $\Sigma \neq 0$, Degenerate mass $m_u = m_d \neq 0$

$O(4)$

$\Rightarrow$ 9 pNGBs π $\quad + 1$ "*pNGB*" η'

Further explicit breaking:

Coupling to $U_D(1)$ → $U(1) \times U(1)$ — **No stable DM**

Coupling to $U_D(1)$ → $SU(2) \times U(1)$ — Stable pions

Charge assignment

Action of flavor symmetry $SO(4) \cong SU_L(2) \times SU_R(2)$

$$\begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} \longmapsto U_L \begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} U_R^\dagger$$

We gauge a 1-parameter subgroup of $SO(4)$ such that:

- $U_D(1)$ gauge theory is consistent
- Pion currents are non anomalous
- Non-abelian flavor symmetry remains

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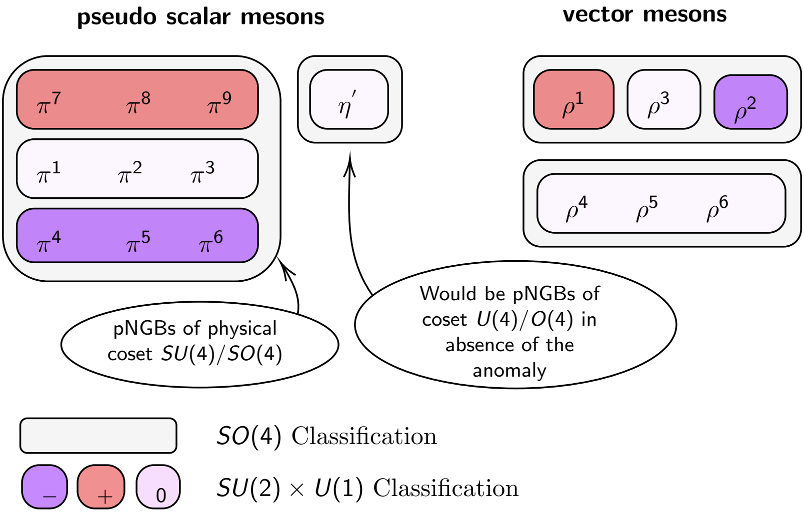
$\Rightarrow$ Anomaly cancellation $\mathcal{Q}^2 = \mathbb{1}$

Charge assignment: $\mathcal{Q} \propto \sigma_{3,R}$

Explict breaking: $SO(4) \to SU(2) \times U(1)$

(Relevant) Particle content



Coset representative and non-linear representation

Coset representative of
$U(4)/O(4)$

$$U := \exp\left(-i\frac{\eta'}{\sqrt{8}f_\eta}\right)\underbrace{\exp\left(-i\frac{\pi}{f_\pi}\right)}_{\in SU(4)/SO(4)}$$

Global $g \in U(4)$ transformation:

$$U \mapsto g\ U\ \underbrace{h[\pi, \eta', g]}_{\in O(4)}$$

Physical interpretation as vacuum fluctuations.

$$\Sigma = U \Sigma_0 U^\top$$

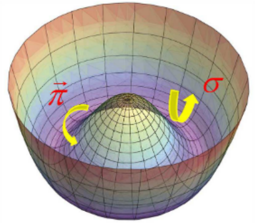


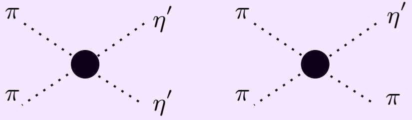
Figure: Taken from [arXiv:hep-ph/1804.05664]

(Non-anomalous) Low energy effective Lagrangian

$$\mathcal{L}_{eff} \;=\; \frac{f_\pi^2}{4}\; Tr\left\{\left(\partial_\mu U^2\right)^\dagger \partial^\mu U^2\right\} \;-\; \frac{f_\eta^2-f_\pi^2}{2f_\eta^2}\partial_\mu\eta'\;\partial^\mu\eta'$$

$$+\;\frac{\mu^3 m}{2}\,Tr\left\{U^2 \;+\; \left(U^2\right)^\dagger\right\} \;-\; \frac{\Delta m_{\eta'}^2}{2}\;\eta'\eta'$$

Contact terms:



Gell-Mann-Oakes-Renner:

$$m_\pi^2 = \frac{m\mu^3}{2\; f_\pi^2}$$

$$m_{\eta'}^2 = m_\pi^2 + \Delta m_{\eta'}^2$$

Topological terms and coset geometry

The symmetric coset space $SU(4)/SO(4)$ has non vanishing 4th homotopy group:

$$\pi_4(SU(4)/SO(4)) \neq 0$$

Problem:

- The standard construction/classification of topological terms in non-linear sigma model by Witten, Weinberg and d'Hoker requires $\pi_4(G/H) = 0$.

  [Witten, Nucl. Phys. B 223 (1983)]

  [D'Hoker and Weinberg, Physical Review D 50 (1994)]

  [Brauner and Kolešová, Nuclear Physics B 945 (2019)]

- Modern approaches give more general classification but no practical construction.

  [Davighi and Gripaios, Journal of High Energy Physics 2018 (2018)]

  [Lee, Ohmori, and Tachikawa, SciPost Physics 10 (2021)]

't Hooft anomaly matching argument

Using the gauge principle to detect anomalous terms in the effective field theory.

Anomaly equation: $\delta S_{eff}^{gauged} = \mathcal{A}$ and $S_{eff} = \lim_{A\to 0} S_{eff}^{gauged}$

Step 1

$$S_{UV} \xrightarrow{\text{Gauge } SU(4)} S_{UV}^{gauged} \xrightarrow{\delta_{SU(4)}} \text{Anomaly}$$

Step 2

$$S_{UV} + S_{Free} \longrightarrow S_{UV}^{gauged} + S_{gauged} \longrightarrow \text{No Anomaly}$$

Step 3

$$S_{eff} + S_{Free} \longrightarrow S_{eff}^{gauged} + S_{gauged} \longrightarrow \text{No Anomaly}$$

Step 4

$$S_{eff} \longrightarrow S_{eff}^{gauged} \longrightarrow \text{Anomaly}$$

The Wess-Zumino term

Chu, Ho, and Zumino, Nuclear Physics B 475 (1996)

Wess and Zumino derived an effective action that solves the anomaly equation. [Wess and Zumino, Physics Letters B 37 (1971)]

Wess-Zumino effective action

$$S_{WZ}[\xi = (\eta', \pi)] = \frac{D_C}{48\pi^2 f_\pi} \int_0^1 \mathrm{d}t \int_{S^4} \mathrm{Tr}\left\{\xi \left((U[t\xi])^{-1} \mathrm{d}U[t\xi]\right)^4\right\}$$

$$\approx \frac{D_C}{250 f_\pi \pi^2} \epsilon^{\mu\nu\sigma\rho} \int_{S^4} \mathrm{Tr}\left\{\pi \partial_\mu \pi \partial_\nu \pi \partial_\sigma \pi \partial_\rho \pi\right\}$$

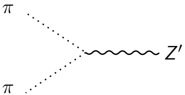
- Does not depend on η' in lowest order χPT.
- Removes superfluous $\pi(t,x) \mapsto -\pi(t,x)$ symmetry in S_{eff}.
- Incorporates $3 \to 2$ process.

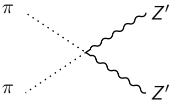
Inclusion of the dark photon Z'

Non-Anomalous contributions:

Exchange derivatives:

$$\partial_\mu U^2 \longmapsto \mathrm{D}_\mu U^2 = \partial_\mu U^2 + ie_D A_\mu \left(\mathcal{Q} U^2 + U^2 \Sigma_0^\dagger \mathcal{Q}^\top \Sigma_0 \right)$$

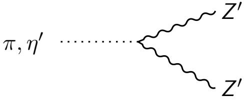




Anomalous contributions:

Anomaly equation:

$$S_{eff} = \lim_{A \to Z'} S_{eff}^{gauged}$$

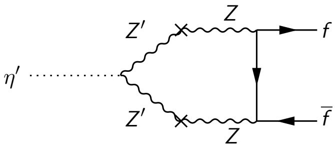


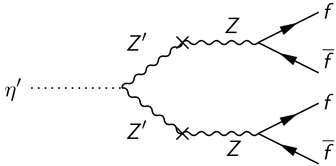
$$\propto \begin{cases} Tr\left\{\pi \ Q^2\right\} & = 0 \\ Tr\left\{\eta' \ Q^2\right\} & \neq 0 \end{cases}$$

η'-decay to Standard Model

Hochberg, Kuflik, and Murayama, Physical Review D 99 (2019)

Depending on lifetime $\tau_{\eta'}$ (and mass m'_η) of the η' the dark matter scenario might be significantly altered or spoiled.





η'-decay may lower the calculated relic abundance of dark matter significantly.

Limiting cases:

- η' decays almost instantly
  $\Rightarrow$ NO DARK MATTER
- $\tau_{\eta'} \approx$ age of universe.
  $\Rightarrow$ Pion abundance not affected.

Summary

- Investigated a class of models with potential explanation for dark matter, thermally coupled to the standard model.
- Worked out IR description including pions and the pseudo scalar singlet.
- Neutral dark pions provide the dark matter candidates.
- The lifetime and mass of the pseudo scalar singlet η' may substantially influence the relic abundance, once the dark photon is coupled.

Bonus content

Discrete symmetries

Charge conjugation C:

UV $q \mapsto \Omega C \bar{q}^{\top}$

IR $\pi \mapsto \pi^{\top}$

Spatial parity P:

UV $q(t, \vec{x}) \mapsto \eta_P \gamma^0 q(t, -\vec{x})$

IR $\pi(t, \vec{x}) \mapsto -\pi(t, -\vec{x})$

The choice $\eta_P = \pm i$ is adopted:

- Parity and flavor symmetries commute.
- All (pseudo) Nambu-Goldstone bosons (pNGB) are pseudo scalars.

Interpolating operators for pions

Bennett et al., Phys. Rev. D 101 (2020)

	$\overline{\Psi_{\mathcal{C}}}\ T_n^{\Psi}\Psi + \left(\overline{\Psi_{\mathcal{C}}}\ T_n^{\Psi}\Psi\right)^*$	J^D		$\overline{\Psi_{\mathcal{C}}}\ T_N^{\pi}\Psi + \left(\overline{\Psi_{\mathcal{C}}}\ T_N^{\pi\top}\Psi\right)^*$	I_3	e_D
π_1	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 d + \overline{d}\gamma^5 u\right)$	1^-	π^A	$\overline{u}\gamma^5 d$	1	0
π_2	$\frac{\imath}{\sqrt{2}}\left(\overline{u}\gamma^5 d - \overline{d}\gamma^5 u\right)$	1^-	π^B	$\overline{d}\gamma^5 u$	-1	0
π_3	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 u - \overline{d}\gamma^5 d\right)$	1^-	π^C	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 u - \overline{d}\gamma^5 d\right)$	0	0
π_4	$\frac{1}{2}\left(\overline{u_{\mathcal{C}}}\gamma^5 u + \overline{u}\gamma^5 u_{\mathcal{C}}\right)$	1^-	π^D	$\frac{1}{\sqrt{2}}\overline{u_{\mathcal{C}}}\gamma^5 u$	-1	-1
π_5	$\frac{1}{2}\left(\overline{d_{\mathcal{C}}}\gamma^5 d + \overline{d}\gamma^5 d_{\mathcal{C}}\right)$	1^-	π^E	$\frac{1}{\sqrt{2}}\overline{d}\gamma^5 d_{\mathcal{C}}$	1	-1
π_6	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 d_{\mathcal{C}} + \overline{u_{\mathcal{C}}}\gamma^5 d\right)$	1^-	π^F	$\overline{u}\gamma^5 d_{\mathcal{C}}$	0	-1
π_7	$\frac{\imath}{2}\left(\overline{u_{\mathcal{C}}}\gamma^5 u - \overline{u}\gamma^5 u_{\mathcal{C}}\right)$	1^-	π^G	$\frac{1}{\sqrt{2}}\overline{u}\gamma^5 u_{\mathcal{C}}$	1	1
π_8	$\frac{\imath}{2}\left(\overline{d}\gamma^5 d_{\mathcal{C}} - \overline{d_{\mathcal{C}}}\gamma^5 d\right)$	1^-	π^H	$\frac{1}{\sqrt{2}}\overline{d}_{\mathcal{C}}\gamma^5 d$	-1	1
π_9	$\frac{\imath}{\sqrt{2}}\left(\overline{u}\gamma^5 d_{\mathcal{C}} - \overline{u_{\mathcal{C}}}\gamma^5 d\right)$	1^-	π^I	$\overline{u_{\mathcal{C}}}\gamma^5 d$	0	1

4th Homotopy group of SU(4)/SO(4)

	π_3	π_4	π_5
$SO(4)$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$SU(4)$	$\mathbb{Z}$	0	$\mathbb{Z}$

Fibration:

$$SO(4) \to SU(4) \to SU(4)/SO(4)$$

Long exact sequence:

$$\begin{array}{ccccccc} \pi_4(SU(4)) & \xrightarrow{h_1} & \pi_4(SU(4)/SO(4)) & \xrightarrow{h_2} & \pi_3(SO(4)) & \xrightarrow{h_3} & \pi_3(SU(4)) \\ 0 & \xrightarrow{h_1} & ? & \xrightarrow{h_2} & \mathbb{Z} \oplus \mathbb{Z} & \xrightarrow{h_3} & \mathbb{Z} \end{array}$$

- $Ker(h_2) = Img(h_1) = 0 \to h_2$ is injective
- $Ker(h_3) = Img(h_2) \cong \pi_4(SU(4)/SO(4))$
- $Ker(h_3) \neq 0$ because $\mathbb{Z} \oplus \mathbb{Z} > \mathbb{Z}$

$\Rightarrow$ $\pi_4(SU(4)/SO(4))$ cannot be trivial

Full UV Lagrangian

Hochberg, Kuflik, and Murayama, Journal of High Energy Physics (2016)

$-\frac{1}{4}\ G^{\alpha}_{\mu\nu}\ G^{\mu\nu}_{\alpha}$ Yang-Mills term for dark gluons : Based on $G_D = Sp(4)$

$-\frac{1}{4}\ F'_{\mu\nu}\ F'^{\mu\nu}$ Yang-Mills term for dark photon : Based on $U_D(1)$

$+\ \overline{u}\ i\ \gamma^{\mu} D_{\mu}\ u\ +\ m\ \overline{u}u$

$+\ \overline{d}\ i\ \gamma^{\mu} D_{\mu}\ d\ +\ m\ \overline{d}d$

Dirac term of dark quarks
2 flavors: u and d quarks
Charged under $G_D \times U_D(1)$

$+\ \left(D'_{\mu}\phi\right)^{\dagger} D'^{\mu}\phi\ +\ V\left[\phi^{\dagger}\phi\right]$ Dark scalar charged under $U_D(1)$

$+\ \frac{\epsilon}{2\cos(\theta_W)}\ F'_{\mu\nu}\ B^{\mu\nu}$ Kinetic mixing of $U_D(1)$ with SM hypercharge

Meson spectrum in 2 flavor Sp(4)-fund. gauge theory

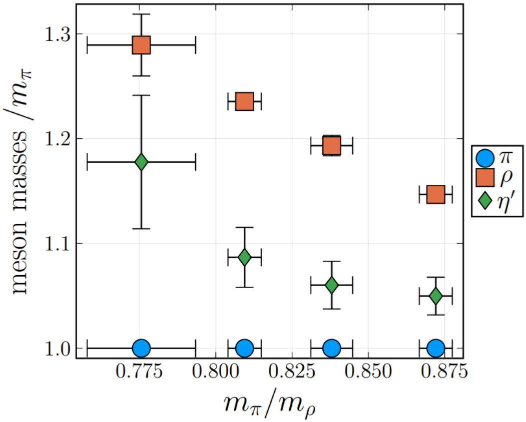


Figure: Meson spectrum in $Sp(4)$-fundamental with 2 flavors.
Taken from [Zierler et al. (2022), arXiv:hep-lat/2210.11187]