A simple way to explain phenomena at the event horizon of a static Black Hole

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Topic overview

1. Advanced concepts of relativistic kinematics
   - Spacetime as manifold
   - Concept of Observer

2. A thought experiment
   - In Schwarzschild coordinates
   - In Kruskal-Szekeres coordinates

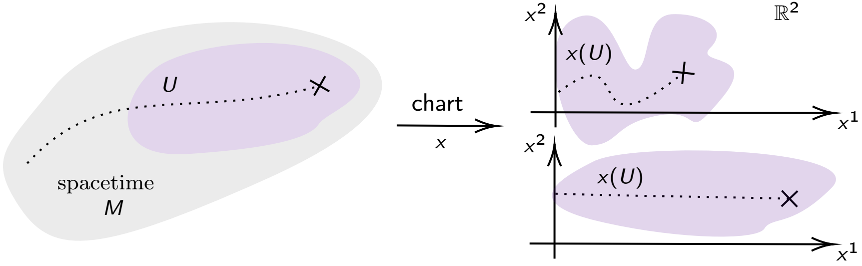
Spacetime as smooth manifold

A spacetime is a smooth manifold $(M, \tau, \mathcal{A}, T, g)$ where

- M a point set (physical events in spacetime)
- τ a topology (a notion of neighborhood and continuity)
- $\mathcal{A}$ an oriented atlas (a collection of charts)
- g a Lorentzian metric (notion of size and shape e.g. geometry)
- T time orientation (global vector field of time flow)

Charts: coordinate systems

A chart is a 1 to 1 (bi-)continuous map, mapping a portion of physical spacetime onto a picture in $\mathbb{R}^d$.

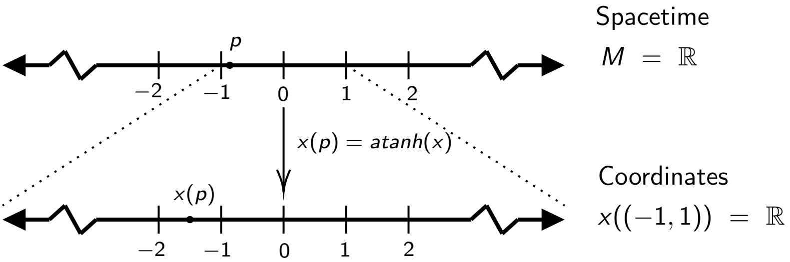


$$x: \; U \subset M \to x(U) \subset \mathbb{R}^2$$
$$p \mapsto (x^1(p), \; x^2(p))$$

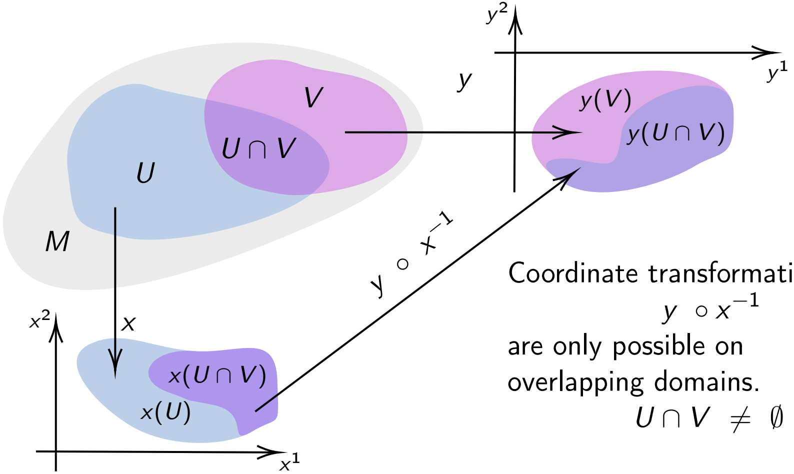
Physical descriptions happen mostly in charts e.g. a specific choice of coordinate system.

Chart ambiguities: Coordinate Stretch

In principle all coordinate space $\mathbb{R}^d$ might be used to only parametrize a portion $U \subset M$ of spacetime.



Change of charts: coordinate transformations



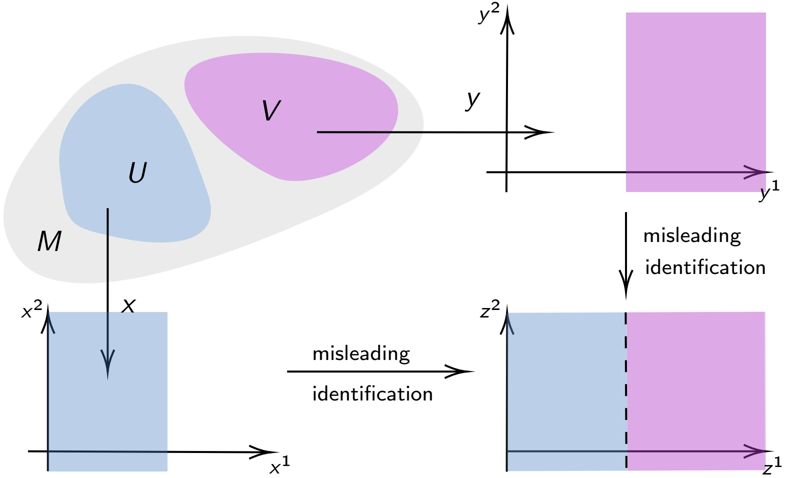
Coordinate transformations

$$y \circ x^{-1}$$

are only possible on overlapping domains.

$$U \cap V \neq \emptyset$$

Chart ambiguities: disconnected charts



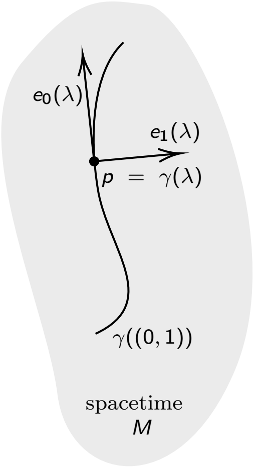
Observer

An **observer** is a tuple $(\gamma, e_0, e_1, \dots)$ where

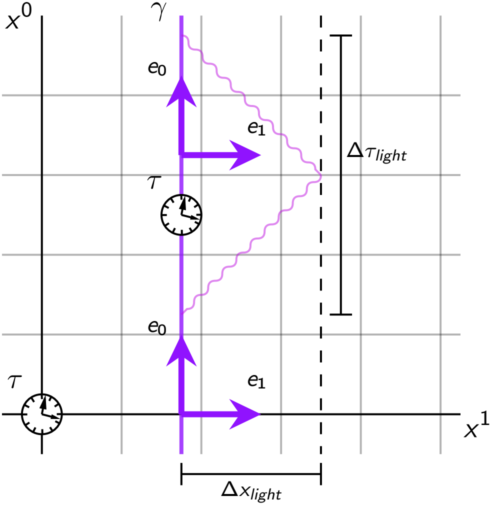
- γ a smooth curve on the manifold $\gamma : (0,1) \to M$
- e_i a orthogonormal frame at every point $g(e_i, e_j) = \eta_{ij}$
- e_0 is tangent to the curve e.g. $e_0 = v_\gamma$

The **observers eigentime** is defined as curve length

$$\tau(\lambda) = \int_0^\lambda \sqrt{g_x(e_0(x), e_0(x))}\,\mathrm{d}x$$



Example: Not so special relativity



Statements :

(M) $M \cong \mathbb{R}^4$

(P) g is **flat** lorentzian metric

(C) $g = c^2\, \mathrm{d}x^0 \mathrm{d}x^0 - \mathrm{d}x^1 \mathrm{d}x^1$

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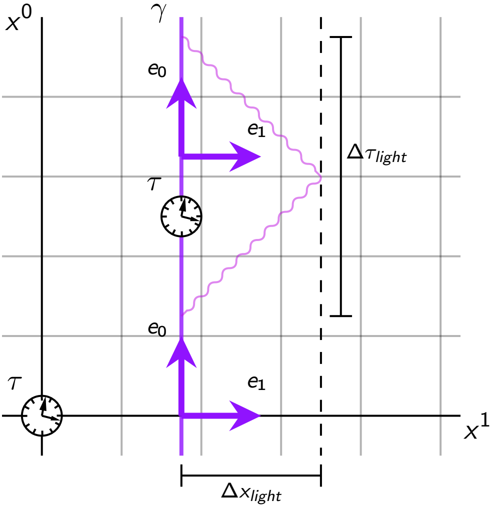

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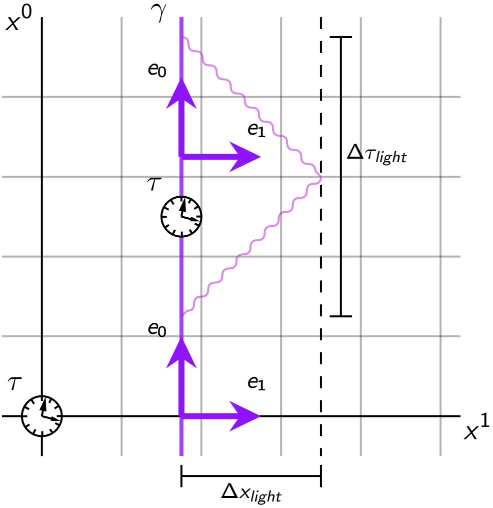

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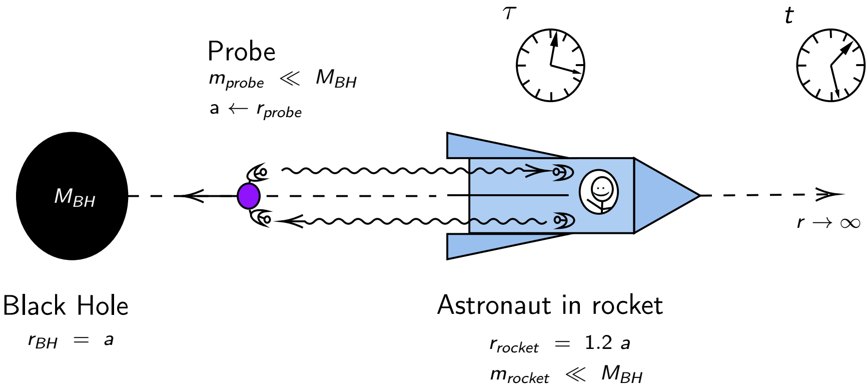
(C) $x^0 = \tau$

(P) speed of light $= c =$ const.

(C) $\Delta x_{light} = \frac{\Delta\tau_{light}}{2} c$

A thought experiment



Scharzschild solution

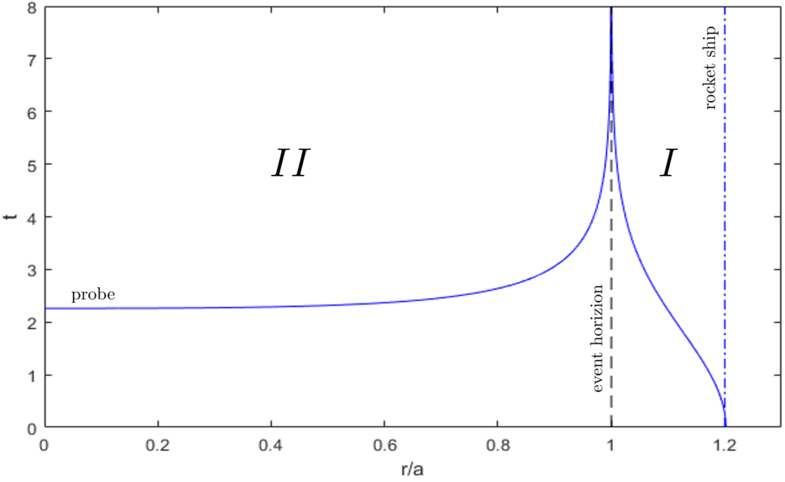
Schwarzschild metric

$$\mathrm{d}s^2 = \frac{r-a}{r}\,\mathrm{d}t^2 - \frac{r}{r-a}\,\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin(\theta)^2\,\mathrm{d}\phi^2\right)$$

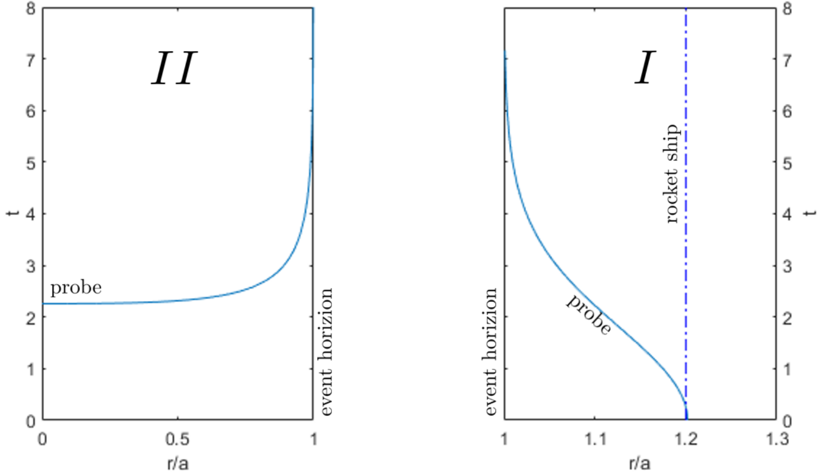
- Neglect angular motion $\rightarrow$ reduce to $2D$ problem
- metric is singular at $r = a$ $\rightarrow$ 2 separate domains

$$\begin{array}{c|c|c} \text{Domain } I & r \in (a, \infty) & t \in (-\infty, \infty) \\ \text{Domain } II & r \in (0, a) & t \in (-\infty, \infty) \end{array}$$

Free falling particle in Schwarzschild chart



Free falling particle in Schwarzschild chart correctly



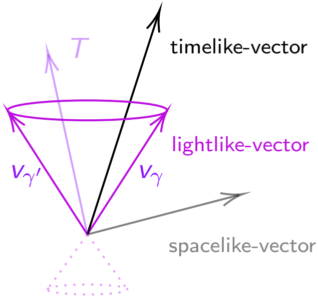
Null geodesics

The trajectories of light are give by the null geodesics

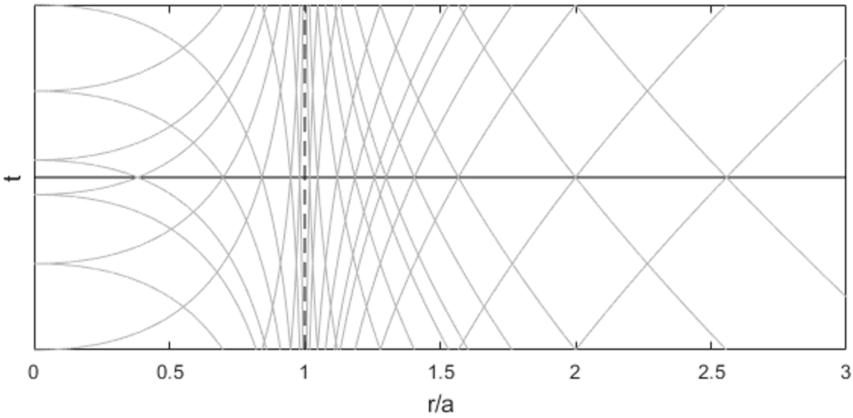
- $\gamma : (0, 1) \to M$ is a geodesic
- $g(v_\gamma, v_\gamma) = 0$

Light (double)cone :

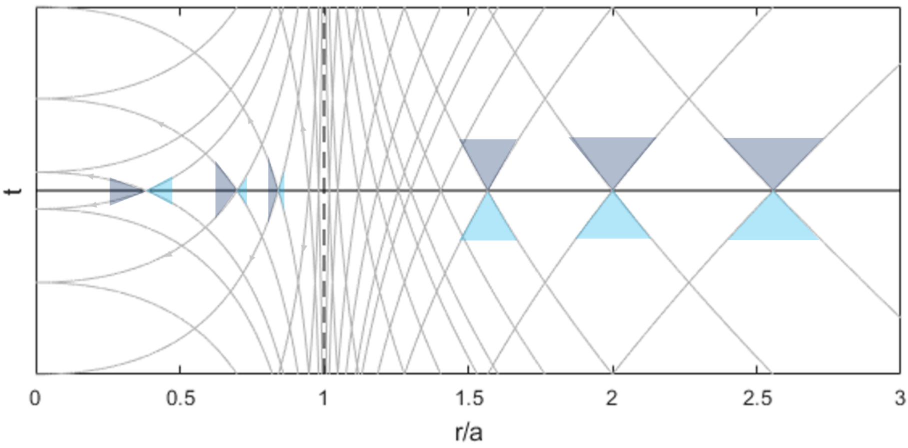
- Defined by null tangent vectors v_γ
- time orientation selects future-cone
- **timelike** vectors inside cone
- **spacelike** vectors outside cone



Null geodesics of Schwarzschild spacetime



Lightcones for Schwarzschild spacetime



Kruskal-Szekeres lightcone-coordinates

Metric in Kruskal-Szekeres lightcone-coordinates

$$g = \underbrace{\frac{a}{r(u,v)} \exp\left(\frac{a - r(u,v)}{a}\right)}_{=:\Omega(u,v)} \, \mathrm{d}u\mathrm{d}v$$

Explict equation for $t(u,v)$

$$t(u,v) = \frac{a}{2} \ln\left(\frac{u^2}{v^2}\right)$$

Implict equation for $r(u,v)$

$$uv = -4a^2 \frac{r-a}{a} \exp\left(\frac{r-a}{a}\right)$$

Extended domains

Discuss solubility of implicit equation for $r(u, v)$

Extended domain *III*

$0 < u < \infty$

$-\infty < v < 0$

Schwarzschild domain *II*

$0 < u < \infty$

$0 < v < \infty$

Solvable for $r(u, v)$ iff $\quad uv < \dfrac{4a^2}{\mathrm{e}}$

Extended domain *IV*

$-\infty < u < 0$

$-\infty < v < 0$

Schwarzschild domain *I*

$-\infty < u < 0$

$0 < v < \infty$

Kruskal-Szekeres coordinates

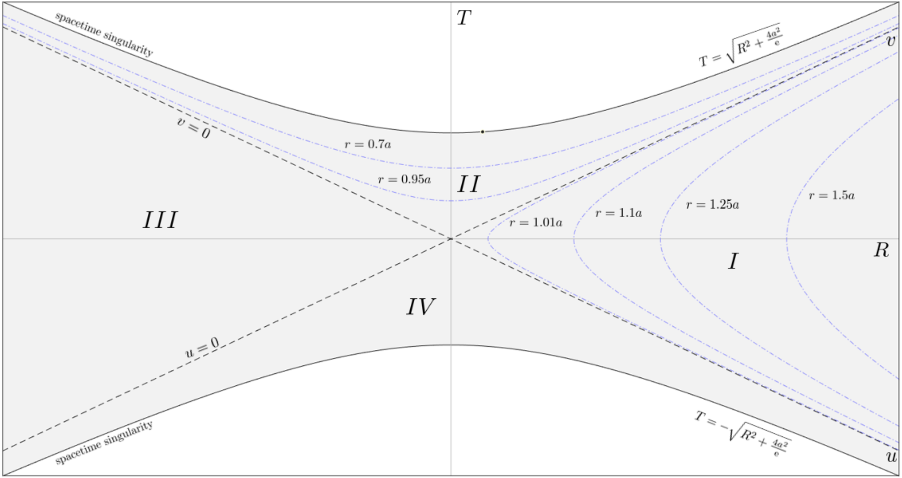
Introduce new coordinates :

- timelike coordinate
$$T = \frac{v + u}{2}$$
- spacelike coordinate
$$R = \frac{v - u}{2}$$

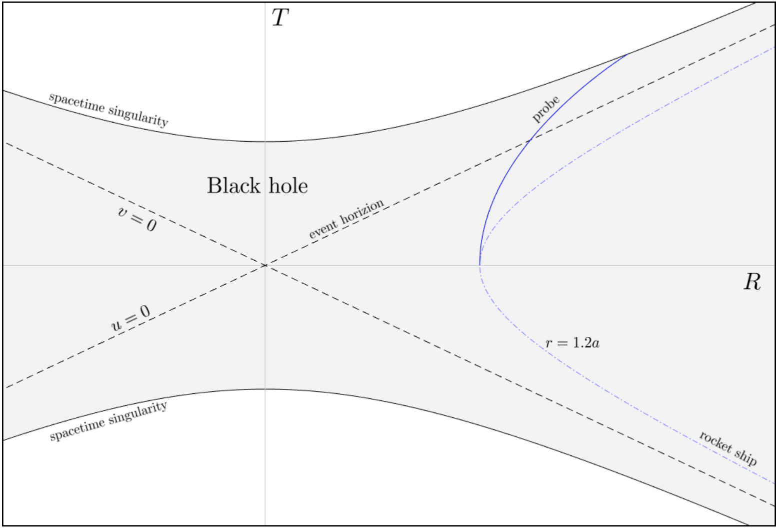
Metric in Kruskal-Szekeres coordinates

$$g = \Omega(T, U)\left(\mathrm{d}T^2 - \mathrm{d}R^2\right)$$

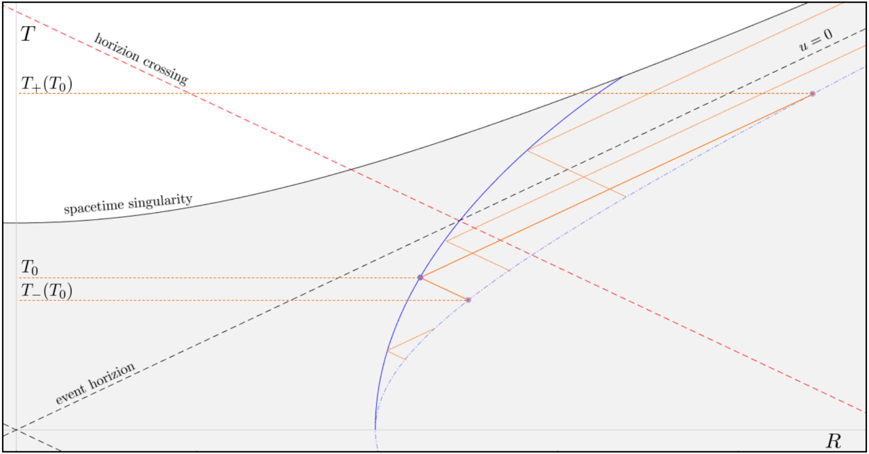
Kruskal-Szekeres-Diagramm



Thought experiment in Kruskal-Szekeres coordinates



Transmission of light signals



Signal transmission times

Transition time measured from by observer

$$\Delta\tau_{Rocket} = \sqrt{\Omega C^2}\left(\sinh^{-1}\left(\frac{T_+}{C}\right) - \sinh^{-1}\left(\frac{T_-}{C}\right)\right)$$

- C Position where the probe is dropped ($C = R_{Rocket}(T = 0)$)
- T_- Kruskal time when the signal is sent.
- T_+ Kruskal time when the signal is recieved.

$$\Delta\tau_{Rocket}(T_+) \xrightarrow[T_+\to\infty]{} \infty$$

Redshifts

What is the redshift of the probes light signal?

Transition time measured by observer

$$z(r_{rocket}, r_{probe}) = \sqrt{\frac{g_{00}(r_{rocket})}{g_{00}(r_{probe})}} - 1 = \sqrt{\frac{(r_{rocket} - a)r_{probe}}{(r_{probe} - a)r_{rocket}}} - 1$$

The redshift diverges when the probe is too close to the horizion

$$z \xrightarrow[r_{probe} \to a]{} \infty \qquad \Rightarrow \quad \textbf{information loss}$$

Consistent time orientation

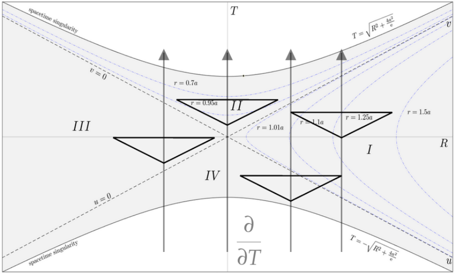
We want to establish an over all consistent notion of **future**.

We can choose

$$\mathcal{T} = \frac{\partial}{\partial T}$$

This fulfills the condition

$$g(\mathcal{T}, \mathcal{T}) > 0$$



Then we select light cones in direction of $\mathcal{T}$ as future light cones

Summary

- Chart ambiguities in Schwarzschild coordinates
- A non pathological chart for black hole spacetime
- What a probe will measure while falling into the black hole
- Light signal transmission to an outside observer
- Consistent notion of future and time orientation