

# Number Theory (Core)

Click on a question number to see how your answers were marked and, where available, full solutions.

Question Number	Score
<b>Modular Arithmetic</b>	
Question 1	1 / 1
Question 2	1 / 1
Question 3	1 / 1
Question 4	1 / 1
<b>Euclidean Algorithm</b>	
Question 5	2 / 3
Question 6	2 / 3
<b>Factorisation</b>	
Question 7	4 / 4
Question 8	4 / 4
<b>Base Conversion</b>	
Question 9	1 / 1
Question 10	1 / 1
Question 11	1 / 1
<b>Total</b>	<b>19 / 21 (90%)</b>

Congratulations, you passed this quiz with a sufficient score. You may include this attempt as part of your self-assessment evidence.

Make sure that you click on "Print this results summary" and save to pdf, so that everything can be read clearly. Do not navigate away from this page before you have saved your result.

# Performance Summary

Exam Name:	Number Theory (Core)
Session ID:	16048924886
Exam Start:	Sat Mar 16 2024 00:07:37
Exam Stop:	Sat Mar 16 2024 00:30:10
Time Spent:	0:22:33

## Question 1

### Modular Arithmetics

Please solve the following modular operations.

$$26 \pmod{4} =$$



Expected answer: 2



Your answer is correct. You were awarded 1 mark.

You scored 1 mark for this part.

Score: 1/1

## Question 2

### Modular Arithmetics

Please solve the following modular operations.

$$-152 \pmod{6} = \boxed{4} \quad \checkmark$$

Expected answer: 4

When  $p$  is negative, we need to find a number  $k$  such that  $p + k \times m$  is between 0 and  $m - 1$ . That number is the modulus.

(Your score will not be affected.)

 Your answer is correct. You were awarded **1** mark.

You scored **1** mark for this part.

**Score: 1/1** 

## Question 3

# Modular Arithmetics

Please solve the following modular operations.

## Modular Arithmetic

Find a number between 42 and 48 which satisfies this equation:

$$\boxed{42} \quad \checkmark$$

Expected answer: 42  $\pmod{7} = 0$

When  $p$  is positive, the result of the modulo operation is the remainder of the deviation of  $p$  by  $m$ .

(Your score will not be affected.)

 Your answer is correct. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1 ✓

## Question 4

### Modular Arithmetics

Please solve the following modular operations.

#### Modular Arithmetic

Find a number which satisfies this equation:

$$148 \pmod{\boxed{148}} \quad \checkmark \quad \left( \text{Expected answer: } \underline{4} \right) = 0$$

✓ Your answer is correct. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1 ✓

## Question 5

### Euclidean Algorithm

Apply the Euclidean algorithm to find the greatest common denominator between 2001 and 246. Show the steps.

#### Part a) Algorithm

Please enter all the steps of the Euclidean algorithm below (please enter the number of steps you need in the "Rows:" box.)

Rows:  Columns:

(     ) **×**

Expected answer:

2001	8	246	33
246	7	33	15
33	2	15	3
15	5	3	0

Wrong number of steps.

You scored **0** marks for this part.

Score: 0/1 **×**

### Part b) gcd

gcd(2001,246)=



Expected answer:

✓ Your answer is correct. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1 **✓**

### Part c) LCM

## Lowest Common Multiple

Use the information above to calculate the lowest common multiple between 2001 and 246.

$\text{lcm}(2001, 246) =$ 

164082

Expected answer: 164082

✓ Your answer is correct. You were awarded 1 mark.  
You scored 1 mark for this part.

Score: 1/1 ✓

## Advice

In order to apply the Euclidean algorithm to find the gcd, you need to apply integer division repeatedly, until the remainder is 0.

Once you know the gcd between two numbers  $p$  and  $q$  you can find their lcm using the following formula:

$$\text{lcm}(p, q) = \frac{p \times q}{\text{gcd}(p, q)}$$

## Question 6

### Euclidean Algorithm

Apply the Euclidean algorithm to find the greatest common denominator between 544 and 248. Show the steps.

#### Part a) Algorithm

Please enter all the steps of the Euclidean algorithm below (please enter the number of steps you need in the "Rows:" box.)

Rows:  Columns:

(     ) **×**

Expected answer:  $\begin{pmatrix} \frac{544}{248} & \frac{2}{5} & \frac{248}{48} & \frac{48}{8} \\ \frac{48}{6} & \frac{8}{8} & \frac{0}{0} \end{pmatrix}$

Wrong number of steps.

You scored **0** marks for this part.

Score: 0/1 **×**

Part b) gcd

gcd(544,248)=



Expected answer: 8



Your answer is correct. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1 **✓**

Part c) LCM

## Lowest Common Multiple

Use the information above to calculate the lowest common multiple between 544 and 248.

lcm(544,248)=



Expected answer: 16864

✓ Your answer is correct. You were awarded 1 mark.

You scored 1 mark for this part.

Score: 1/1 ✓

## Advice

In order to apply the Euclidean algorithm to find the gcd, you need to apply integer division repeatedly, until the remainder is 0.

Once you know the gcd between two numbers  $p$  and  $q$  you can find their lcm using the following formula:

$$\text{lcm}(p, q) = \frac{p \times q}{\text{gcd}(p, q)}$$

## Question 7

### Factorisation

In this question we will compute the gcd and lcm of two numbers using their prime factorisations.

To enter the prime factorisation of a number, use  $*$  to denote multiplication and  $^$  for power. For example, the prime factorisation of 12 would be entered as  $3*2^2$ .

p

Enter the prime factorisation (in exponent form) of 240.

$2^4*3*5$   $2^4 \times 3 \times 5$  ✓

Expected answer:  $2^4*3^1*5^1$   $2^4 \times 3^1 \times 5^1$

✓ Your answer is numerically correct. You were awarded 1 mark.



You scored **1** mark for this part.

Score: 1/1 ✓

q

Enter the prime factorisation (in exponent form) of 72.

$$\boxed{2^3 \times 3^2} \quad \checkmark$$

Expected answer:  $2^3 \times 3^2$

✓ Your answer is numerically correct. You were awarded **1** mark.  
You scored **1** mark for this part.

Score: 1/1 ✓

c)

Using the information above, enter the gcd and the lcm of 240 and 72.

$$\text{gcd}(240, 72) = \boxed{24} \quad \checkmark$$

Expected answer: 24

$$\text{lcm}(240, 72) = \boxed{720} \quad \checkmark$$

Expected answer: 720

Gap 0

Gap 1

Score: 2/2

## Advice

To find the prime factorisation of a number, you can start from finding all the prime numbers below the square root of that number.

## Question 8

# Factorisation

In this question we will compute the gcd and lcm of two numbers using their prime factorisations.

To enter the prime factorisation of a number, use  $*$  to denote multiplication and  $^$  for power. For example, the prime factorisation of 12 would be entered as  $3*2^2$ .

p

Enter the prime factorisation (in exponent form) of 660.

$2^2*3*5*11$   $2^2 \times 3 \times 5 \times 11$  ✓

Expected answer:  $2^2*3^1*5^1*11^1$   $2^2 \times 3^1 \times 5^1 \times 11^1$

✓ Your answer is numerically correct. You were awarded 1 mark.  
You scored 1 mark for this part.

Score: 1/1 ✓

q

Enter the prime factorisation (in exponent form) of 150.

$2*3*5^2$   $2 \times 3 \times 5^2$  ✓

Expected answer:  $2^1*3^1*5^2$   $2^1 \times 3^1 \times 5^2$

✓ Your answer is numerically correct. You were awarded 1 mark.  
You scored 1 mark for this part.

Score: 1/1 ✓

c)

Using the information above, enter the gcd and the lcm of 660 and 150.

$$\gcd(660, 150) = \boxed{30} \quad \checkmark$$

Expected answer: 30

$$\text{lcm}(660, 150) = \boxed{3300} \quad \checkmark$$

Expected answer: 3300

Gap 0

Gap 1

Score: 2/2

## Advice

To find the prime factorisation of a number, you can start from finding all the prime numbers below the square root of that number.

## Question 9


Convert 8661 to base 5

$$8661_{10} = \boxed{234121} \quad \checkmark$$

Expected answer: 234121<sub>5</sub>

 Your answer is correct. You were awarded 1 mark.

You scored 1 mark for this part.

Score: 1/1 

## Advice

To solve this problem you can use one of two methods: either you divide by the powers of 5 continually, or divide by 5 itself. For example:

$$8661 \div 5 = 1732 \quad R1$$

$$1732 \div 5 = 346 \quad R2$$

$$346 \div 5 = 69 \quad R1$$

$$69 \div 5 = 13 \quad R4$$

$$13 \div 5 = 2 \quad R3$$

$$2 \div 5 = 0 \quad R2$$

Then, using the remainders, we find that  $8661_{10} = 234121_5$ .

## Question 10

Convert 6984 to base 6

$$6984_{10} =$$

52200



Expected answer: 52200<sub>6</sub>



Your answer is correct. You were awarded 1 mark.

You scored 1 mark for this part.

Score: 1/1

## Advice

To solve this problem you can use one of two methods: either you divide by the powers of 6 continually, or divide by 6 itself. For example:

$$6984 \div 6 = 1164 \quad R0$$

$$1164 \div 6 = 194 \quad R0$$

$$194 \div 6 = 32 \quad R2$$

$$32 \div 6 = 5 \quad R2$$

$$5 \div 6 = 0 \quad R5$$

Then, using the remainders, we find that  $6984_{10} = 52200_6$ .

# Question 11

Convert  $1001_3$  from base 3 to base 10.

$$1001_3 = \boxed{28}$$



Expected answer: 28<sub>10</sub>



Your answer is correct. You were awarded **1** mark.

You scored **1** mark for this part.

**Score: 1/1** 

## Advice

To solve this problem you can just multiply each digit by the corresponding powers of 3:

=28

$$1 \cdot 3^3 + 0 \cdot 3^2 + 0 \cdot 3^1 + 1 \cdot 3^0$$

Then, using the remainders, we find that  $1001_3 = 28_{10}$ .

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