

18/ $-5n^2 + 9n + 7$ is odd for any $n \in \mathbb{N}$.

Ans n is even : $n = 2k$

$$\begin{aligned} -5n^2 + 9n + 7 &= -5(2k)^2 + 9(2k) + 7 \\ &= -20k^2 + 18k + 7 \\ &= 2 \times (-10k^2 + 9k) + 7 \end{aligned}$$

n is odd : $n = 2k+1$,

$$\begin{aligned} -5n^2 + 9n + 7 &= -5(2k+1)^2 + 9(2k+1) + 7 \\ &= -5(4k^2 + 4k + 1) + 18k + 9 + 7 \\ &= -20k^2 - 20k - 5 + 18k + 16 \\ &= -20k^2 - 2k + 11 \\ &= 2 \times (-10k^2 - k) + 11 \end{aligned}$$

Sum of even and odd is odd.

19/ $-5n^2 - n + 2$ is even for any $n \in \mathbb{N}$

n is odd : $n = 2k+1$

$$\begin{aligned} -5n^2 - n + 2 &= -5(2k+1)^2 - (2k+1) + 2 \\ &= -5(4k^2 + 4k + 1) - 2k - 1 + 2 \\ &= -20k^2 - 20k - 5 - 2k - 1 + 2 \\ &= -20k^2 - 22k - 4 \\ &= 2 \times (-10k^2 - 11k - 2) \end{aligned}$$

n is even : $n = 2k$

$$\begin{aligned} -5n^2 - n + 2 &= -5(2k)^2 - (2k) + 2 \\ &= -20k^2 - 2k - 2 \\ &= 2 \times (-10k^2 - k - 1) \end{aligned}$$

Which is even

20. Show that $n^2 - 5n - 10$ is even for any $n \in \mathbb{N}$

n is even: $n = 2k$

$$\begin{aligned}n^2 - 5n - 10 &= (2k)^2 - 5 \times (2k) - 10 \\&= 4k^2 - 10k - 10 \\&= 2 \times (2k^2 - 5k - 5)\end{aligned}$$

n is odd: $n = 2k + 1$

$$\begin{aligned}n^2 - 5n - 10 &= (2k + 1)^2 - 5 \times (2k + 1) - 10 \\&= (4k^2 + 4k + 1) - 10k - 5 - 10 \\&= 4k^2 + 4k + 1 - 10k - 15 \\&= 4k^2 - 6k - 14 \\&= 2 \times (2k^2 - 3k - 7)\end{aligned}$$

~~Therefore, $n^2 - 5n - 10$ is even for any $n \in \mathbb{N}$.~~

\therefore Which is even.