Date

Recurrence

Q1/ Prove that $n^2 - 4n + 3 \geq 0$ for any $n \geq 3$

Base step

prove case for $n = 4$

$4^2 - 4 \times 4 + 3 = 16 - 16 + 3 = 3 > 0$

Since 3 is greater than 0, the base case holds true.

We need to prove $(k+1)^2 - 4(k+1) + 3 > 0$

$(k+1)^2 + 4(k+1) + 3 = k^2 + 2k + 1 - 4k - 4 + 3$

$k^2 + 2k + 1 - 4k - 4 + 3 = k^2 - 4k + 3 + 2k - 1$

$k^2 - 4k + 3 > 0$. There for we can write

$k^2 - 4k + 3 + 2k - 1 > 2k - 1$

For $k > 3$, $2k - 1$ is always positive

$(k+1)^2 - 4(k+1) + 3 > 0$

We have proven that $n^2 - 4n + 3 > 0$ for all $n > 3$

Recurrence

Q2. Prove that $9^n + 7$ is divible by 8 for any $n \geq 0$

Base case

1st prove the base case $n = 0$

$9^0 + 7 = 1 + 7 = 8$

Since 8 is divisible by 8 the base case holds true

Induction step

prove that $9^{k+1} + 7$ is divisble by 8

$9^{k+1} + 7 = 9 \times 9^k + 7$

$9^k = 8m - 7$

$9^{k+1} + 7 = 9 \times (8m - 7) + 7$

$9 \times (8m - 7) + 7 = 72m - 63 + 7 = 72m - 56$

$72m - 56 = 8(9m - 7)$

Since $8(9m - 7)$ is clearly divisble by 8, $9^{k+1} + 7$ is divisible by 8

We have proven that $9^n + 7$ is divislble by 8 for all $n \geq 0$