No. 01

Date 2024.03.28

DFCS|DK|C2|203

1. Problems

Exercise 1

1/ Prove or disprove $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} : x - y^3 = 0$

- This statement is true

  * To prove the first statement true ($P(x)$), we need to show that for any x chosen from the real numbers, there exists a real number y such that $x - y^3 = 0$. This can be proved by providing a general argument that ensures there's always a y satisfying the equation for any x.

Since $Q(x,y)$ is true for the given $y = \sqrt[3]{x}$, and $P(x)$ is true for any x due to the nature of cube roots, both statements are statements are indeed true.

2/ Prove or disprove $\exists y \in \mathbb{R}, \forall x \in \mathbb{R} : x - y^3 = 0$

- This statement is false

  * To prove the secound statement false ($Q(x,y)$) we need to provie a counter examples showing that there exists at least one real number x for which the equation. $x - y^3 = 0$ does not hold true for all y. This counter example can depends on the value of y

To show $Q(x,y)$ false, consider $x = y^3 + 1$. If we plug this into the equation $x - y^3 = 0$, we get $(y^3 + 1) - y^3 = 1 \neq 0$ so $Q(x,y)$ is false for this x and y

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Prove or disprove $\forall x \in \mathbb{N}, \exists y \in \mathbb{N} : x - y^3 = 0$

* This statement is false
  * We can take $x = 2$. For any y in $\mathbb{N}$, y^3 cannot be equal to 2 because the cube root of 2 is not a natural number.

To prove the statement false $(Q(x,y))$, we need to show that there's no single natural number y for which the equation $x - y^3 = 0$ holds true for all natural numbers x. This is because the cube root of some natural numbers may not be a natural number itself.

Prove or disprove $\exists y \in \mathbb{N}, \forall x \in \mathbb{N} : x - y^3 = 0$

This statement is false.

* To prove the statement false $(Q(x,y))$, we need to provide a counter example showing thats there's at least one natural number x for which the equation $x - y^3 = 0$ does not hold true for a particular natural number y.

To show $Q(x,y)$ false, consider $x = y^3 + 1$. If we plug this into the equation $x - y^3 = 0$, we get $(y^3 + 1) - y^3 = 1 \neq 0$; so $Q(x,y)$ is false for this particular x and y

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09/ Prove or disprove $\forall x \in N, \exists y \in N : (x+y)^2 = x^2 + 6x + y^2$

- This statement is true

* To prove statement true ($Q(x,y)$), we just need to provide an example of such a y for any given x, and this example can depend on the value of x. Here, the example provided is $y=3$

Since $(x+y)^2 = (x+3)^2 = x^2 + 6x + 9$ we see that $Q(x,y)$ is true for $y=3$.

06/ Prove or disprove $\exists y \in N, \forall x \in N : (x+y)^2 = x^2 + 6x + y^2$

- This statement is true

* To prove statement true ($Q(x,y)$) we need to provide a general argument showing that for any natural number x, the equation $(x+y)^2 = x^2 + 6x + y^2$ holds true for particular natural number

This equation holds true for all natural numbers x, Therefore, $Q(x,y)$ is true for $y=3$

07/ Prove or disprove $\exists y \in N, \exists x \in N : x - y = 0$

- This statement is true.

* To prove the statement true ($Q(x,y)$) we can provide an example where $x - y = 0$ holds. We take $x=0$, and indeed for any y; $0 - y = 0$ so $Q(x,y)$ is true

* ($P(y)$), we can provide an example where $x - y = 0$ holds. We take $y = 0$ and indeed for $x = 0$, $0 - 0 = 0$ so $P(y)$ is true.

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08/ Prove or disprove $\forall y \in \mathbb{N}, \exists x \in \mathbb{N}: x-y=0$

- This statement is true.

* To prove this statement true we need to show that for any natural number y. there exists a natural number x such that $x-y=0$. This is true because we can always choose $x=y$. Then $x-y=y-y=0$.

(Q(x,y)) we can provide an example where $x-y=0$ holds. We can take $x=y$ and indeed $x-y=y-y=0$ so $Q(x,y)$ is true for $x=y$.

09/ Prove or disprove $\forall y \in \mathbb{N}, \forall x \in \mathbb{N}: x-y=0$

- This statement is false.

* To prove this statement fake (P(y)), we need to find a countexample where $x-y\neq 0$ for some natural number y. We can take $y=0$. For any x in $\mathbb{N}$, $x-0=x$ and it's evident that not all numbers equal zero.

To prove the secound statemet false (Q(x,y)), we need to find a counter example where $x-y\neq 0$ for some natural number x any $y=1$. We can take $x=1$. Then $Q(1,1)$ becomes $1-1=0$

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10/ Prove or disprove $\exists x \in \mathbb{N}, \forall y \in \mathbb{N} : x - y = 0$

- This statement is false

* To prove this statement false ($Q(x,y)$) we need to provide a counter example where $x - y \neq 0$ for some natural number x and $y = x - 1$. We can take $y = x - 1$ Then $Q(x, x-1)$ becomes $x - (x-1) = 1$ which is false.

11/ Prove or disprove $\exists y \in \mathbb{R}, \exists x \in \mathbb{N} : x - y = 0$

- This statement is true.

* To prove this statement true ($Q(x,y)$) we can provide an example where $x - y = 0$ holds. We can take $x = 0$, and indeed for any real number y, $0 - y = 0$ so $Q(x,y)$ is true for $x = 0$

12/ Prove or disprove $\forall y \in \mathbb{R}, \exists x \in \mathbb{N} : x - y = 0$

- This statement is false.

* To prove this statement false ($Q(x,y)$) we need to provide a general argument showing that $x - y = 0$ is always false. For $(x, 1/2)$, the equation $x - \frac{1}{2} = 0$ implies $x = 1/2$. But $1/2$ is not a natural number ($1/2 \notin \mathbb{N}$), so the statement is always false.

13/ Prove or disprove $\forall y \in \mathbb{R}, \forall x \in \mathbb{N} : x - y = 0$

- This statement is false.

* To prove this statement false ($Q(x,y)$), we need to provide a counter example where $x - y \neq 0$ for some natural number x and $y = 1$. We can take $x = 1$. Then $Q(1,1)$ becomes $1 - 1 = 0$ which is false.

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14/ Prove or disprove $\exists x \in N, \forall y \in \mathbb{R} : x - y = 0$

. This statement is false.

* To prove the statement false $(Q(x,y))$, we need to find a counter example where $x - y \neq 0$ for some real number y. We can take $y = x - 1$ Then $Q(x, x-1)$ bezomes $x - (x-1) = 1$

15/ Prove or disprove $\exists y \in \mathbb{N}, \exists x \in \mathbb{R} : x - y = 0$

. This statement is true

* To prove the statement true $(Q(x,y))$, we can provide an example where $x - y = 0$ holds. We can take $x = 0$ and indeed for any real number y, $0 - y = 0$ so $Q(x,y)$ is true for $x = 0$

16/ Prove or disprove $\forall y \in \mathbb{N}, \exists x \in \mathbb{R} : x - y = 0$

. This statement is true.

*. To prove the statement $(P(y))$, we need to show that for any natural number y. there exists a real number x such that $x - y = 0$. This is indeed true because for any y in $\mathbb{N}$, we can simply choose $x = y$. Then $x - y = y - y = 0$

17/ Prove or disprove $\forall y \in \mathbb{N}, \forall x \in \mathbb{R} : x - y = 0$

. This statement is false.

* To prove the secound statement false $(Q(x,y))$, we need to provide a counter example where $x - y \neq 0$ for some ral number x and $y = 1$. We can take $x = 1$ Then $Q(1,1)$ becoomse $1 - 1 = 0$ which false.

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1s/ Prove or disprove $\exists x \in \mathbb{R}, \forall y \in \mathbb{N} : x - y = 0$

- This statement is false.

* To prove the statement false $(Q(x, y))$, we need to provide a counter example where $x - y \neq 0$ for some natural number y. We can take $y = x - 1$. Then $Q(x, x-1)$ becomes $x - (x - 1) = 1$ which is false.