Complexity

Q1/ Show that $6 \cdot \log(x) + x^2 + 3 \cdot x^x = O(x^x)$

$\log(x) \leq x$ for $x > 4 \Rightarrow 4 \times \log(x) \leq 6 \times x$

$x^2 \leq x \times x = x^2 \leq x^x$ for $x > 4$

$3 \times x^x = 3 \times x^x$ for $x > 0$

Step 2

$6 \times \log(x) \leq 6 \times x \leq 6 \times x^2$

$x^2 < x^x$

$3 \times x^x = 3 \times x^x$

Step 3

$6 \times \log(x) + x^2 + 3 \times x^x \leq 6 \times x^x + x^x + 3 \times x^x$

$6 \times \log(x) + x^2 + 3 \times x^x \leq 10 \times x^x$

Step 4

$6 \times \log(x) + x^2 + 3 \times x^x \leq 10 \times x^x$ for $x > 4$

~~for~~ $f(x) = 6 \times \log(x) + x^2 + 3 \times x^x = O(x^x)$

Constants $C = 10$ and $k = 4$

Date

7/ Show that $-3 \cdot \log(x) + 6 \times x^2 + 6 \times x^x = O(x^x)$

For $-3 \cdot \log(x)$ $|\log(x)| \le x$ for $x > 4 \rightarrow -3 \cdot \log(x) \le 3 \cdot x$

for $6 \cdot x^2$ -

$x^2 < x \cdot x = x^2 \le x^x$ for $x > 4 \rightarrow 6 \cdot x^2 \le 6 \cdot x^x$

For $\cdot x^x$ $\quad 6 \cdot x^x = 6 \cdot x^x$ for $x > 0$

Step 3

$-3 \cdot \log(x) + 6 \cdot x^2 + 6 \cdot x^x \le 3 \cdot x^x \le 3 \cdot x^x + 6 \cdot x^x + 6 x^x$

$-3 \cdot \log(x) + 6 \cdot x^2 + 6 \cdot x^x \le 15 \cdot x^x$

Step 4 -

$-3 \cdot \log(x) + 6 \cdot x^2 + 6 \cdot x^x \le 15 \cdot x^x$ for $x > 4$

$f(x) = -3 \cdot \log(x) + 6 \cdot x^2 + 6x^x = O(x^x)$

3/ Show that $\log(x) - 8 \cdot x \log(x) + 3 \cdot x^x = O(x^x)$

Step 1

$\log(x) \le x^x$ for $x > 4$

$x \log(x) \le x \cdot x = x^x$ for >4

$x^x = x^2$ for $x > 0$

Step 3

$\log(x) - 8 \cdot x \log(x) + 3 \cdot x^x \le x^x + 8 \cdot x^x + 3 \cdot x^x$

$\log(x) - 8 \cdot x \log(x) + 3 \cdot x^x \le 12 \cdot x^2$

Step 4

$\log(x) - 8 \cdot x \log(x) + 3 \cdot x^x \le 12 \cdot x^x$ for $x \ge 4$

$f(x) = \log(x) - 8 \cdot x \log(x) + 3 \cdot x^2 = O(x^x)$