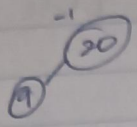
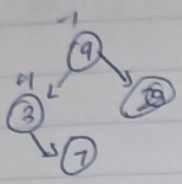


1/ insert (20):



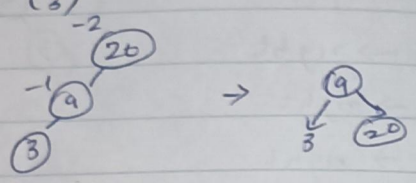
$9 < 20$   
 $\rightarrow$  to left subtree  
 $\rightarrow$  left is null, insert 20 to left.

2/ insert (7):



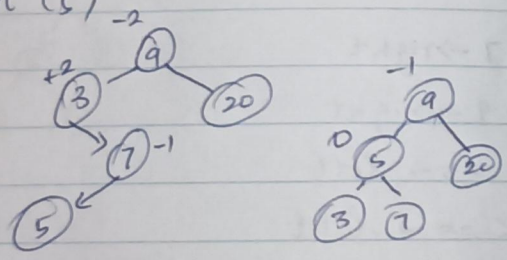
$7 < 9 \rightarrow$  to left  
 $7 > 3 \rightarrow$  to right  
 3's right is null, insert 7

3/ insert (3):



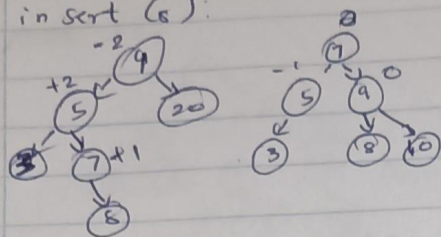
$3 < 20 \rightarrow$  to left subtree  
 $3 < 9 \rightarrow$  to left subtree  
 the tree's left is heavier  
 $\rightarrow$  rotate right.

in serc (5)



$5 < 9 \rightarrow$  to left  
 $5 > 3 \rightarrow$  to right  
 $5 < 7 \rightarrow$  to left  
 7's left is null, insert 5  
 3's right is right heavy  
 $\rightarrow$  double rotate left.

6/ insert (6):



$8 < 9 \rightarrow$  to left

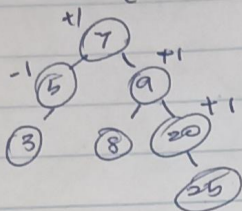
$8 \geq 5 \rightarrow$  to right

$8 \geq 7 \rightarrow$  to right

⑦'s right is null, insert ⑧

tree is left heavy Rotate left right

7/ insert (25)



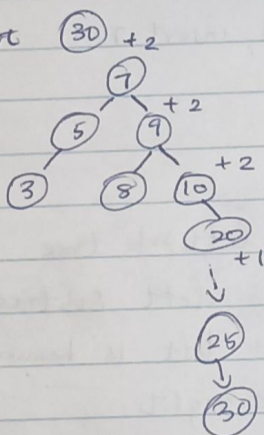
$25 \geq 7 \rightarrow$  right

$25 \geq 9 \rightarrow$  right

$25 \geq 20 \rightarrow$  right

②0's right is null, insert ②5

8/ insert (30)



$30 \geq 7 \rightarrow$  right

$30 \geq 9 \rightarrow$  right

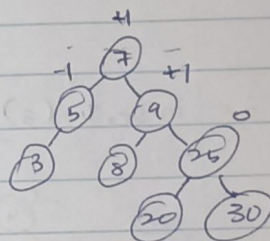
$30 \geq 20 \rightarrow$  right

$30 \geq 25 \rightarrow$  right

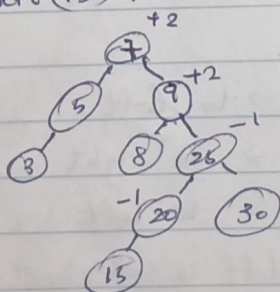
$30 \geq 25 \rightarrow$  right.

②0's tree is right heavy

$\rightarrow$  1 rotate left.



9/ insert (15):



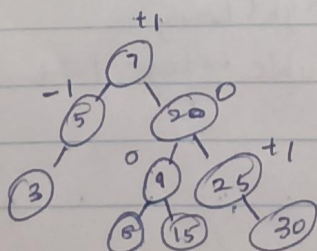
$15 \geq 7 \rightarrow$  right

$15 \geq 9 \rightarrow$  right

$15 < 25 \rightarrow$  left

$15 < 20 \rightarrow$  left

②0's left is null, insert 15

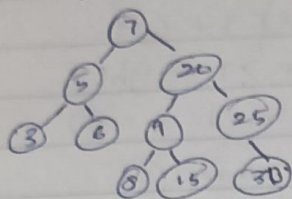


⑨'s tree is right weight

Rotate left right.



10/ insert (6):

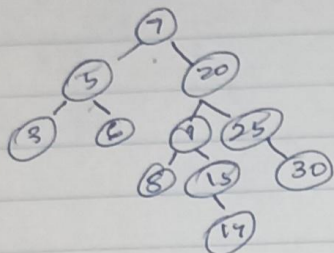


$6 < 7 \rightarrow \text{left}$

$6 \geq 5 \rightarrow \text{right}$

5's right is null  $\rightarrow$  insert 6

11/ insert (17):



$17 \geq 7 \rightarrow \text{Right}$

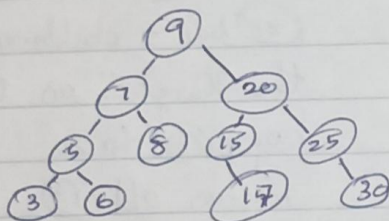
$17 < 20 \rightarrow \text{left}$

$17 \geq 9 \rightarrow \text{Right}$

$17 \geq 15 \rightarrow \text{Right}$

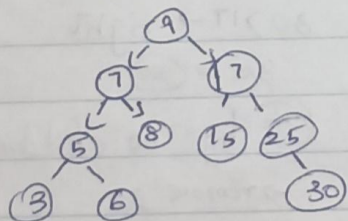
15's Right is null  $\rightarrow$  insert 17

12/



the tree is right heavy  
 $\rightarrow$  double rotate left

13/ Delete 20



$20 > 9 \rightarrow \text{right}$

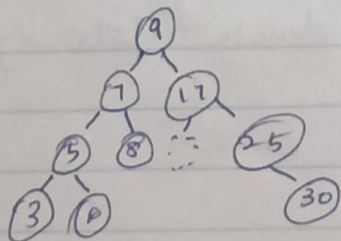
$20 = 20$

node 20 has 2 children

$\rightarrow$  the largest in the left sub tree is 17

$\rightarrow$  copy 17 to 20, remove the old 17

14/ Delete 15

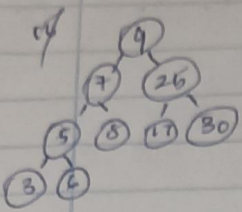


$15 > 9 \rightarrow \text{right}$

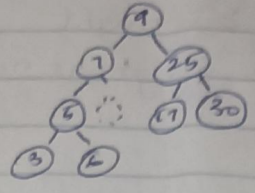
$15 < 17 \rightarrow \text{left}, 15 = 15$

remove 15

17's tree is right heavy simple rotate left

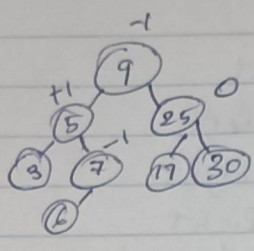


8/ Delete (8):

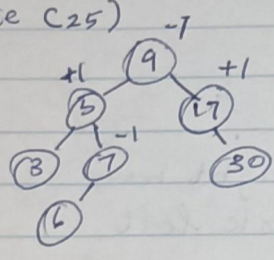


$8 < 9 \rightarrow \text{left}$   
 $8 > 7 \rightarrow \text{right}$   
 found - 8 = 8  
 remove 8

⑦ tree is left heavy  
 Single rotate right



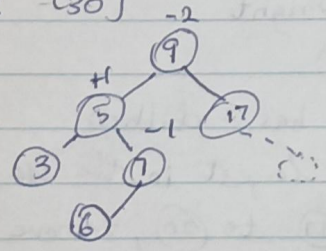
17/ Delete (25)



$25 > 9 \rightarrow \text{right}$   
 $25 = 25$

②⑤ has 2 children  
 the largest on the left  
 copy ①⑦ to 25  
 remove old ①⑦

18/ Delete (30)



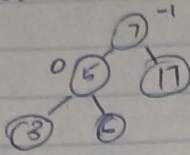
$30 > 9 \rightarrow \text{right}$   
 $30 > 17 \rightarrow \text{right}$   
 $30 = 30$

③⑦ has no children  
 $\rightarrow \text{remove } 30$

The tree is left heavy  
 double rotate right



19/ Delete (7)



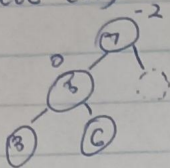
$7 > 5 \rightarrow \text{right}$

$9 = 9$

7 has no left child

Set 17 rotate right child in 5's position

20/ Delete (17)



$17 > 7 \rightarrow \text{right}$

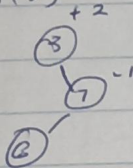
$17 = 17$

17 has no children

$\rightarrow$  remove 17 Tree is left heavy

$\rightarrow$  simple rotate right

21/ Delete (5)

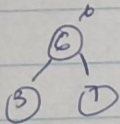


$5 = 6$

5 has 2 children

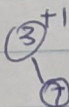
copy the largest of the left (3) to 5

remove old 5



Tree is right heavy rotate left

22/ Delete (6)



(6)

6 has 2 children

copy the largest on the left (3)

~~4~~ remove old 6

Remove 3

Remove 7

$3 > 3$

found 7

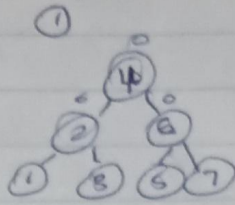
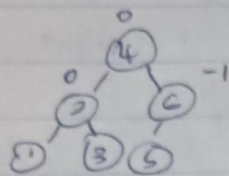
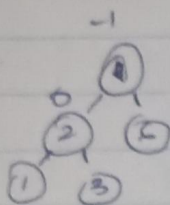
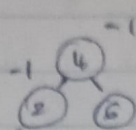
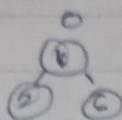
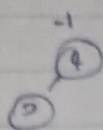
3 don't have left child

remove 3

set the right child as parent

1 2 3 4 5 6 7

order to insert : 4, 2, 6, 1, 3, 5, 7



no rotation.